Disentangling Mass and Mixing Hierarchies

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(Received 16 July 2015; published 15 October 2015)

We present a fully perturbative mechanism that naturally generates mass hierarchies for the standard model (SM) fermions in a flavor-blind sector. The dynamics generating the mass hierarchies can therefore be independent from the source of flavor violation, and hence this dynamics may operate at a much lower scale. This mechanism works by dynamically enforcing simultaneous diagonalization—alignment—among a set of flavor-breaking spurions, as well as generating highly singular spectra for them. It also has general applications in model building beyond the SM, wherever alignment between exotic and SM sources of flavor violation is desired.

DOI: 10.1103/PhysRevLett.115.161803 PACS numbers: 12.15.Ff

Introduction.—The origin of the large mass and mixing hierarchies among the standard model (SM) fermions—the flavor puzzle—is a significant open problem in particle physics. Attempts to resolve this problem have taken a variety of approaches. The most well known is perhaps the Froggatt-Nielsen mechanism [1], which assigns different charges of a pseudoanomalous symmetry among the SM generations. It thereby can physically distinguish fermion flavors and generate a hierarchy of masses and relative mixings for them. There exist multiple alternate formulations or extensions of this general idea that assign various types of horizontal dynamics to the SM generations (see, e.g., Refs. [2–9], among many others).

Approaches of this type intrinsically link the origin of the mass and mixing hierarchies. This can lead to flavor model-building challenges. For instance, considering the first two quark generations, the Cabibbo-Kobayashi-Maskawa (CKM) quark mixing matrix element $|V_{cd}| \sim 0.2$, while the mass hierarchy is $m_{u,d}/m_c \sim 10^{-3}$. Similarly, in the lepton sector, the charged leptons exhibit a large mass hierarchy, while the Pontecorvo-Maki-Nakagawa-Sakata (PMNS) mixing matrix elements are all $\mathcal{O}(1)$.

In this Letter we present a mechanism that dynamically and naturally generates SM mass hierarchies without charging the SM fermions under any Froggatt-Nielsen style horizontal symmetries. The SM fermions need only be charged under their $U(3)$ flavor symmetries, and they couple universally to the physics that generates their mass hierarchies. This means that the scale at which the mass hierarchies are generated, Λ_H, can be independent from the scale of flavor breaking, which could have interesting phenomenological consequences. For example, Λ_H may be low enough to be detectable at the LHC; if Λ_H is near the electroweak scale, the Jarlskog invariant can be large during the era of sphaleron transitions, opening up a new avenue for significant electroweak baryogenesis.

Strategy.—Specifically, we show how to dynamically generate vacuum expectation values—spurions—for a set of bifundamental flavon fields, $\{\lambda_\alpha\}$, with the "aligned, spectrally disjoint and rank-1" pattern

$$\begin{aligned}\langle\lambda_1\rangle &= U\mathrm{diag}\{r_1, 0, \ldots\}V^\dagger,\\ \langle\lambda_2\rangle &= U\mathrm{diag}\{0, r_2, \ldots\}V^\dagger,\end{aligned} \tag{1}$$

and so on, with U and V being unitary matrices, the same for each flavon. (This is inherently different from a rank-1 projector approach. See, e.g., Ref. [10].) Applied to the SM with three generations, these spurions each break $U(3)\times U(3)$–type flavor symmetry down to a different subgroup, such that collectively the flavor symmetry is broken down to baryon or lepton number. With spurions of the form in Eq. (1), one may then naturally construct mass hierarchies among the SM fermions by assigning extra symmetries or dynamical effects *horizontally among the flavons*. For instance, for a set of three up-type flavons $\{\lambda_{t,c,u}\}$, bifundamental under $U(3)_Q\times U(3)_U$, the up-type SM Yukawa terms could be generated from the irrelevant operator

$$H^\dagger\bar{Q}_L\left\{\frac{s_t}{\Lambda_H}\frac{\lambda_t}{\Lambda_F}+\frac{s_c}{\Lambda_H}\frac{\lambda_c}{\Lambda_F}+\frac{s_u}{\Lambda_H}\frac{\lambda_u}{\Lambda_F}\right\}U_R, \tag{2}$$

where $\Lambda_F \sim \langle\lambda_\alpha\rangle$ is the scale of flavor breaking. The s_α's are $U(3)\times U(3)$ singlet operators—in this sense, they are *flavor blind*—that encode a hierarchy $\langle s_t\rangle \gg \langle s_c\rangle \gg \langle s_u\rangle$, generated at the scale Λ_H. The up-type quark mass hierarchies follow immediately from the pattern (1), independently of the structure of the matrices, U and V, that encode flavor-violating effects. (We focus here on up-type quarks, but the generalization to the down-type quarks and leptons follows analogously.)

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This approach contrasts with Froggatt-Nielsen style horizontal charges, which are assigned directly to the SM fermions. Instead, the SM fermions are coupled universally to the flavor-blind, hierarchy-generating operators s_α. While $\langle s_\alpha\rangle/\langle s_\beta\rangle$ is fixed by the observed SM mass hierarchies, and while the flavor-breaking scale, Λ_F, is bounded below by precision flavor constraints, the hierarchy scale Λ_H is unconstrained by these effects and could be quite low.

The particular scenario we have in mind is to consider three sectors: a SM sector, a "flavor" sector, and a "hierarchy" sector. The dynamics of the flavor sector breaks the $U(3)_Q \times U(3)_U$ flavor symmetries at a scale Λ_F by generating spurions of the form (1) for the three flavons $\lambda_{t,c,u}$. Suppose that these flavons also carry parity symmetries $P_\alpha\colon \lambda_\alpha \to -\lambda_\alpha$. These are broken by P_α-odd, flavor singlet spurions $\langle s_\alpha\rangle$, generated in the hierarchy sector at Λ_H. The SM, flavor, and hierarchy sectors then interact through the three-way portal in Eq. (2). We show a schematic representation of this scenario in Fig. 1. We also show a sample UV completion, which generates the operator (2) at tree level, in which s_α is a set of scalar fields. [Note that other operators like $H^\dagger \bar{Q}_L s_\alpha \lambda_\alpha \lambda_\beta^\dagger \lambda_\beta U_R$ are annihilated by the pattern (1).]

In the remainder of this Letter we will first specify the algebraic conditions that automatically ensure that a set of matrices is aligned, spectrally disjoint, and rank 1. We proceed to show that the most general renormalizable potential for the λ_α has a minimum that enforces these algebraic conditions. With this mechanism in hand, we will present an example of a set of horizontal discrete symmetries on the spurions that can generate the SM mass hierarchies. We will also show how to construct an approximate CKM matrix within this framework.

Algebraic conditions.—Consider two tensors λ_{iI} and ξ_{iI} charged under the bifundamental of a $U(n) \times U(n)$ flavor symmetry group. That is $i, I = 1, \ldots, n$ are indices of the (anti-)fundamental representations. We adopt a matrix notation that encodes contractions of indices of the same type. For instance, $(\lambda\xi^\dagger)_{ij} \equiv \sum_I \lambda_{iI}\xi^*_{jI}$ and $(\lambda\xi^\dagger)^\dagger_{ij} \equiv (\xi\lambda^\dagger)_{ij}$, and similarly for uppercase indices. Hereafter, we shall not distinguish between the two index types, but we will remember instead that λ_α can contract on the right or the left with $\lambda_\beta^\dagger$, but not with λ_β, and that $\lambda^\dagger\lambda$ lives in a different space than $\lambda\lambda^\dagger$, etc.

FIG. 1. (Left panel) Schematic representation of the low energy effective theory. The lines represent the irrelevant interactions in Eq. (2). (Right panel) Sample UV completion, in which Λ_H and Λ_F are identified with the mass scales of ϕ_α and χ_α, respectively.

Alignment: Suppose that

$$[\lambda, \xi]_1 \equiv \lambda^\dagger\xi - \xi^\dagger\lambda = 0, \qquad [\lambda, \xi]_2 \equiv \lambda\xi^\dagger - \xi\lambda^\dagger = 0, \quad (3)$$

so that $\lambda^\dagger\xi$ and $\lambda\xi^\dagger$ are both Hermitian. This is necessary and sufficient to ensure that λ and ξ may be simultaneously biunitarily real diagonalized by the same two unitary matrices. In other words,

$$\lambda = UD_\lambda V^\dagger \quad \text{and} \quad \xi = UD_\xi V^\dagger, \quad (4)$$

with D_λ and D_ξ diagonal and real: we say λ and ξ are "aligned." This result extends to a set of $k \geq 2$ tensors λ_α that all satisfy the condition (3) pairwise. We include a proof in the Supplemental Material [11]. (See also Ref. [12], which proves a more general statement.)

Spectrally disjoint: Suppose we further require

$$\lambda^\dagger\xi = 0 \quad \text{and} \quad \lambda\xi^\dagger = 0. \quad (5)$$

This condition subsumes Eq. (3), so λ and ξ must be aligned. When combined with Eq. (4), this condition further implies $D_\lambda D_\xi = 0$, or in index notation $d_{\lambda i}d_{\xi_i} = 0$, for each i, where $d_{\lambda i}$ and d_{ξ_i} are the real diagonal elements of D_λ and D_ξ. Hence, under the condition (5), λ and ξ are required to be aligned and "spectrally disjoint," in the sense that $d_{\xi_i} = 0$ whenever $d_{\lambda i} \neq 0$, and vice versa. The converse statement follows trivially. Equation (5) extended pairwise to a set of k tensors is, therefore, sufficient and necessary for them all to be aligned and spectrally disjoint.

Rank 1: A maximal set of n linearly independent tensors that satisfy Eq. (5) pairwise are automatically also "rank 1," in the sense that each tensor must have a single nonzero eigenvalue. More generally, any single tensor λ is rank 1 if and only if

$$\mathrm{Tr}(\lambda^\dagger\lambda)^2 - \mathrm{Tr}(\lambda^\dagger\lambda\lambda^\dagger\lambda) = 0. \quad (6)$$

In index notation, this becomes $\sum_{i<j}|d_{\lambda i}|^2|d_{\lambda j}|^2 = 0$, and the only nontrivial solution is $|d_{\lambda i_0}| > 0$ and $d_{\lambda i \neq i_0} = 0$ for some $i_0 \in \{1, \ldots, n\}$. Hence, λ is rank 1, and the converse argument is trivial. A set of tensors, therefore, has the aligned, spectrally disjoint and rank-1 structure (1) if and only if the algebraic conditions (5) and (6) are satisfied pairwise and individually on the set, respectively.

Potential.—We now proceed to construct a potential that ensures that Eqs. (5) and (6) hold dynamically for a set of up-type flavons, $\lambda_\alpha \in \{\lambda_t, \lambda_c, \lambda_u\}$. The parities $P_\alpha\colon \lambda_\alpha \to -\lambda_\alpha$ restrict the form of the renormalizable potential, such that it may only involve terms containing, at most, two different flavons. The full potential for $k \geq 2$ flavons can therefore be constructed from a sum of single-field potentials and two-field potentials.

Single-field potential: The most general flavor-invariant renormalizable potential for a single flavon is

$$\begin{aligned} V_{1f}^{\alpha} &= \mu_1^{\alpha}|\mathrm{Tr}(\lambda_\alpha^\dagger\lambda_\alpha) - r_\alpha^2|^2 \\ &\quad + \mu_2^{\alpha}[\mathrm{Tr}(\lambda_\alpha^\dagger\lambda_\alpha)^2 - \mathrm{Tr}(\lambda_\alpha^\dagger\lambda_\alpha\lambda_\alpha^\dagger\lambda_\alpha)] \\ &= \mu_1^{\alpha}\Big|\sum_i |d_{\alpha i}|^2 - r_\alpha^2\Big|^2 + 2\mu_2^{\alpha}\sum_{i<j}|d_{\alpha i}|^2|d_{\alpha j}|^2. \end{aligned} \tag{7}$$

Both operators are positive semidefinite. For $\mu_{1,2}^{\alpha} > 0$, Eq. (6) is therefore satisfied at the minimum of this potential. The particular solution is $|d_{\alpha i_0}| = r_\alpha$ for some $i_0 \in \{1, \dots, n\}$ and $d_{\alpha i \neq i_0} = 0$. Hence, λ_α is rank 1.

Two-field potential: The most general CP-conserving, flavor- and parity-invariant renormalizable potential for two fields λ_α and λ_β can be written as

$$\begin{aligned} V_{2f}^{\alpha\beta} &= \mu_3^{\alpha\beta}[\mathrm{Tr}(\lambda_\alpha^\dagger\lambda_\alpha) + \mathrm{Tr}(\lambda_\beta^\dagger\lambda_\beta) - r_\alpha^2 - r_\beta^2]^2 \\ &\quad + \sum_{\pm}\mu_{4,\pm}^{\alpha\beta}|\mathrm{Tr}[\lambda_\alpha^\dagger\lambda_\beta \pm \lambda_\beta^\dagger\lambda_\alpha]|^2 \\ &\quad + \sum_{i=1,2}\mu_{5,i}^{\alpha\beta}\mathrm{Tr}[[\lambda_\alpha, \lambda_\beta]_i^\dagger[\lambda_\alpha, \lambda_\beta]_i] \\ &\quad + \mu_6^{\alpha\beta}\{\mathrm{Tr}[(\lambda_\alpha\lambda_\beta^\dagger)^\dagger(\lambda_\alpha\lambda_\beta^\dagger)] + \mathrm{Tr}[(\lambda_\alpha^\dagger\lambda_\beta)^\dagger(\lambda_\alpha^\dagger\lambda_\beta)]\}. \end{aligned} \tag{8}$$

The operators in Eq. (8) are all manifestly positive semidefinite. With all of the coefficients positive, the global minimum of the potential is thus $V_{2f} = 0$. The operator corresponding to μ_3 vanishes if both λ_α and λ_β are in the vacua of their single field potentials (7). The operator corresponding to μ_6 is nonzero *if and only if* $\lambda_\alpha^\dagger\lambda_\beta = \lambda_\alpha\lambda_\beta^\dagger = 0$, and all remaining operators also vanish at this condition. Hence, the global minimum of V_{1f} and V_{2f} together is located at the aligned, spectrally disjoint, and rank-1 conditions (5) and (6).

Extended to a set of k fields, $\{\lambda_\alpha\}$, the pairwise potential

$$V_{\mathrm{pp}} = \sum_\alpha V_{1f}^{\alpha} + \sum_{\alpha<\beta} V_{2f}^{\alpha\beta}, \tag{9}$$

with couplings all positive thus dynamically generates a set of spurions of the desired pattern (1). The flat directions of its minimum are parametrized solely by the unitary matrices U and V, which simultaneously rotate $\{\lambda_\alpha\}$ as in Eq. (1). Although the potential appears to contain a very large number of free parameters, the only significant parameters for the low energy physics are the radial norms r_α, as long as all of the other parameters are positive.

Parity breaking effects: Breaking of the P_α symmetries in the hierarchy sector can radiatively induce parity-odd operators in the potential, e.g., $\mathrm{Tr}(\lambda_\alpha^\dagger\lambda_\beta)$. Since all such operators are invariant under the simultaneous rotation of the set $\{\lambda_\alpha\}$, they do not destabilize the flat directions of the vacuum. In addition, these parity-odd terms are suppressed by $(\langle s_\alpha\rangle\langle s_\beta\rangle/\Lambda_H^2)$ for every pair of parity symmetries $P_{\alpha,\beta}$ that they break, and they can be further two-loop suppressed by the SM portal (2) (see the UV completion in Fig. 1 for an example). For the SM quarks, the largest parity-odd contribution is then $\sim(m_c m_t/v^2)/(16\pi^2)^2 \ll m_u/m_t$, the largest hierarchy in the system. All such terms may then be neglected.

Two-sector potential: Now consider a second set of three down-type flavons $\lambda_{\hat\alpha} \in \{\lambda_b, \lambda_s, \lambda_d\}$ that are charged under flavor $U(3)_Q \times U(3)_D$. We distinguish these from the up-type flavons by their hatted index. The common $U(3)_Q$ group admits up-down cross terms

$$\begin{aligned} V_{\mathrm{mix}}^{\alpha\hat\alpha} &= \nu_1^{\alpha\hat\alpha}\mathrm{Tr}[(\lambda_\alpha^\dagger\lambda_{\hat\alpha})^\dagger(\lambda_\alpha^\dagger\lambda_{\hat\alpha})] \\ &\quad + \nu_2^{\alpha\hat\alpha}[\mathrm{Tr}(\lambda_\alpha^\dagger\lambda_\alpha) + \mathrm{Tr}(\lambda_{\hat\alpha}^\dagger\lambda_{\hat\alpha}) - r_\alpha^2 - r_{\hat\alpha}^2]^2, \end{aligned} \tag{10}$$

into the most general CP-, flavor-, and parity-invariant potential, i.e., $V_{\mathrm{pp}}^{\mathrm{up}} + V_{\mathrm{pp}}^{\mathrm{down}} + V_{\mathrm{mix}}$. Both operators are positive semidefinite, and we assume $\nu_{1,2} > 0$. The ν_2 term vanishes at the vacua of V_{pp}, but the ν_1 terms cannot vanish simultaneously with the μ_6 terms since one cannot nontrivially satisfy $\lambda_\alpha^\dagger\lambda_\beta = \lambda_\alpha^\dagger\lambda_{\hat\alpha} = \lambda_{\hat\alpha}^\dagger\lambda_{\hat\beta} = 0$.

Since V_{mix} respects $\lambda_\alpha\lambda_\beta^\dagger \to -\lambda_\alpha\lambda_\beta^\dagger$ for $\alpha \neq \beta$, it cannot introduce tadpoles that shift the nontrivial stationary points of V_{pp} from the $\{\lambda_\alpha\lambda_\beta^\dagger = 0\}_{\alpha\neq\beta}$ contour. Moreover, the ν_1 term has curvature $\partial^2 V_{\mathrm{mix},\nu_1}/\partial\lambda_\alpha\partial\lambda_\beta^\dagger \propto \delta_{\alpha\beta}$. Provided ν_1 is somewhat small compared to μ_6 and $\mu_{4,+}$, this term cannot destabilize an existing V_{pp} minimum. No symmetries, however, forbid tadpoles that shift the location of the radial vacuum $\mathrm{Tr}(\lambda_\alpha\lambda_\alpha^\dagger)$. Hence, the total potential retains local nontrivial minima somewhere on the $\{\lambda_\alpha\lambda_\beta^\dagger = 0\}_{\alpha\neq\beta}$ contour, i.e., at the aligned, spectrally disjoint configuration. For $\nu_1^{\alpha\hat\alpha} > 0$, the cross terms typically squeeze the location of the radial vacuum to $\mathrm{Tr}(\lambda_\alpha\lambda_\alpha^\dagger) = \bar r_\alpha^2 < r_\alpha^2$. Provided $\nu_1^{\alpha\hat\alpha}$ are not too large compared to the μ_1^α terms, the vacuum remains nontrivial, i.e., $\langle\lambda_\alpha\rangle \neq 0$.

It remains for us to check that the unit rank of $\langle\lambda_\alpha\rangle$ is not spoiled. The desired configuration (1) is explicitly

$$\lambda_\alpha = U_U D_\alpha V_U^\dagger, \qquad \lambda_{\hat\alpha} = U_D D_{\hat\alpha} V_D^\dagger, \tag{11}$$

where we choose D_α to be the rank-1 diagonal matrix whose αth diagonal entry, $d_\alpha \neq 0$. At this configuration, the cross terms become

$$V_{\mathrm{mix}}^{\alpha\hat\alpha} = \nu_1^{\alpha\hat\alpha} d_\alpha^2 d_{\hat\alpha}^2 |\mathcal{V}_{\mathrm{ckm}}^{\alpha\hat\alpha}|^2, \tag{12}$$

where $\mathcal{V}_{\mathrm{ckm}}^{\alpha\hat\alpha}$ is the $\alpha\hat\alpha$th element of $\mathcal{V}_{\mathrm{ckm}} \equiv U_D^\dagger U_U$, the usual unitary up-down mixing matrix. Unitarity forbids all of these terms from simultaneously being zero. This term also lifts the U_U and U_D flat directions of the potential (9). That is, it determines the texture of $\mathcal{V}_{\rm ckm}$.

Perturbing the βth diagonal zero entry of D_α by ϵ corresponds to perturbing the rank-1 (or disjoint) configuration. From Eqs. (10) and (11), this generates only an $\mathcal{O}(\epsilon^2)$ correction $\delta V^{\alpha\hat{\alpha}}_{\rm mix} = \epsilon^2 \nu_1^{\alpha\hat{\alpha}} d_{\hat{\alpha}}^2 |\mathcal{V}^{\beta\hat{\alpha}}_{\rm ckm}|^2$. One may similarly check that in the vacuum of Eq. (12), $\mathcal{O}(\epsilon)$ perturbations of the alignment condition arise in $V_{\rm mix}$ at $\mathcal{O}(\epsilon^2)$, in concordance with the argument above. Thus, provided μ_6, $\mu_{4,+} \gtrsim \nu_1 > 0$, there remains a local minimum at the aligned, spectrally disjoint, rank-1 configuration for each set. In contrast, note that perturbing the nonzero element $d_\alpha \to d_\alpha + \epsilon$ leads to a $\mathcal{O}(\epsilon)$ tadpole, as above, that shifts the radial vacuum away from r_α.

SM hierarchies.—Quark sector: In general, one is free to choose the mechanism at work in the hierarchy sector. We present here an example which makes use of horizontal discrete symmetries to generate the SM quark hierarchies.

We assign an integer charge p_α ($p_{\hat{\alpha}}$) to each λ_α ($\lambda_{\hat{\alpha}}$) under its own individual discrete symmetry $\mathbb{Z}_{2p_\alpha}$ ($\mathbb{Z}_{2p_{\hat{\alpha}}}$), except for λ_t. These discrete symmetries act as the parity symmetries P_α on the flavons, required to secure the potential in Eqs. (9) and (10) [13]. The suppression of parity-odd terms is not spoiled if only a single flavon—λ_t in this case—in each set does not carry a parity. For each symmetry $\mathbb{Z}_{2p_\alpha}$ ($\mathbb{Z}_{2p_{\hat{\alpha}}}$), we further assign a field σ_α ($\sigma_{\hat{\alpha}}$), belonging to the hierarchy sector, with unit discrete charge. This produces the irrelevant operators

$$\begin{aligned} &H^\dagger \bar{Q}_L \left\{ \frac{\lambda_t}{\Lambda_F} + \left[\frac{\sigma_c}{\Lambda_H}\right]^{p_c} \frac{\lambda_c}{\Lambda_F} + \left[\frac{\sigma_u}{\Lambda_H}\right]^{p_u} \frac{\lambda_u}{\Lambda_F} \right\} U_R \\ &\quad + H\bar{Q}_L \left\{ \left[\frac{\sigma_b}{\Lambda_H}\right]^{p_b} \frac{\lambda_b}{\Lambda_F} + \left[\frac{\sigma_s}{\Lambda_H}\right]^{p_s} \frac{\lambda_s}{\Lambda_F} + \left[\frac{\sigma_d}{\Lambda_H}\right]^{p_d} \frac{\lambda_d}{\Lambda_F} \right\} D_R. \end{aligned} \tag{13}$$

There is neither $\mathbb{Z}_{2p_t}$ nor σ_t, so that the top Yukawa coupling is unsuppressed. Applying the pairwise potential (9) and (10) to both up- and down-type flavons, we obtain a complete set of aligned, spectrally disjoint, and rank-1 spurions, as in Eq. (11). In other words, $D_t = \mathrm{diag}\{0, 0, \bar{r}_t\}$ and so on, with $\bar{r}_\alpha \lesssim \Lambda_F$ being the radial location of the vacuum.

If we further assume an approximately uniform scale of breaking for all of the discrete symmetries $\langle \sigma_\alpha \rangle / \Lambda_H \sim \varepsilon$—a natural assumption—then an $\langle s_\alpha \rangle \sim \Lambda_H \varepsilon^{p_\alpha}$ hierarchy is generated by the discrete charges p_α alone. For example, one could make the discrete charge choices

$$\begin{aligned} p_c = 2, \qquad p_u = 5, \qquad p_b = 2, \\ p_s = 3, \qquad p_d = 5. \end{aligned} \tag{14}$$

For $\varepsilon \sim 0.1$, this approximately reproduces the SM quark mass hierarchies.

For anarchic $\nu_1^{\alpha\hat{\alpha}} > 0$, the potential (12) ensures that the flavor mixing matrix settles to a sparse unitary matrix. One can, however, generate an approximation of the observed CKM matrix with some special choices. Suppose there exists a symmetry which requires the couplings $\nu_i^{\alpha\hat{\alpha}}$ and $\mu_i^{\alpha\beta}$ to be universal in α and $\hat{\alpha}$, while $r_t > r_u = r_c$ and $r_b > r_d = r_s$. One may show the potential is minimized for a mixing matrix

$$\mathcal{V}_{\rm ckm} = \begin{pmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{pmatrix} \tag{15}$$

which has a single, arbitrarily large mixing angle for the first two generations. Reproducing the rest of the CKM matrix likely requires the introduction of further small perturbations, perhaps arising from irrelevant operators or interactions coupling to $U(3)_U \times U(3)_D$. We emphasize that this flavor-violating physics is independent from the dynamics of the hierarchy sector.

Lepton sector: A similar process may be applied to the SM leptons, for $H\bar{L}_L E_R$ and $H^\dagger \bar{L}_L N_R$ Yukawa couplings analogous to Eq. (2). For $\nu_i^{\alpha\hat{\alpha}}$, $\mu_i^{\alpha\beta}$, and r_α all universal in α and $\hat{\alpha}$, the PMNS mixing matrix may have arbitrary $\mathcal{O}(1)$ entries. (A different mechanism, however, may be responsible for the extreme overall suppression of the neutrino Yukawa couplings.)

The degeneracy of two of the neutrino masses in the case of an inverted hierarchy [14] can be explained if this sector has only two flavons, λ and ξ, but with $\mu_2^\xi < 0$, $2\mu_{4,+} + \mu_6 \gg |\mu_2^\xi|/2$, and, still, $\mu_2^\lambda > 0$. In this scenario, $\mu_2^\xi < 0$ relaxes the rank-1 condition such that the ξ spurion eigenspectrum prefers instead to be degenerate. When combined with the more energetically favored disjoint condition (5) enforced by μ_6 and $\mu_{4,+}$, one finds

$$\begin{aligned} \langle \lambda \rangle / \Lambda_F &\sim U \mathrm{diag}\{0, 0, 1\} V^\dagger, \\ \langle \xi \rangle / \Lambda_F &\sim U \mathrm{diag}\{1, 1, 0\} V^\dagger. \end{aligned} \tag{16}$$

One may then obtain two degenerate Dirac neutrino masses and one much lighter.

BSM applications.—In the context of beyond the SM (BSM) model building, it is often desirable to obtain new physics (NP) whose flavor-breaking effects are aligned with, but not proportional to, the SM Yukawa couplings. This is more general than minimal flavor violation, and it can be achieved dynamically with the $V_{\rm pp}$ potential.

Assume the existence of a SM spurion $\lambda_{\rm sm} \sim U\mathrm{diag}\{\delta', \delta, 1\} V^\dagger$, with $\delta' \ll \delta \ll 1$, and a second field, $\lambda_{\rm np}$, whose vacuum expectation value represents a flavor-breaking NP spurion. We apply the potential (9) for these two spurions, but treat $\lambda_{\rm sm}$ as a static background field, fixed by some high scale physics. For $\lambda_{\rm sm}$ and $\lambda_{\rm np}$ to be aligned, it suffices that the condition (3) is satisfied: if

$\mu_{5,i} > 0$, it is energetically favorable for $\lambda_{\rm np}$ to settle such that the corresponding operators vanish. This automatically results in alignment with $\lambda_{\rm sm}$.

Since $\lambda_{\rm sm}$ has maximal rank, it is not possible for the two spurions to be spectrally disjoint. In the limit $\mu_1^{\rm np} > |\mu_2^{\rm np}| \gg \mu_{4,+}, \mu_6$, taking all constants to be positive except $\mu_2^{\rm np}$, the vacuum solution is either one of

$$\begin{aligned}\langle\lambda_{\rm np}\rangle/\Lambda_F &\sim U{\rm diag}\{1,0,0\}V^\dagger,\\ \langle\lambda_{\rm np}\rangle/\Lambda_F &\sim U{\rm diag}\{1,1,1\}V^\dagger,\end{aligned} \tag{17}$$

corresponding to whether $\mu_2^{\rm np} > 0$ or $\mu_2^{\rm np} < 0$, respectively. These two spurions are linearly independent and aligned. As such, one can span the whole space of possible aligned NP spurions by taking linear combinations of these two spurions and $\lambda_{\rm sm}$.

Conclusions.—We have shown that the SM fermion mass and mixing angle hierarchies may have autonomous origins, such that they may arise at vastly different physical scales. This result is a consequence of a new mechanism, in which the vacuum of the general flavon field potential dynamically generates an aligned, spectrally disjoint, and rank-1 structure for the $U(3) \times U(3)$ flavor-breaking spurions. Of particular significance, this mechanism permits the physics responsible for the SM quark mass or Yukawa hierarchies to operate close to the electroweak scale, without being in conflict with precision flavor constraints. It may, therefore, be experimentally accessible at LHC. It also may have broad applications in the construction of flavor-safe, natural BSM theories or for electroweak baryogenesis.

We thank Bob Cahn, Yuval Grossman, Lawrence Hall, Zoltan Ligeti, Aneesh Manohar, Yasunori Nomura, Duccio Pappadopulo, and Michele Papucci for helpful conversations. The work of S. K. was supported by the LDRD Program of LBNL under U.S. Department of Energy Contract No. DE-AC02-05CH11231. D. R. is supported by the NSF under Grant No. PHY-1002399.

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