Effect of New Physics in Astrophysical Neutrino Flavor

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Astrophysical neutrinos are powerful tools for investigating the fundamental properties of particle physics through their flavor content. In this Letter, we perform the first general new physics study on ultrahigh energy neutrino flavor content by introducing effective operators. We find that, at the current limits on these operators, new physics terms cause maximal effects on the flavor content; however, the flavor content on the Earth is confined to a region related to the assumed initial flavor content. Furthermore, we conclude that a precise measure of the flavor content on the Earth will provide orders of magnitude improvement on new physics bounds. Finally, we discuss the current best fits of flavor content of the IceCube data and their interplay with new physics scenarios.

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Introduction.—The existence of extraterrestrial ultrahigh energy neutrinos has been confirmed by the IceCube neutrino observatory [1,2], opening the possibility for studying ultrahigh energy particle production mechanisms as well as new neutrino physics [3,4]. The nature of these neutrinos from 35 TeV to 2 PeV is still a puzzle; at the moment, there are many astrophysical and beyond the standard model candidate sources [5–10] that may produce these neutrinos. Currently, there is no statistically significant spatial correlation between observed neutrinos and potential sources [11,12].

Even though the sources of these neutrinos remain unknown, it is still possible to find evidence of new physics. The vacuum neutrino propagation Hamiltonian is linearly proportional to the neutrino square mass differences and inversely proportional to the neutrino energy. For astrophysical ultrahigh energy neutrinos, this operator is suppressed, allowing us to look for extremely tiny new physics effects which, otherwise, cannot be seen. In the standard oscillation scenario, for any given initial flavor composition, the final composition, after the propagation, lies in a small region on the flavor triangle close to $(\phi_e:\phi_\mu:\phi_\tau) = (1:1:1)$. The flavor content of the astrophysical neutrinos has been studied in [1,2,13–17]. These analyses find that flavor content is statistically consistent with the standard oscillations expectations. Future data will clarify the IceCube astrophysical event flavor composition.

In this Letter, we perform the first general new physics study of the astrophysical neutrino flavor content by introducing effective operators in the standard three neutrino scenario with unitary evolution. This is, by far, the most general approach for studying new physics in astrophysical neutrino flavors, and this approach covers many exotic particle physics models. There are few cases we do not consider in this Letter. First, the model with which we work is limited within lepton number conservation, and we do not consider models such as the neutrino-antineutrino oscillations [18,19]. Second, we do not consider the neutrino decay model which violates unitary evolution and was discussed elsewhere [20]. Similarly, we also do not consider models with sterile neutrino states [21]. The sterile neutrino mixing matrix elements are known to be minuscule compared with the active neutrino mixing elements [21–25], and the contribution to the transition probability due to the sterile neutrinos is suppressed by the sterile-active matrix element to the fourth power.

Ultrahigh energy astrophysical neutrino oscillations.—Neutrinos change lepton flavors as they propagate macroscopic distances. This is due to the fact that the neutrino propagation eigenstates are not the eigenstates of the charged current weak interaction. In the presence of a dense medium, the decoherent scattering interactions are important [26], but in this Letter, we assume vacuum propagation.

In general, the relation between the propagation eigenstates $|\nu_i\rangle$ and the flavor eigenstates $|\nu_\alpha\rangle$ is given by a unitary transformation $V(E)$

$$|\nu_\alpha\rangle = \sum_i V_{\alpha i}(E)|\nu_i\rangle. \tag{1}$$

For astrophysical neutrinos, the propagation distance is much longer than the oscillation length, and in this limit, the oscillation from flavor state $|\nu_\alpha\rangle$ to a flavor state $|\nu_\beta\rangle$ can be averaged

$$\bar{P}_{\nu_\alpha\to\nu_\beta}(E) = \sum_i |V_{\alpha i}(E)|^2 |V_{\beta i}(E)|^2, \tag{2}$$

where the probability depends only on the mixing matrix elements $|V_{\alpha i}(E)|$, which is, in general, energy dependent.

Using the probability given in this equation and the flux at production ϕ_α^p, we can calculate the neutrino flux on the Earth, $\phi_\beta^\oplus(E)$, for a flavor β. It is more convenient to define the energy averaged flavor composition as

$$\bar{\phi}_\beta^\oplus = \frac{1}{|\Delta E|}\int_{\Delta E}\sum_\alpha \bar{P}_{\nu_\alpha\to\nu_\beta}(E)\phi_\alpha^p(E)dE, \qquad (3)$$

where we assume E^{-2} power law for the production flux and $\Delta E = [10\ \mathrm{TeV}, 10\ \mathrm{PeV}]$. Note, however, that our main results are largely insensitive to the spectral index. We also assume that all flavors have the same energy dependence at the source.

In astrophysics, charged pion decay from proton-proton collisions is one of the preferred neutrino production channels. In this scenario, the initial flavor composition is $(\phi_e:\phi_\mu:\phi_\tau) = (1:2:0)$. Other scenarios, such as rapid muon energy loss, produce $(0:1:0)$; neutron decay dominated sources produce $(1:0:0)$ and are of interest, while compositions such as $(0:0:1)$ are not expected in the standard particle astrophysics scenarios. In order to plot the flavor content in a flavor triangle, we introduce the flavor fraction, $\alpha_\beta^\oplus = \bar{\phi}_\beta^\oplus/\sum_\gamma \bar{\phi}_\gamma^\oplus$.

For the vacuum propagation, the Hamiltonian of the standard neutrino oscillation only depends on the neutrino mass term

$$H = \frac{1}{2E}U\begin{pmatrix}0 & 0 & 0\\ 0 & \Delta m_{21}^2 & 0\\ 0 & 0 & \Delta m_{31}^2\end{pmatrix}U^\dagger = \frac{1}{2E}UM^2U^\dagger, \qquad (4)$$

where E is neutrino energy, $\Delta m_{ij}^2 = m_i^2 - m_j^2$, and U is the standard lepton mixing matrix U. Throughout this Letter, we assume the normal mass ordering. We also performed the same study by assuming the inverted mass ordering; however, differences are minor, and mass ordering does not affect any of our main conclusions.

The current measurements of the standard neutrino oscillation experiments allow us to determine the astrophysical neutrino flavor content at detection given an assumption of the neutrino production. In Fig. 1, we show allowed regions of the flavor content on the Earth, where we use the standard mixing angles and their errors from the global fits [27] in order to produce probability density distributions for the flavor content. Since the CP phase is not strongly constrained by either terrestrial [28,29] or astrophysical [30] neutrinos, we assume a flat distribution from 0 to 2π. Note that, for simplicity, we use the larger of the asymmetric errors and implement them as Gaussian. In the left plot, we assume four different production flavor composition hypotheses. We observe that all the allowed regions of astrophysical neutrino flavor content on the Earth are close to $(1:1:1)$, except when the initial flavor content is $(1:0:0)$ [31]. In the right plot, we show the allowed region of the flavor content of the astrophysical neutrinos with all possible astrophysical production

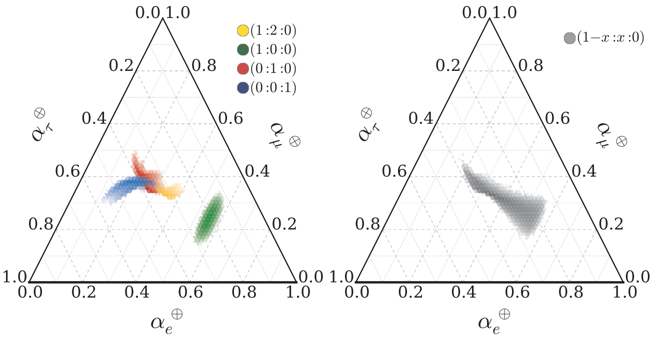

FIG. 1 (color). Allowed regions of the flavor content on the Earth using the priors on the mixing angles and errors given from the current neutrino oscillation measurements. In the left plot, the different colors correspond to different assumptions on flavor content at the production. The color intensity is proportional to the probability density. In the right plot, we further sample the initial flavor content as $(x:1-x:0)$.

mechanisms; i.e., the production flavor composition is sampled with $(x:1-x:0)$ uniformly on x [32]. Therefore, this rather narrow band covers all possible scenarios of the standard neutrino oscillations with the standard astrophysical neutrino production mechanisms.

New physics in effective Hamiltonians.—An effective way of introducing new physics in neutrino oscillations is by introducing new operators. The full Hamiltonian that incorporates the new physics operators, in the flavor basis, can be expressed as

$$H = \frac{1}{2E}UM^2U^\dagger + \sum_n\left(\frac{E}{\Lambda_n}\right)^n\tilde{U}_nO_n\tilde{U}_n^\dagger = V^\dagger(E)\Delta V(E),$$

where $O_n = \mathrm{diag}(O_{n,1}, O_{n,2}, O_{n,3})$ and $\Delta = \mathrm{diag}(\Delta_1, \Delta_2, \Delta_3)$. O_n and Λ_n set the scale of the new physics and $\tilde{U}_n$ is the mixing matrix that describes the new physics flavor structure. In the effective theory approach, lower order operators are more relevant; thus, in this Letter, we will only study the first terms in the expansion, namely $n = 0$ and $n = 1$.

Although, in this Letter, we will study $n = 0$ and $n = 1$, results can be extended to higher orders. These new operators can be interpreted in different new physics contexts. Some examples for $n = 0$ new physics are couplings between neutrinos and spacetime torsion [33], CPT-odd Lorentz violation [34–37], and nonstandard neutrino interactions [38–41]. As for $n = 1$ new physics operators, CPT-even Lorentz violation [42,43] and equivalence principle violation [44,45] are possible examples.

There are some constraints from neutrino oscillation experiments to these effective operators in the context of Lorentz and CPT violation [46]. The most stringent limits on certain parameters are obtained from Super-Kamiokande and IceCube atmospheric neutrino analyses [47,48]. In this context, the CPT-odd and CPT-even Lorentz violation coefficients are constrained to be $\sim 10^{-23}$ GeV and $\sim 10^{-27}$ depending on the flavor structure $\tilde{U}_n$. These constraints can be used to set the scales of $n = 0$ and $n = 1$ operators introduced in this Letter. For example, we set $O_0 = 1 \times 10^{-23}$ GeV as a current limit of the $n = 0$ operator, and $O_1 = 1 \times 10^{-23}$ GeV with $\Lambda_1 = 1$ TeV as a current limit of $n = 1$ operators, where $(O_1/\Lambda_1) = 10^{-27}$. Throughout this Letter, we have assumed the scale of O_1 is of the order of O_0 without loss of generality.

Anarchic sampling prediction and IceCube results.—In order to predict the flavor composition on the Earth in the presence of new physics, the values of the mixing matrices $\tilde{U}_n$ should be specified. In order to show a prediction with new physics operators, we have to account for all the free parameters in the mixing matrix; we use a random sampling scheme to construct the mixing matrix. A well established schema is the anarchic sampling [49–52], which samples a flat distribution given by the Haar measure

$$d\tilde{U}_n = d\tilde{s}_{12}^2 \wedge d\tilde{c}_{13}^4 \wedge d\tilde{s}_{23}^2 \wedge d\tilde{\delta}, \tag{5}$$

where, $\tilde{s}_{ij}$, $\tilde{c}_{ij}$, and $\tilde{\delta}$ correspond to sines, cosines, and phase for the new physics n-operator mixing angles. We omit the Majorana phases since they do not affect neutrino oscillations.

In Fig. 2, we show the allowed regions using anarchic sampling in the case where $H = (E/\Lambda_n)^n \tilde{U}_n O_n \tilde{U}_n^\dagger$. In this case, we neglect the mass term, we are considering that the Hamiltonian has only one operator, i.e., $V = \tilde{U}_n$, and the result does not depend on n. Each plot in this figure corresponds to a different production flavor composition. We show the pion decay production $(1:2:0)$ [yellow], beta decay $(1:0:0)$ [green], muon cooling $(0:1:0)$ [red], and for completeness, we show the exotic ν_τ dominant model $(0:0:1)$ [blue]. The color density in these plots is a representation of the probability given by the anarchic sampling.

In Fig. 3, we show the case where we have a mass term and the $n = 0$ operators. In the top plot, we set $O_0 = 1.0 \times 10^{-23}$ GeV, corresponding to the order of the current best limit on this operator. In the bottom left plot, we set $O_0 = 3.6 \times 10^{-26}$ GeV, and in the bottom right plot we set $O_0 = 6.3 \times 10^{-28}$ GeV. These values are chosen because they have the same magnitude as the mass term with neutrino energy of $E_\nu = 35$ TeV and $E_\nu = 2$ PeV, respectively. In this plot, the colors represent different assumptions in the production flavor content, and the color intensity is the probability given by the anarchic sampling as in Fig. 2.

In Fig. 4, we show the case for the $n = 1$ operators. The color notations and their intensities have the equivalent meaning as Fig. 3. As before, in the top plot, we set the new physics operator to the current best limit $(O_1/\Lambda_1) \sim 10^{-27}$. This is achieved by choosing $O_1 = O_0 = 1.0 \times 10^{-23}$ GeV and $\Lambda_1 = 1$ TeV. In the bottom left plot, $O_1 = 3.6 \times 10^{-26}$ GeV and $\Lambda_1 = 35$ TeV are used, and in the bottom right plot, the parameters are $O_1 = 6.3 \times 10^{-28}$ GeV and $\Lambda_1 = 2$ PeV. These choices make new physics to be the same magnitude as the mass term with a neutrino energy of $E_\nu = 35$ TeV and $E_\nu = 2$ PeV,

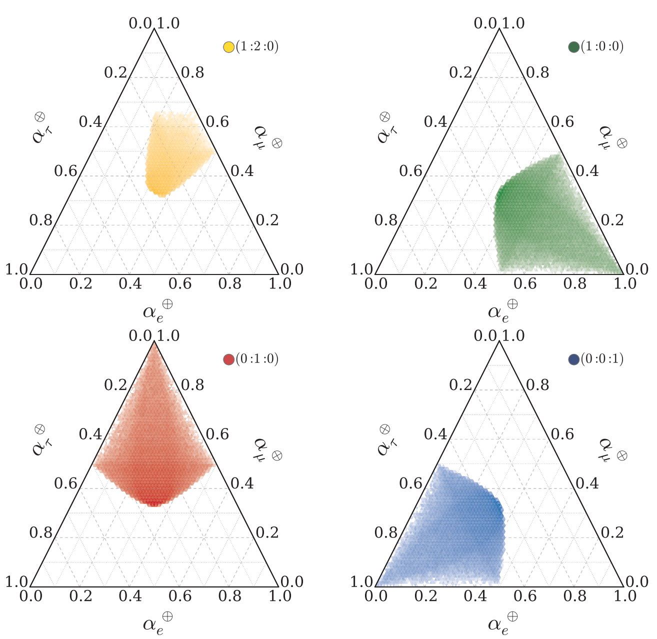

FIG. 2 (color). Allowed region using anarchic sampling on the mixing angles for the new physics operator when the mass term in the Hamiltonian is neglected. The different plots correspond to different assumptions on flavor content at production. The color intensity is proportional to the probability predicted by anarchic sampling.

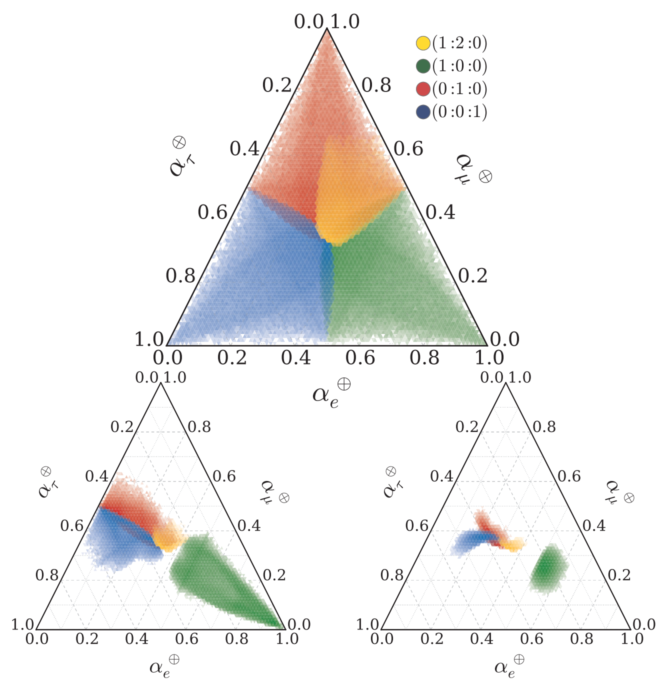

FIG. 3 (color). Allowed region using anarchic sampling on the mixing angles for the new physics $n = 0$ operators. The top plot corresponds to the current limits on $n = 0$ operator; the bottom left plot corresponds to $O_0 = 3.6 \times 10^{-26}$ GeV, while the bottom right plot corresponds to $O_0 = 6.3 \times 10^{-28}$ GeV.

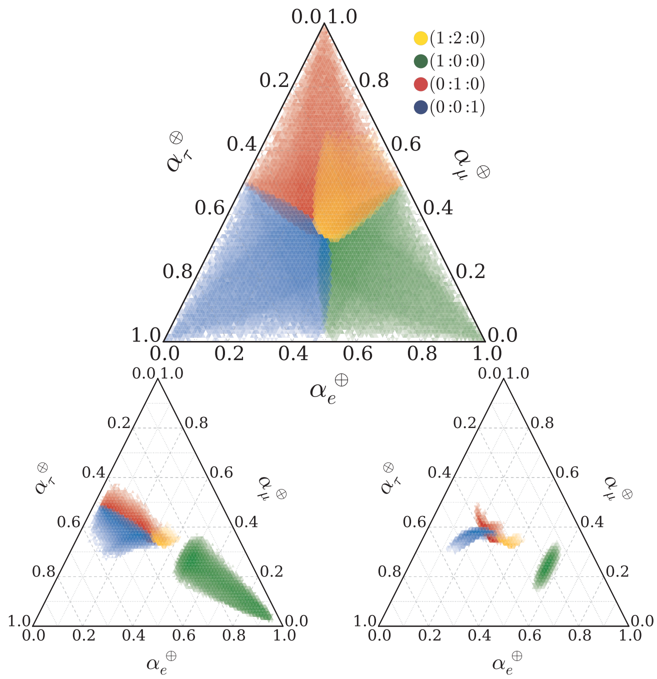

FIG. 4 (color). Allowed region using anarchic sampling on the mixing angles for the new physics $n = 1$ operators. The top plot corresponds to the current limits on $n = 1$ operator; the bottom left plot corresponds to $O_1 = 3.6 \times 10^{-26}$ GeV and $\Lambda_1 = 35$ TeV ($(O_1/\Lambda_1) = 1.0 \times 10^{-30}$), while the bottom right plot corresponds to $O_1 = 6.3 \times 10^{-28}$ GeV and $\Lambda_1 = 2$ PeV ($(O_1/\Lambda_1) = 3.2 \times 10^{-34}$).

respectively. In other words, these choices explore new physics down to $(O_1/\Lambda_1) = 1.0 \times 10^{-30}$ and $(O_1/\Lambda_1) = 3.2 \times 10^{-34}$. This can be compared, for example, to the aforementioned best limits of Lorentz and *CPT* violation in the neutrino sector [47,48]. The potential limits from astrophysical neutrino flavor content can be well beyond what terrestrial neutrino experiments can achieve.

From Figs. 3 and 4, we observe that the allowed regions in the flavor triangle change in a similar way to a function of the energy scale. This is true for any higher operators, because what matters is the scale where they dominate over standard neutrino mass terms, and these two operators are sufficient to predict behaviors of any higher order operators. Comparing Figs. 3 and 4 with respect to Fig. 2, where the allowed regions are more symmetric, there is a preferred region along the vacuum oscillation triangle shown in Fig. 1. It is interesting to notice that, due to the unitary evolution and the fact that the oscillations are averaged, for a given production flavor content, only a subset of the flavor triangle is accessible. The pion decay production mechanism $(1:2:0)$ is one of the most natural astrophysical scenarios for high energy neutrino production. From Figs. 3 and 4, the allowed region for this case is the smallest, which means that, if future measurements exclude this region, the pion production dominant mechanism is excluded regardless of the presence of new oscillation physics.

In the analyses of the IceCube high energy neutrino events, different results have been shown. The first result [53], using the IceCube result [2], showed a best fit at $(1:0:0)$ disfavoring $(1:1:1)$ at 92% C.L. Later, the same authors did an improved analysis [14] including energy dependence and extra systematic errors, finding that the best fit may move considerably depending on the features of the energy spectrum such as including an energy cutoff or not. The IceCube Collaboration later published an analysis of the flavor ratio above 30 TeV [15] finding a best fit at $(0:\frac{1}{5}:\frac{4}{5})$, as well as excluding $(1:0:0)$ and $(0:1:0)$ at more than 90% C.L. This IceCube result shows a best fit dominated by the ν_τ component, which can be explained by the correlation between the energy cutoff and the Glashow resonance, as noted by [14]. In obtaining this best fit, the IceCube Collaboration has assumed an equal amount of neutrinos and antineutrinos, which best corresponds to a proton-proton source. On the other hand, if the neutrino source is proton-photon dominated, then the neutrino-antineutrino ratio weakens, making the previous conclusion. It is interesting to notice that, if this IceCube best fit does not change considerably after adding more data, the production mechanism has to include a ν_τ component. This is because the new physics in the propagation can not give the best fit value for any plausible astrophysical scenarios. This implies not only new physics in the neutrino oscillations, but also new physics in the production mechanism.

Conclusions.—We performed the first new physics study on the astrophysical neutrino flavor content using effective operators in the standard three neutrino scenario. These operators can represent a variety of models such as Lorentz and *CPT* violation, violation of equivalent principle, cosmic torsion, nonstandard interactions, etc., making this Letter the most general study of new physics in astrophysical neutrino flavor content to date.

We found that large effects in the flavor content on the Earth are still allowed with given terrestrial bounds on new physics in the neutrino sector. This implies that an accurate measurement of the flavor content will provide stronger bounds on new physics. Furthermore, there are regions on the flavor triangle that cannot be accessed even in the presence of new physics in the neutrino oscillations for any of the plausible astrophysical mechanisms. Interestingly, the most natural astrophysical mechanism, pion decay, has the smallest region in the flavor triangle even when new physics is considered. The real astrophysical neutrino production mechanism in nature may be the combination of channels, but our results hold for such a case. Therefore, a higher statistics measurement by future neutrino telescopes, such as IceCube-Gen2 [54], could reveal not only the initial neutrino flavor ratios, but also the presence of new physics in neutrinos.

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