

## Importance of the Bulk Viscosity of QCD in Ultrarelativistic Heavy-Ion Collisions

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(Received 21 February 2015; published 22 September 2015)

We investigate the consequences of a nonzero bulk viscosity coefficient on the transverse momentum spectra, azimuthal momentum anisotropy, and multiplicity of charged hadrons produced in heavy ion collisions at LHC energies. The agreement between a realistic 3D hybrid simulation and the experimentally measured data considerably improves with the addition of a bulk viscosity coefficient for strongly interacting matter. This paves the way for an eventual quantitative determination of several QCD transport coefficients from the experimental heavy ion and hadron-nucleus collision programs.

DOI: 10.1103/PhysRevLett.115.132301

PACS numbers: 12.38.Mh, 25.75.-q, 47.75.+f

*Introduction.*—Ultrarelativistic heavy ion collisions realized at the Relativistic Heavy Ion Collider (RHIC) and the Large Hadron Collider (LHC) are able to reach energies high enough to create and study the quark-gluon plasma (QGP), a novel state of nuclear matter where the quark and gluon degrees of freedom become manifest [1]. This hot and dense nuclear medium was found to behave like an almost perfect fluid, with one of the smallest shear viscosity to entropy density ratios  $\eta/s$  in nature [2–6]. Currently, one of the main theoretical challenges in nuclear physics is to model such collisions and extract from experiment the transport properties of this new phase of nuclear matter.

Fluid-dynamical models have been highly successful in describing the production of hadrons in heavy ion collisions. The azimuthal momentum anisotropy of hadrons in particular has been shown to be a sensitive probe of the shear viscosity of the QGP [7], and has been used repeatedly to estimate this transport coefficient [8]. One limitation of this extraction procedure is the uncertainty associated with the early time dynamics of the collisions: the azimuthal momentum distribution of hadrons is known to be closely related to the initial shape of the medium [9–11]. Therefore, an accurate determination of the shear viscosity and other transport properties of QCD matter demands further improvements in the modeling of the earliest stages of the collisions.

Recent improvements in modeling the early time dynamics of heavy ion collisions [12,13] using the IP-Sat model of the nucleon wave function [14] followed by a classical Yang-Mills evolution of the gluon fields [15] led to unprecedented success [16] in describing charged hadron azimuthal momentum distributions as characterized by their harmonic coefficients  $v_n$  ( $n = 2, 3, 4, \dots$ ). Further support for this initial state model, known as the IP-Glasma model, was provided by the remarkable agreement with data of its prediction for the event-by-event distributions of  $v_n$  measured by the ATLAS Collaboration [17].

The same approach, however, had less success in describing the full transverse momentum distribution of hadrons, showing clear tension with data in the low transverse momentum region [16]. In this Letter we show that the inclusion of bulk viscosity, which was neglected in previous studies, can relieve this tension. In principle, the bulk viscosity of QCD matter should not be zero for the temperatures achieved at the RHIC and the LHC and it may become large enough to affect the evolution of the medium. In fact, simulations of heavy ion collisions that include the effect of bulk viscosity have already been performed [18–28] and have demonstrated that bulk viscosity can have a non-negligible effect on heavy ion observables.

In addition to the early time description of the collision provided by the IP-Glasma model, our calculations include a phase of hadronic rescatterings after the hydrodynamic evolution, implemented using the ultrarelativistic quantum molecular dynamics simulation UrQMD [29,30]. Moreover, the intermediate fluid-dynamical evolution is resolved using a more complete version [31] of the Israel-Stewart theory [32] that takes into account all the second order terms that couple the shear-stress tensor and bulk viscous pressure. This hybrid approach with IP-Glasma initial conditions is found to be capable of describing simultaneously the multiplicity and average transverse momentum of pions, kaons, and protons when a finite bulk viscosity, of the order  $\zeta/s \approx 0.3$ , is included near the QCD phase transition region. Such a finite bulk viscosity also considerably reduces, by almost 50%, the value of the shear viscosity needed to describe the harmonic flow coefficients.

*Model.*—The initial state of the medium is determined using the IP-Glasma model with the thermalization time set to  $\tau_0 = 0.4$  fm. The system then evolves following the conservation law  $\partial_\mu T^{\mu\nu} = 0$ , where the stress-energy tensor  $T^{\mu\nu}$  is composed of the ideal part  $T_{\text{id}}^{\mu\nu} = \varepsilon u^\mu u^\nu - \Delta^{\mu\nu} P_0(\varepsilon)$  and the dissipative part  $T_{\text{diss}}^{\mu\nu} = \pi^{\mu\nu} - \Delta^{\mu\nu} \Pi$ . Here,  $\varepsilon$  is the local energy density,  $P_0(\varepsilon)$  is the thermodynamic pressure

according to the equation of state,  $u^\mu$  is the fluid velocity,  $\Pi$  is the bulk viscous pressure, and  $\pi^{\mu\nu}$  is the shear-stress tensor. We further introduced the projection operator  $\Delta^{\mu\nu} = g^{\mu\nu} - u^\mu u^\nu$  onto the 3D space orthogonal to the fluid velocity. The equation of state  $P_0(\varepsilon)$  is the chemical equilibrium one taken from Ref. [33]. It is a parametrization of a lattice QCD calculation matched onto a hadron resonance gas calculation at lower temperatures. We assume that the baryon number density and diffusion are zero at all space-time points and our metric convention is  $g^{\mu\nu} = \text{diag}(1, -1 - 1 - 1)$ .

The time-evolution equations satisfied by  $\Pi$  and  $\pi^{\mu\nu}$  are relaxation-type equations derived from kinetic theory [34,35]. These are solved numerically within the MUSIC hydrodynamics simulation [36–38]. Explicitly, we solve

$$\tau_\Pi \dot{\Pi} + \Pi = -\zeta\theta - \delta_{\Pi\Pi}\Pi\theta + \lambda_{\Pi\pi}\pi^{\mu\nu}\sigma_{\mu\nu}, \quad (1)$$

$$\begin{aligned} \tau_\pi \dot{\pi}^{(\mu\nu)} + \pi^{\mu\nu} &= 2\eta\sigma^{\mu\nu} - \delta_{\pi\pi}\pi^{\mu\nu}\theta + \varphi_\gamma\pi_\alpha^{(\mu}\pi^{\nu)\alpha} \\ &\quad - \tau_{\pi\pi}\pi_\alpha^{(\mu}\sigma^{\nu)\alpha} + \lambda_{\pi\Pi}\Pi\sigma^{\mu\nu}. \end{aligned} \quad (2)$$

The above equations include all the nonlinear terms that couple the bulk viscous pressure and shear-stress tensor and have recently been shown to be in good agreement with solutions of the 0 + 1 Anderson-Witting equation in the massive limit [39] and of the 1 + 1 Anderson-Witting equation in the massless limit [40,41]. For the sake of simplicity, the transport coefficients  $\tau_\Pi$ ,  $\delta_{\Pi\Pi}$ ,  $\lambda_{\Pi\pi}$ ,  $\tau_\pi$ ,  $\eta$ ,  $\delta_{\pi\pi}$ ,  $\varphi_\gamma$ ,  $\tau_{\pi\pi}$ , and  $\lambda_{\pi\Pi}$  are fixed using formulas derived from the Boltzmann equation near the conformal limit [35]. The shear viscosity coefficient is assumed to be proportional to the entropy density, i.e.,  $\eta \propto s$ . The bulk viscosity coefficient employed is the same one introduced in Ref. [24], which corresponds to a parametrization of calculations from Ref. [42] for the QGP phase and from Ref. [43] for the hadronic phase. These two calculations are matched at  $T_c = 180$  MeV and the value of  $\zeta/s$  at this temperature is  $\zeta/s(T_c) \approx 0.3$ . This parametrization is plotted in Fig. 1 as the blue solid curve. The results shown in this Letter have a small sensitivity to the value of  $\zeta/s$  near the matching temperature, which can be doubled without leading to major modifications in our results.

At an isothermal hypersurface specified by the switching temperature  $T_{\text{switch}}$ , the simulation switches from a fluid-dynamical description to a transport description [44], modeled using the UrQMD simulation. The momentum distribution of hadrons at each hypersurface element is calculated via the usual Cooper-Frye formalism [45]. The multiplicity of each hadron species is sampled assuming that every fluid element is a grand-canonical ensemble while the momentum of each hadron is obtained by sampling the momentum distribution using the rejection method. We note that the Cooper-Frye formalism requires as an input the nonequilibrium momentum distribution of

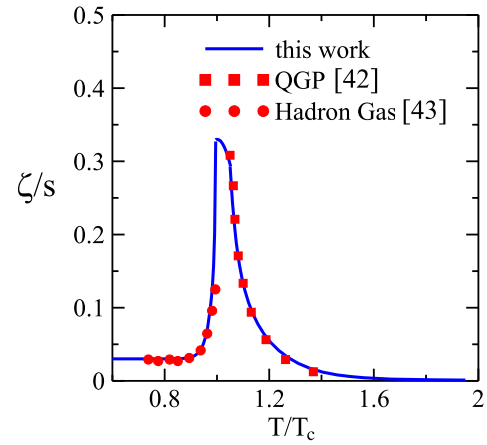


FIG. 1 (color online). The bulk viscosity over entropy density parametrization used in our simulations as a function of  $T/T_c$ .

each hadron inside the fluid elements. For the correction related to bulk viscous pressure, we employ the distribution derived from the Boltzmann equation using the relaxation time approximation, as described in Ref. [46]. For the shear-stress tensor nonequilibrium correction, we employ the usual ansatz obtained from the 14-moment approximation [23,47]. The details of how the UrQMD simulation is matched to MUSIC will be presented in an upcoming paper.

We emphasize that the nonequilibrium corrections to the momentum distribution of hadrons at the moment of switching are still not completely understood from a theoretical point of view and represent a source of uncertainty in simulations of heavy ion collisions. However, the differential observables carry most of these uncertainties since they are more sensitive to the details of how the momentum of hadrons is distributed when converting from a hydrodynamic to a transport description. For this reason, we fix all the free parameters of our model using  $p_T$ -integrated observables.

*Results and discussion.*—In our simulations, the value of the shear viscosity coefficient is adjusted to provide a good agreement with the integrated flow harmonic coefficients  $v_n$  up to  $n = 4$ . For the simulations that include both bulk and shear viscosity, this procedure led to the value  $\eta/s = 0.095$ . For the simulations that include only the shear viscosity, our baseline calculation is carried out with  $\eta/s = 0.16$ . The larger value of  $\eta/s$  compensates for the reduction of momentum anisotropy due to the effect of the bulk viscosity.

The pion and kaon multiplicity  $v_n$  and, to a lesser extent, their average  $p_T$  are only mildly sensitive to the choice of switching temperature between the hydrodynamic and UrQMD phases. Proton observables, on the other hand, do depend significantly on the choice of  $T_{\text{switch}}$ . The switching temperature used in the following calculations is fixed such that a good description of the proton multiplicity and average  $p_T$  is achieved for the simulation

with both shear and bulk viscosities. This value is  $T_{\text{switch}} = 145$  MeV.

In Figs. 2(a)–2(c), we show the multiplicity, the average transverse momentum of pions, kaons, and protons, and the integrated flow harmonics of charged hadrons, as a function of centrality class. The  $v_n\{2\}$  coefficients are calculated following the cumulant method [48] using the same  $p_T$  cuts employed by the ALICE Collaboration [49]. The multiplicity and average transverse momentum are calculated without a lower  $p_T$  cut [50]. All resonances and hadrons included in the UrQMD simulation are considered in our analyses and we neglect all weak decays. The solid curves correspond to the simulations that include bulk and shear viscosities, while the dashed lines correspond to the calculations with only the shear viscosity. The band around the dashed curves shows how the results are modified when  $T_{\text{switch}}$  is varied from 135 to 165 MeV. For  $\langle p_T \rangle$  and  $v_n$ , the upper section of the band corresponds to the calculations with the lowest  $T_{\text{switch}}$  while for multiplicity it corresponds to ones with the highest  $T_{\text{switch}}$ . The points correspond to measurements by the ALICE Collaboration [49,50].

As expected, the simulations without bulk viscosity are still able to well describe the centrality dependence of the flow harmonic coefficients  $v_{2,3,4}\{2\}$ . However, these calculations overestimate  $\langle p_T \rangle$  of pions, kaons, and protons by almost 30%. This happens because the IP-Glasma model gives rise to an initial state with large gradients of pressure and the subsequent fluid-dynamic expansion accordingly produces a significant radial flow. Therefore, in order to describe the data the transverse momentum of the produced particles must be considerably reduced.

Including hadronic rescatterings by itself does not reduce  $\langle p_T \rangle$ , modifying mostly the intermediate  $p_T$  region of the pion spectra [51,52]. Moreover, we can see from the bands around the dashed lines in Fig. 2 that increasing the switching temperature will not help in fixing the

multiplicity of pions, and is not enough to reproduce the correct values of  $\langle p_T \rangle$ . Finally, reducing  $\eta/s$  alone not only is unable to sufficiently suppress  $\langle p_T \rangle$ , but also ends up destroying the good description of the flow harmonic coefficients.

Including bulk viscosity leads to a suppression of  $\langle p_T \rangle$  and can improve our description of the data. This is because the bulk viscous pressure acts as a resistance to the expansion or compression of the fluid. In heavy ion collisions, the expansion rate is mostly large and positive, leading to a bulk viscous pressure that reduces the effective pressure of the system and, consequently, slows down the acceleration of the fluid.

As shown in Fig. 2, the calculations with bulk viscous pressure are indeed able to provide a good description of all the  $p_T$ -integrated observables. The calculated average transverse momenta of pions, kaons, and protons are within the error bars of the ALICE measurements [50] for most of the centrality classes considered. The pion and proton multiplicities measured by ALICE [50] are well described by the model, which however systematically overpredicts the multiplicity of kaons by  $\sim 10\%$ . Finally, we see that the inclusion of bulk viscosity does not spoil the description of the flow harmonic coefficients  $v_{2,3,4}\{2\}$  as a function of centrality. We note that the bulk viscosity reduces  $v_{2,3,4}\{2\}$  by more than 10% but this effect is compensated for by decreasing the shear viscosity over entropy density ratio from  $\eta/s = 0.16$  to  $\eta/s = 0.095$ , leading to a very similar quality of description. Within this study, the inclusion of bulk viscosity can therefore reduce the value of shear viscosity extracted from data by almost 50%.

We now study  $p_T$ -differential observables within the best fit configuration including shear and bulk viscosities. Figure 3 shows the  $p_T$  spectra of pions, kaons, and protons and  $v_{2,3,4}\{2\}(p_T)$  of charged hadrons for the 0%–5% and 30%–40% centrality classes. The solid lines correspond to

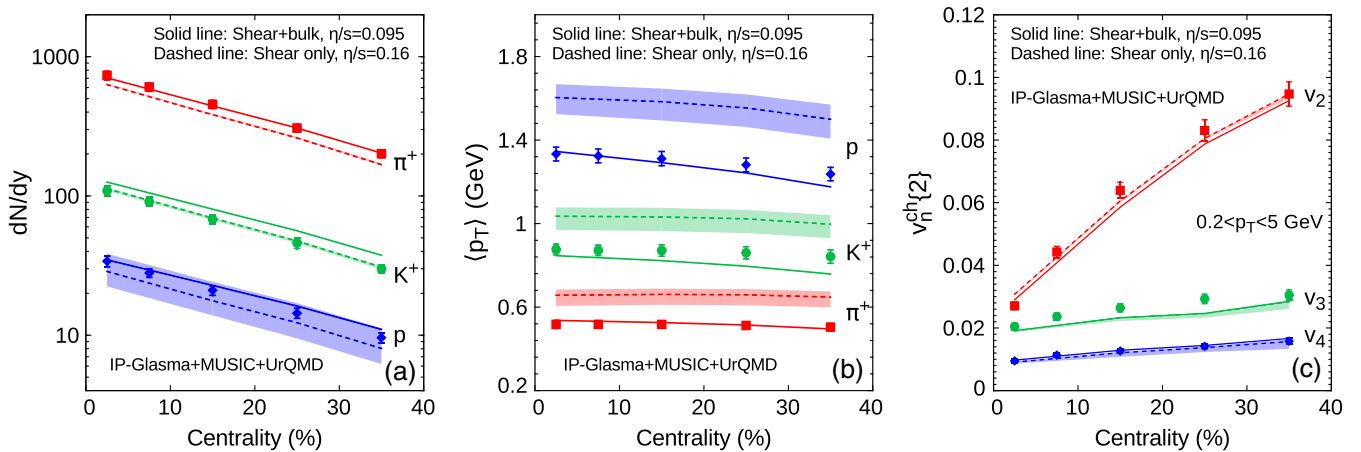


FIG. 2 (color online). Multiplicity (a), average transverse momentum (b), and flow harmonic coefficients (c) as a function of centrality. The bands around the dashed lines show the effect of  $T_{\text{switch}}$  on the observables. The points correspond to measurements by the ALICE Collaboration [49,50], with bars denoting the experimental uncertainty.

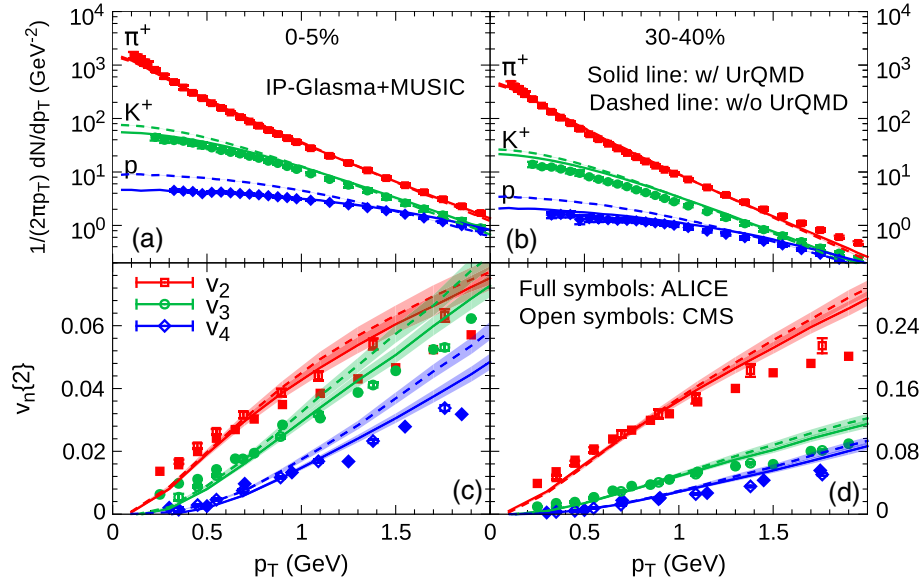


FIG. 3 (color online). Transverse momentum spectra (upper panels) of pions, kaons, and protons and harmonic flow coefficients (lower panels) as a function of the transverse momentum. Two centrality classes are considered: 0%–5% (left panels) and 30%–40% (right panels). The bands denote the statistical uncertainty of the calculation. The full and open symbols correspond to measurements by the ALICE Collaboration [49] and CMS Collaboration [53,54], respectively, with bars denoting the experimental uncertainty.

the calculations with bulk and shear viscosity discussed above while the dashed lines correspond to the same calculations without the effect of hadronic rescatterings. Note that the  $p_T$  spectra display reasonable agreement with the data, which is in line with the good description of the multiplicity and  $\langle p_T \rangle$  of pions, kaons, and protons, displayed in Fig. 2. The values of  $v_n\{2\}(p_T)$  of charged hadrons show more deviations from the data, in particular the ALICE data [49], which are systematically smaller than the CMS measurements [53,54] at high  $p_T$ .

We find that hadronic rescatterings have an almost negligible effect on pion spectra (the difference between the red dashed curve and the solid one is barely visible in the plot) and only affects the differential flow harmonics of charged hadrons at high  $p_T$ . On the other hand, they play an important role in the description of kaon and, especially, proton spectra. Without taking into account all of these effects, it would not be possible to globally describe these observables. These findings are consistent with those from Refs. [51,52].

*Conclusions.*—In this Letter, we discussed the effect of bulk viscous pressure on the multiplicity, average transverse momentum, and azimuthal momentum anisotropy of charged hadrons using a state-of-the-art simulation of ultrarelativistic heavy ion collisions. It includes IP-Glasma initial conditions, which in combination with hydrodynamics are known to provide a good description of the flow harmonic coefficients, and the UrQMD simulation, which models the hadronic rescatterings that follow the fluid-dynamical evolution of the system. This fluid-dynamical evolution also considers

several nonlinear terms absent from several previous studies. The inclusion of bulk viscosity was found to have a large effect on the average transverse momentum of charged hadrons and on the elliptic flow coefficient. In fact, when using the IP-Glasma initial conditions, the bulk viscosity is essential to describe the  $p_T$  spectra of charged hadrons, and leads to a considerably better description of the data. A similar quality of description involving only shear viscosity could not be obtained in our current model.

This work constitutes the first phenomenological investigation that shows that the bulk viscosity of QCD matter is not small, at least around the phase transition region. Our calculations suggest that  $\zeta/s \approx 0.3$  or larger around  $T_c$ . We also showed that the inclusion of bulk viscosity considerably modifies the optimum value of shear viscosity required to describe the data, reducing it by almost 50%. Therefore, the effects of bulk viscosity cannot be neglected when extracting any transport coefficient from the data. The effects of bulk viscosity on ultracentral collisions, already briefly investigated in Ref. [46], and on several other experimental observables will be the subject of future studies.

The authors thank M. Luzum, J. B. Rose, P. Huovinen, J. Noronha, R. Snellings, and A. Kalweit for useful discussions. This work was supported in part by the Natural Sciences and Engineering Research Council of Canada, and by U.S. DOE Contract No. DE-SC0012704. G. S. D acknowledges support through a Banting Fellowship of the Natural Sciences and Engineering Research Council of Canada. B. S. acknowledges support from a DOE Office of Science Early Career Award. Computations were made on

the Guillimin supercomputer at McGill University, managed by Calcul Québec and Compute Canada. The operation of this supercomputer is funded by the Canada Foundation for Innovation (CFI), Ministère de l'Économie, de l'Innovation et des Exportations du Québec (MEIE), RMGA. and the Fonds de recherche du Québec–Nature et technologies (FRQ-NT).

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