Does the Collapse of a Supramassive Neutron Star Leave a Debris Disk?

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One possible channel for black hole formation is the collapse of a rigidly rotating massive neutron star as it loses its angular momentum or gains excessive mass through accretion. It was proposed that part of the neutron star may form a debris disk around the black hole. Such short-lived massive disks could be the sources of powerful jets emitting cosmological gamma-ray bursts. Whether the collapse creates a disk depends on the equation of state of the neutron star. We survey a wide range of equations of states allowed by observations and find that disk formation is unfeasible. We conclude that this channel of black hole formation is incapable of producing powerful jets, and discuss implications for models of gamma-ray bursts.

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A canonical mass of neutron stars born in supernova explosions is $M \approx 1.4 M_\odot$. The distribution of M around $1.4 M_\odot$ might, however, extend above $2 M_\odot$, especially if the neutron star is born spinning fast, with a period approaching the minimum ("breakup") $P_{\rm spin} \sim 1$ ms. The additional centrifugal support allows a stable hydrostatic configuration with mass M that would be forbidden for nonrotating stars.

Neutron stars in binary systems have additional chances to gain mass through accretion. The two most massive known pulsars are in binary systems, although not currently accreting. If future accretion is capable of spinning up these stars to $P_{\rm spin} \sim 1$ ms, they could stably increase their mass above the maximum mass for nonrotating neutron stars.

Such rigidly rotating centrifugally supported "supramassive" neutron stars (SMNSs) may also be created in mergers of neutron star binaries. Recent observations of $\approx 2 M_\odot$ pulsars, J1614-2230 and J0348+0432 [1,2], indicate that the equation of state (EOS) of dense nuclear matter is relatively stiff at several times the nuclear saturation density, and therefore some mergers may initially result in a stable object supported by pressure and fast rotation (e.g., Ref. [3]). Numerical simulations show that the object will initially rotate differentially [4–8], but that solid body rotation will be rapidly established following outwards transport of angular momentum via magnetic stresses and gravitational waves. The time scale for differential rotation to be removed could be as short as tens of ms [9], and will almost certainly be much shorter than 10 s (e.g., Ref. [10]). The heat stored in the merger product is also mostly lost to neutrino emission within seconds (e.g., Ref. [11]).

The SMNS is fated to collapse to a black hole. Its lifetime is controlled by the eventual loss of angular momentum (spindown-induced collapse) or excessive mass growth (accretion-induced collapse). The collapse is associated with a huge release of gravitational energy and could produce a bright transient event—a burst of electromagnetic radiation, such as a cosmological gamma-ray burst (GRB).

This GRB trigger is plausible if the equatorial part of the neutron star is not immediately swallowed by the black hole but forms a compact, massive, centrifugally supported disk around it. Jets of hot plasma and radiation are expected to emerge from the debris disk and power the burst (e.g., Ref. [12]).

In the merger scenario, the SMNS eventually collapses due to its gradual spin-down, which removes the rotational support in minutes to hours. The spin-down time scale depends on the magnetic field of the merger product, which is likely amplified to $B \sim 10^{15}$ G during the merger [13–15]. This implies a moderate delay of the collapse-powered burst following the gravitational waves that are emitted during the merger and hopefully detected by Advanced Laser Interferometer Gravitational-Wave Observatory [16,17].

The goal of this Letter is to assess if the key condition for this burst scenario—a massive debris disk after the collapse—can be satisfied. The structure of the SMNS and hence the outcome of its collapse are controlled by the EOS of the dense nuclear matter $P(\rho)$. Available general relativistic simulations of the collapse do not show disk formation [18,19]. These simulations, however, implemented only simplified EOSs. In particular, Ref. [18] used the polytropic $P \propto \rho^{1+1/n}$ with index $n \leq 2$, and found that less than $10^{-3} M_\odot$ remains outside the black hole at the termination of the simulation, comparable to their numerical resolution. They also found that for an extremely soft EOS (with $n = 2.9$ and 3) disks can form; however, such EOSs are incompatible with observations of neutron stars. The remaining open possibility is that a different form of the EOS could lead to disk formation, e.g., soft at high densities (which gives a compact inner core—the seed for a future black hole) and stiff at lower densities (which gives an extended outer core with a high angular momentum).

In this Letter we explore a wide range of EOSs in search for one that could possibly give a debris disk. Instead of carrying out full-fledged and computationally expensive hydrodynamic simulations of SMNS collapse, we employ a simple method. We analyze the equilibrium hydrostatic configuration prior to the collapse and check if it satisfies a necessary condition for formation of a debris disk after the collapse.

Condition for disk formation.—A stringent criterion on disk formation can be derived by assuming that all but an infinitesimal amount of the SMNS's mass and angular momentum are inherited by the newly formed Kerr black hole. Matter at the SMNS equator has the largest specific angular momentum, j_e, and hence is the most likely to form a disk. The angular momentum is conserved during collapse, as long as magnetic and viscous torques are negligible and the spacetime remains axisymmetric. The centrifugal barrier will stop the equatorial matter from plunging into the horizon if j_e exceeds the specific angular momentum of the innermost stable circular orbit (ISCO) in the Kerr metric of the nascent black hole,

$$j_e > j_{\rm ISCO}(a) \Rightarrow \text{disk formation is possible.} \qquad (1)$$

Note that $j_{\rm ISCO}$ depends on the spin parameter $a = Jc/GM^2$ where J is the angular momentum inherited by the black hole from the SMNS. A similar criterion has been employed previously to the collapse of supermassive gas clouds [20].

Maximally rotating maximal mass.—We construct axisymmetric neutron star models using the RNS code [21,22], which calculates relativistic rotating hydrostatic equilibria following the method outlined in Refs. [23,24]. The collapse occurs when the stellar mass exceeds $M_{\rm max}$ at which the star becomes unstable according to the turning-point criterion [25], and no hydrostatic solution is found (Ref. [26] shows that neutral instability is extremely close to the turning point, so the distinction between the two is insignificant).

$M_{\rm max}$ depends on the angular momentum J and the EOS of dense nuclear matter. For a given EOS, we calculate $M_{\rm max}(J)$ and find a and j_e immediately prior to collapse. Disk formation is clearly impossible for a nonrotating star because matter will fall radially into the newly formed Schwarzschild black hole. As J, and hence j_e, are increased, black hole spin a increases and hence $j_{\rm ISCO}$ decreases. Equation (1) could thus in principle be satisfied at some point along the maximal mass sequence.

The maximal mass sequence $M_{\rm max}(J)$ cannot be extended indefinitely as it eventually reaches the mass-shedding limit, beyond which the corotating orbital frequency at the SMNS equator exceeds the SMNS rotation frequency. This point defines the maximally rotating maximum mass (MRMM), $M_{\rm max}(J_{\rm max})$, which is typically 10%–30% higher than $M_{\rm max}(0)$. The collapsing MRMM has the best chance to form a debris disk but this is not guaranteed. Although j_e of the MRMM is just sufficient to orbit the hydrostatic star, the spacetime metric changes after the collapse and the same j_e can fail to sustain Keplerian rotation around the nascent black hole. If Eq. (1) is not met for the MRMM, it will not be met for any slower rotating maximal mass models and we may conclude that disk formation is impossible for this EOS.

The input parameters of the RNS code are the central energy density and the oblateness of the star. For a given oblateness we find the maximal mass model by varying the central energy density. Then we step along the maximum mass sequence toward MRMM by increasing the oblateness parameter. At the end of the sequence we iterate the oblateness until the mass shed limit is found to within a specified accuracy. At each step we check if the disk formation criterion, Eq. (1), is satisfied.

Figure 1 illustrates our procedure for three representative EOSs, labeled EOSA (schematically described as stiff at high densities), EOSB (stiff at low densities), and EOSC (soft at all densities). For EOSA, $j_e < j_{\rm ISCO}$ for any black hole spin a, so disk formation is impossible according to

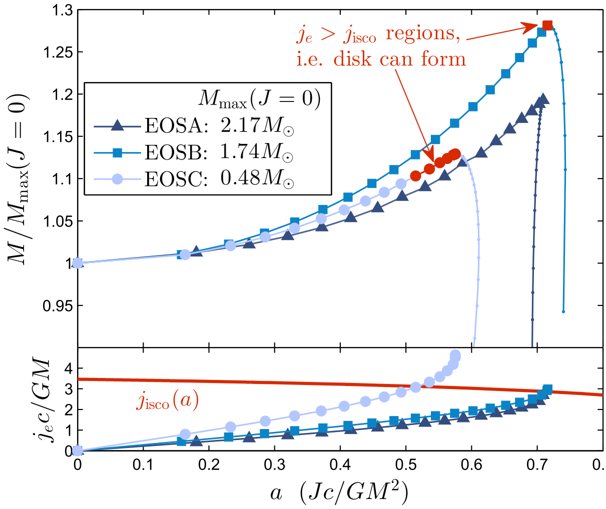

FIG. 1 (color online). Sequence of neutron star mass M and spin parameter a for three sample EOSs, illustrating our method for assessing the possibility of disk formation following SMNS collapse. The top portion of the figure shows the maximal mass sequence (triangles, squares, circles) and mass shed limits (small points) for each EOS. Masses are normalized to the maximal value for a nonrotating star corresponding to each EOS. The bottom portion shows the (dimensionless) specific angular momentum of a test particle at the SMNS equator, $j_e c/GM$, along the maximal sequence curves. A solid red line denotes the minimal angular momentum required to orbit the resulting Kerr black hole with spin parameter a, $j_{\rm ISCO}(a)$. According to the criterion given by Eq. (1), disk formation is ruled out as long as j_e lies below this red curve. These three EOSs are also marked in Fig. 2.

Eq. (1). For EOSC, $j_e > j_{\rm ISCO}$ for $a \gtrsim 0.5$, indicating that a disk could form; however, the maximum nonrotating mass for this unrealistic EOS is only $0.48 M_\odot$. Disk formation is also possible for EOSB, but only for a very narrow range of J near the mass-shedding limit.

Survey of the EOS space.—The possibility of disk formation is controlled by the EOS of dense nuclear matter, which is poorly known. Therefore, below we conduct a survey over a broad range of EOSs. Our goal is to check whether it is possible to simultaneously satisfy the disk formation criterion and current observational constraints on neutron star radii and masses.

We parametrize the EOS at $\rho > \rho_0 = 10^{14.3}$ g cm^{-3} as a broken power law ($\rho_0 \approx$ nuclear saturation density). This choice is motivated by previous works [27] which show that a piecewise polytrope can reliably reproduce a variety of EOS models. The break is fixed at density $\rho_1 = 10^{14.7}$ g cm^{-3}. At densities below ρ_0 we use the SLy EOS [28] with the approximation of Ref. [27], and we fix $P(\rho_0)$ to the SLy value.

With fixed ρ_1 we are left with only two free parameters: $P_1 = P(\rho_1)$ and the power-law index at $\rho > \rho_1$, $\Gamma_2 = d \ln P / d \ln \rho$. Two degrees of freedom in the EOS may be insufficient to predict observables to within $\sim 1\%$ accuracy (e.g., as in Ref. [27]). However, this form of EOS is sufficiently flexible for our purposes, allowing independent variation of the SMNS mass M and radius R. These parameters determine the star's compactness M/R, the key factor for disk formation.

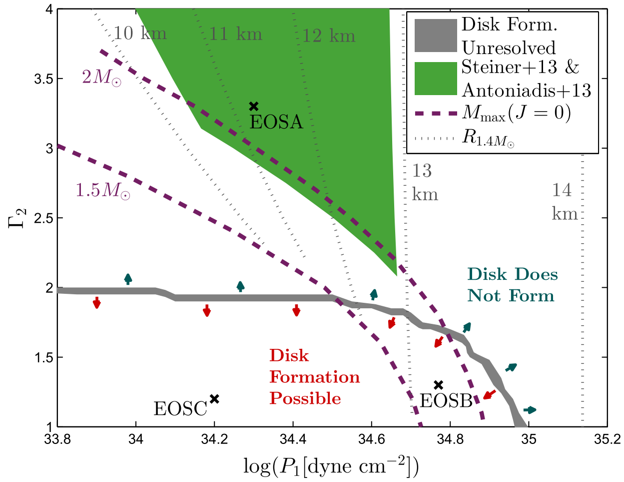

FIG. 2 (color online). Regions of allowed and forbidden disk formation in the EOS parameter space, separated by the grey strip in which formation is unresolved by our numerical procedure. Dashed purple curves show contours of constant maximum mass for nonrotating neutron stars, while dotted black lines indicate constant radius values for a $1.4 M_\odot$ nonrotating star. The green region shows the 2σ allowed parameter space based on observed neutron star masses [2] (bottom boundary) and constraints on neutron star radius [29] (left and right side boundaries).

The results of our numerical survey of the parameter space P_1-Γ_2 are shown in Fig. 2. For a "stiff" EOS above the grey strip even the MRMM configuration fails to meet the criterion of Eq. (1), and thus disk formation is ruled out. The criterion is met by the MRMM below the grey strip (and possibly inside the strip where it is numerically unresolved).

Small P_1 or Γ_2 values are however problematic as they predict low $M_{\rm max}$ while observations demonstrate the existence of neutron stars with $M \approx 2 M_\odot$ [1,2], even at moderate rotation when centrifugal effects may be neglected (the 39 ms spin period of J0348 + 043 is slow enough that it can be treated as essentially nonrotating for the purpose of constraining the maximal neutron star mass).

An additional observationaly accessible parameter is the radius of normal neutron stars with moderate rotation and canonical mass $M \approx 1.4 M_\odot$. For instance using observations of transiently accreting and bursting neutron stars, Ref. [29] reported $R_{1.4 M_\odot} = 10.42$–12.89 km at 2σ. We note that current neutron star radius constraints are subject to uncertainties in both astrophysics and nuclear physics modeling and the radius constraints are not entirely settled yet (cf., e.g., Refs. [30–32]).

For any candidate EOS one should check its prediction for $M_{\rm max}(J \approx 0)$ as well as $R_{1.4 M_\odot}$, which can be tested against observations. Figure 2 shows the contours of constant $M_{\rm max}(J = 0)$ and $R_{1.4 M_\odot}$ on the P_1-Γ_2 plane together with the observational constraints. The condition $M_{\rm max} > 2 M_\odot$ alone excludes almost the entire region where disk formation is possible. A significant gap appears between this region and the allowed region if following Ref. [29] we also require $R_{1.4 M_\odot} < 13$ km.

By tweaking the shape of $P(\rho)$ with additional parameters we have managed to construct EOSs for which the $R_{1.4 M_\odot}$ curves do not exclude the disk formation region. Nevertheless, even below the grey strip, formation of a debris disk requires significant fine-tuning toward the MRMM configuration. Disk formation quickly becomes impossible if M is reduced below MRMM (see Fig. 1, in particular model EOSB). Specifically, for the EOS "smithed" to change the $R_{1.4 M_\odot}$ curves, we find that a fine-tuning in mass of 6×10^{-4} is necessary.

Discussion and astrophysical implications.—Our method employs a simple parametrization for the high density EOS as a piecewise polytrope, and hence may not replicate nuances of realistic EOSs (for instance, Ref. [27] shows that it tends to overestimate the sound speed). This parametrization is, however, sufficient to capture the overall mass distribution of the star, which is most important to our analysis.

In addition to our parameterization, we have applied our method directly to the entire list of "realistic" EOSs given in Ref. [27] (their Table III), and find that none of these support disk formation. Interestingly, the robustness of our main conclusion relies in part on the recent discovery of a

$2M_\odot$ neutron star [1,2] and hence could not have been made with as much confidence prior to 2010, when the largest known mass was $1.74 \pm 0.04 M_\odot$.

Although lower limits on the maximum neutron star mass are well established by dynamical measurements, observational constraints on the neutron star radius are subject to systematic uncertainties (e.g., Ref. [33]). Our conclusion that disk formation is unlikely depends most sensitively on the established maximum mass constraints, and less critically on the neutron star radius. The latter may be varied independently with additional EOS parameters.

Our analysis assumed axisymmetric collapse. This is reasonable since nonaxisymmetric perturbations will likely be damped out via gravitational waves. Furthermore, if the amount of surviving disk mass is determined by deviations from axisymmetry then producing a disk of an interesting mass $\gtrsim 10^{-3} M_\odot$ translates into a radial perturbation of $\gtrsim 2$ km, an unlikely occurrence.

We have additionally assumed that magnetic or viscous torques do not affect the SMNS matter during the collapse. Numerical hydrodynamical simulations consistently show that the SMNS matter collapses on a dynamical time scale with approximate conservation of angular momentum and negligible dissipation effects on fluid streamlines [18]. Magnetic fields could become dynamically important only when they are extremely strong. Such fields could also slightly affect the SMNS structure. Its radius would be increased up to ~16% in the most extreme case of magnetic pressure equal to thermal pressure (e.g., Ref. [34]).

Our results have implications for some GRB models. Electromagnetic emission from SMNSs formed in neutron star binary mergers has been proposed by many authors (e.g., Refs. [35–39]) to explain long-lived x-ray flares ("extended emission") and plateaus observed following short duration GRBs (e.g., Refs. [40–42]), which in some cases have been observed to terminate abruptly in a way suggesting a SMNS that has collapsed to a black hole [43]. These magnetar models have been criticized because it is not clear how to produce the relativistic jet responsible for the initial GRB itself as the result of baryonic pollution from the young neutron star remnant (e.g., Ref. [44]). This has recently led to the suggestion of a "time reversal" scenario [16,17], whereby black hole formation and the GRB is delayed for tens or hundreds of seconds following the merger, but due to light time travel effects is observed before x rays from the SMNS remnant cease. A similar physical situation, which posits the collapse of a SMNS to a black hole following the accretion of matter from a binary companion (accretion-induced collapse; e.g., Refs. [45,46]) is also commonly invoked as an alternative to neutron star merger models for short GRBs.

Both these alluring models (accretion-induced collapse and Time Reversal) require a debris disk after the SMNS collapse in order to power the short GRB. Our results show that this assumption contradicts the stiff nuclear EOS inferred from observations of neutron stars.

This does not necessarily mean that SMNS collapse will have no observational electromagnetic signature. For instance Refs. [47,48] suggest that if the SMNS is initially magnetized, a significant electromagnetic transient could arise regardless of any surrounding accretion disk. However, such a transient is unlikely to last many dynamical times across the black hole horizon and hence may fail to explain the 0.1–1 s duration of observed short GRBs.

Our model assumes solid body rotation and a cold EOS and hence does not rule out a disk if the black hole forms shortly following a binary neutron star merger. Disk formation in fact appears to be a robust outcome of general relativistic simulations of the merger process (e.g., Ref. [9]). Thermal pressure is only sustained for a few seconds after the merger, until neutrino cooling sets in. More importantly, the merger remnant is primarily supported by *differential* rotation, such that the collapse is usually initiated by the outwards redistribution of angular momentum, as is expected to occur on a time scale of tens or hundreds of milliseconds due to magnetic or viscous stresses. Since in this case collapse occurs prior to the establishment of solid body rotation throughout the remnant, disk formation is much more likely than in the case of a delayed collapse.

Finally, our results also render untenable proposed scenarios for long duration GRBs which postulate a long delay (exceeding hours or days) between the core collapse of a massive star and the formation of a black hole with a debris accretion disk [49].

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