Nonchiral Enhancement of Scalar Glueball Decay in the Witten-Sakai-Sugimoto Model

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We estimate the consequences of finite masses of pseudoscalar mesons on the decay rates of scalar glueballs in the Witten-Sakai-Sugimoto model, a top-down holographic model of low-energy QCD, by extrapolating from the calculable vertex of glueball fields and the η' meson that follows from the Witten-Veneziano mechanism for giving mass to the latter. Evaluating the effect on the recently calculated decay rates of glueballs in the Witten-Sakai-Sugimoto model, we find a strong enhancement of the decay of scalar glueballs into kaons and η mesons, in fairly close agreement with experimental data on the glueball candidate $f_0(1710)$.

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The fundamental theory of the strong interactions, quantum chromodynamics (QCD), which has quarks confined in color-neutral bound states, also admits bound states whose valence constituents are all gluons, i.e., the non-Abelian gauge bosons of QCD. This prediction of additional mesons, called gluonia or glueballs, dates back to the early 1970s [1] and has been substantiated by lattice QCD [2], which estimates the mass of the lowest glueball state to be around 1600–1800 MeV. Experimentally, however, their status remains unclear and controversial [3]. The lowest scalar glueball state has quantum numbers of the vacuum and can be expected to mix with scalar mesons made from quarks and antiquarks. To disentangle the contributions, information on decay processes is needed. Theoretical expectations vary greatly; the lowest glueball state may be even so broad that it forms a mere background for the isoscalar meson spectrum [4].

QCD in the limit of a large number of colors N_c [5,6], which in many cases turns out to be a remarkably successful approximation to real QCD with $N_c = 3$, predicts a parametric suppression of decay rates of glueballs compared to light quarkonia by a factor $1/N_c$ as well as a suppression of mixing [7]. If glueballs are indeed narrow and not strongly mixed, one should be able to identify one of the isoscalar-scalar mesons below 2 GeV as a predominantly glueball state. In phenomenological studies the experimentally well-established [11] mesons $f_0(1500)$ and $f_0(1710)$ have been alternatingly identified as possible glueball candidates [12–14]. Both are comparatively narrow states, but their decay patterns are rather different: $f_0(1500)$ decays primarily into four pions and secondly into two pions, with decays into kaons and η mesons suppressed, whereas $f_0(1710)$ instead decays predominantly into two kaons, with a ratio [11] $\Gamma(2\pi)/\Gamma(K\bar{K}) = 0.41^{+0.11}_{-0.17}$, much lower than 3:4 expected from a flavor-blind glueball. In the case of $f_0(1500)$ the strong deviation from flavor blindness is usually attributed to mixing, while for $f_0(1710)$ it has been suggested that glueballs couple more strongly to the more massive pseudoscalar mesons, a mechanism termed "chiral suppression" [15,16]; this could make it possible that $f_0(1710)$ is a nearly unmixed glueball, as most recently argued in [13,14] (see also Ref. [17]).

Since lattice QCD results on glueballs in interaction with quarks are still sparse, in particular concerning decay patterns, it is of interest to employ (top-down) gauge-gravity duality, a string-theoretic approach to studying strongly coupled large-N_c gauge theories, to obtain new insights from first principles [18]. In fact, the spectrum of glueballs has been one of the first applications of a nonsupersymmetric holographic model derived by Witten [20] from type-IIA superstring theory [21–23]. The Witten model has subsequently been extended by Sakai and Sugimoto to include chiral quarks through $D8$-$\overline{D8}$ probe branes [24,25]. With only one free coupling constant at a (Kaluza-Klein) mass scale $M_{\mathrm{KK}} \sim 1$ GeV, this provides a remarkably successful model for low-energy QCD, with quantitative predictions for vector and axial-vector meson spectra and decay rates that agree with real QCD to within 10%–30% [26].

In Ref. [27], the Witten-Sakai-Sugimoto (WSS) model was used for the first time to evaluate the decay rate of the lowest glueball state into pions and to compare this with experimental data for the $f_0(1500)$ meson, although the mass of the lowest holographic glueball mode is obtained as 855 MeV. In Ref. [28] we revisited this calculation with the result that the decay width of the lowest mode is much higher than the one obtained in [27]. Since the lowest mode corresponds to an "exotic polarization" [22] of the gravitational fields of the Witten model, we have proposed to discard the latter and to instead consider the next-lowest, predominantly dilatonic mode with mass 1487 MeV to correspond with the glueball in QCD. Despite the closeness of its mass to that of the $f_0(1500)$ meson, we have found that the decay pattern into two and four pseudoscalar mesons is not reproduced: the decay rate of $f_0(1500)$ into two pions is underestimated by about a factor of 2, while the prediction for the dominant decay mode of $f_0(1500)$, i.e., decay into four pions, is an order of magnitude too small. Extrapolating the mass of the holographic glueball to that of the glueball candidate $f_0(1710)$ (which is within 16% of the mass of the dilatonic mode) [29], we have instead found close agreement with the decay rate into two pions. Since the Witten-Sakai-Sugimoto model is chiral, pions, kaons, and η mesons are predicted simply in flavor-symmetric ratios 3∶4∶1; this thus fails to explain the much stronger decays into two kaons and two η mesons.

In this Letter we study the possible effect of finite quark masses on the decay rate of glueballs in the Witten-Sakai-Sugimoto model in order to see whether a sufficiently strong enhancement of the decay into kaons and η mesons could result. In Refs. [30,31], it has been shown that nonlocal mass terms implementing Gell-Mann–Oakes–Renner relations can be induced by either world sheet instantons or a deformation by a bifundamental field related to the open string tachyon that arises between (parallel) D- and anti-D-branes. Since no complete calculation along these lines exists, the additional coupling of glueballs and pseudoscalar mesons induced by the nonlocal mass term is not known. However, the additional coupling of glueballs to η' mesons due to the part of its mass term that arises from the anomalous breaking of $U(1)_A$ flavor symmetry can be calculated exactly. We propose to use that as a simple and plausible model for how the totality of nonlocal mass terms for the pseudoscalar meson depends on the scalar glueball fields, and, thereby, to extrapolate the results for the glueball decay pattern obtained in [28] to finite pseudoscalar masses.

In the chiral Witten-Sakai-Sugimoto model [24,25], the coupling of pseudoscalar Goldstone bosons as well as vector and axial-vector mesons to scalar and tensor glueball fields are determined by the dependence of the Dirac-Born-Infeld part of the $D8$-brane action on metric and dilaton fields,

$$S_{D8}^{\mathrm{DBI}} = -T_{D8}\mathrm{Tr}\int d^9x e^{-\Phi}\sqrt{-\det(\tilde{g}_{MN} + 2\pi\alpha' F_{MN})}, \quad (1)$$

where Φ and $\tilde{g}_{MN}$ are the dilaton and the nine-dimensional induced metric in the ten-dimensional background given by

$$\begin{aligned} ds^2 &= \left(\frac{u}{R}\right)^{3/2}[\eta_{\mu\nu}dx^\mu dx^\nu + f(u)dx_4^2] \\ &\quad + \left(\frac{R}{u}\right)^{3/2}\left[\frac{du^2}{f(u)} + u^2 d\Omega_4^2\right], \\ f(u) &= 1 - \frac{u_{\mathrm{KK}}^3}{u^3}, \quad (2) \\ e^\Phi &= g_s\left(\frac{u}{R}\right)^{3/4}, \quad (3) \end{aligned}$$

with circle-compactified $x^4 \simeq x^4 + 2\pi/M_{\mathrm{KK}}$ and $M_{\mathrm{KK}} = \frac{3}{2}u_{\mathrm{KK}}^{1/2}R^{-3/2}$. The stacks of N_f $D8$- and anti-$D8$-branes are assumed to be localized at antipodal points, giving rise to trivial embeddings $x^4 = \text{const.}$, which extend from the holographic boundary at $u = \infty$ to the minimal point u_{KK}, where branes and antibranes connect. This breaks the chiral group $U(N_f)_L \times U(N_f)_R$ down to its diagonal group, leading to a nonet (for $N_f = 3$) of pseudoscalar Goldstone bosons described by

$$U(x) = P\exp\left(i\int_{-\infty}^{\infty} dz A_z(z,x)\right) = e^{i\Pi^a\lambda^a/f_\pi}, \quad (4)$$

where $z/u_{\mathrm{KK}} = \sqrt{(u/u_{\mathrm{KK}})^3 - 1}$ parametrizes the radial extent of the joined $D8$- and anti-$D8$-branes. Matching the pion decay constant f_π to 92.4 MeV and the mass of the lowest vector meson mode $A_\mu(z,x) = \rho_\mu(x)\psi_{(1)}(z)$ to the ρ meson mass $m_\rho \approx 776$ MeV fixes $M_{\mathrm{KK}} = 949$ MeV and $\lambda = g_{\mathrm{YM}}^2 N_c = 16.63$ [24,25]; matching instead $m_\rho/\sqrt{\sigma}$, with σ, the string tension of the model, to large-N_c lattice results [26,28] gives a somewhat lower value of the 't Hooft coupling, $\lambda = 12.55$, which we use with the higher value to give a band of variation for the holographic predictions.

In Refs. [24,32] (see also [33]) it was shown that the $U(1)_A$ anomaly requires us to combine the Ramond-Ramond 2-form field strength F_2 with the isoscalar η_0 that is localized on the $D8$-branes in a gauge-invariant combination $\tilde{F}_2$ with bulk action

$$S_{C_1} = -\frac{1}{4\pi(2\pi l_s)^6}\int d^{10}x\sqrt{-g}|\tilde{F}_2|^2 \quad (5)$$

with

$$\tilde{F}_2 = \frac{6\pi u_{\mathrm{KK}}^3 M_{\mathrm{KK}}^{-1}}{u^4}\left(\theta + \frac{\sqrt{2N_f}}{f_\pi}\eta_0\right)du\wedge dx^4, \quad (6)$$

where θ is the QCD theta angle and

$$\eta_0(x) = \frac{f_\pi}{\sqrt{2N_f}}\int dz \mathrm{Tr}A_z(z,x). \quad (7)$$

This gives rise to a Witten-Veneziano [34,35] mass term for η_0 that is local with respect to the effective $(3+1)$-dimensional boundary theory but nonlocal in the bulk, with mass squared

$$m_0^2 = \frac{N_f}{27\pi^2 N_c}\lambda^2 M_{\mathrm{KK}}^2. \quad (8)$$

For $N_f = N_c = 3$, $M_{\mathrm{KK}} = 949$ MeV, and with λ varied from 16.63 to 12.55 one finds $m_0 = 967$–730 MeV.

The other pseudoscalar mesons described by (4) are massless in the Witten-Sakai-Sugimoto model. Current quark masses can, in principle, be introduced through a deformation by a bulk field $\mathcal{T}$ in the bifundamental representation of the chiral symmetry group [30] that is related to tachyon condensation or, alternatively, through world sheet instantons [31]. Both introduce nonlocal mass terms for the pseudoscalar mesons, which one may qualitatively write as

$$\int d^4x \int_{u_{\mathrm{KK}}}^{\infty} du h(u) \mathrm{Tr}(\mathcal{T}(u) P e^{-i\int dz A_z(z,x)} + \mathrm{H.c.}), \quad (9)$$

where $h(u)$ includes metric fields. Choosing appropriate boundary conditions for $\mathcal{T}$, the quark mass matrix arises through

$$\int_{u_{\mathrm{KK}}}^{\infty} du h(u) \mathcal{T}(u) \propto \mathcal{M} = \mathrm{diag}(m_u, m_d, m_s), \quad (10)$$

thereby realizing a Gell-Mann–Oakes–Renner relation.

Integration over u leads to mass terms for all Goldstone bosons, including one for the flavor singlet η_0 in addition to the Witten-Veneziano mass term. The flavor octet η_8 and η_0 can be diagonalized to mass eigenstates η and η'. With $\mathcal{M} = \mathrm{diag}(m, m, m_s)$ and $m = (m_u + m_d)/2$, fixing $m_\pi = 140$ MeV and $m_K = 497$ MeV, this diagonalization yields, for $\lambda = 16.63$–12.55,

$$m_\eta = 518\text{–}476 \text{ MeV}, \qquad m_{\eta'} = 1077\text{–}894 \text{ MeV}, \quad (11)$$

$$\theta_P = -14.4° \text{ to } -24.2°, \quad (12)$$

with θ_P the octet-singlet mixing angle; this shows that the above holographic result for m_0 is in the correct ballpark [36].

In order to determine how mass terms affect the coupling of glueballs and mesons worked out in Ref. [28], we would need to know the dependence on the dilaton and metric fields of $h(u)$ as well as the profile of the bifundamental field $\mathcal{T}(u)$. Absent this information, we turn to the fully known nonlocal mass term produced by (5) and (6) for η_0. Inserting the mode expansion of glueball fields G_D and G_E defined in Ref. [28], we find

$$S_{\eta_0}^{\mathrm{eff}} = -\frac{1}{2} \int d^4x m_0^2 \eta_0^2 (1 - 3d_0 G_D + 5\check{c}_0 G_E) + \cdots, \quad (13)$$

with

$$d_0 = 3u_{\mathrm{KK}}^3 \int_{u_{\mathrm{KK}}}^{\infty} H_D(u) u^{-4} du \approx \frac{17.915}{\lambda^{1/2} N_c M_{\mathrm{KK}}}, \quad (14)$$

$$\check{c}_0 = \frac{3}{4} u_{\mathrm{KK}}^3 \int_{u_{\mathrm{KK}}}^{\infty} H_E(u) u^{-4} du \approx \frac{15.829}{\lambda^{1/2} N_c M_{\mathrm{KK}}}, \quad (15)$$

where the latter is given for completeness only, since we are going to discard the "exotic" mode G_E given the results in [28]. Here $H_{D,E}(u)$ are the radial profile functions of the glueball modes, normalized in order to give a canonical kinetic term for $G_{D,E}(x)$.

Given its similarity to how a nonlocal mass term is generated through world sheet instantons, this result seems to be a reasonable first guess as to how nonlocal mass terms couple in general. Concerning the bifundamental field $\mathcal{T}$ associated with tachyon condensation, a plausible guess would be that the metric dependence derives from the integration measure of $D8$-branes, $d^9 x e^{-\Phi}\sqrt{-\tilde{g}}$. For the predominantly dilatonic glueball field [43], this turns out to have exactly the same dependence on terms linear in $G_D(x)H_D(u)$, as follows from (5) and (6), namely a factor $[1 - 3G_D(x)H_D(u)]$. In order to calculate the coupling constant analogous to d_0 in (14), one would need to know the holographic profile of $\mathcal{T}$, of which we only know that it will be concentrated around $u = u_{\mathrm{KK}}$. As a simplistic guess one could try a function that mimics the profile of the term $A_z \partial_\mu^2 A_z$ in the D-brane action when A_z equals the zero mode describing the Goldstone bosons. This would simply determine the analog of d_0 to be equal to the coupling d_1 that appears in the chiral $G_D\pi\pi$ term [28],

$$\mathcal{L}_{G_D\pi\pi}^{\mathrm{chiral}} = \frac{1}{2} d_1 \mathrm{Tr}(\partial_\mu \pi \partial_\nu \pi) \left(\eta^{\mu\nu} - \frac{\partial^\mu \partial^\nu}{M^2}\right) G_D, \quad (16)$$

where $d_1 \approx 17.226 \lambda^{-1/2} N_c^{-1} M_{\mathrm{KK}}^{-1}$. This differs from d_0 by a mere 4%.

We shall, therefore, continue with the working hypothesis that the overall coupling of the glueball field to the mass term for the pseudoscalar mesons is universal. This essentially assumes that the mixing of singlet and octet mesons is invariant under a holographic renormalization group evolution, which in particular implies the absence of a direct coupling $G\eta\eta'$.

With this assumption, i.e., adding

$$-\frac{1}{2} \sum_i m_i^2 P_i^2 (1 - 3d_0 G_D), \quad (17)$$

with P_i the mass eigenstates of the pseudoscalar mesons, to (16), we obtain the following modification factor for the decay rate to two pseudoscalar mesons of mass m_P:

$$\left(1 - 4\frac{m_P^2}{M^2}\right)^{1/2} \left(1 + \alpha \frac{m_P^2}{M^2}\right)^2, \quad (18)$$

with

$$\alpha = 4(3d_0/d_1 - 1) \approx 8.480 \quad \text{for } G_D. \quad (19)$$

An analogous calculation for the exotic scalar glueball using the results of [28] for the chiral contributions gives

$$\alpha = 4(5\check{c}_0 - \check{c}_1)/(c_1 + 2\check{c}_1) \approx 2.630 \quad \text{for } G_E. \quad (20)$$

[Note that, to leading order, the dependence on λ and M_{KK} drops out in (19) and (20).] In (18) the first factor represents a simple kinematical suppression, which is overcome by the coupling of the glueball field to the mass term of the pseudoscalar fields. A similar result, but with $\alpha = 1$, was obtained in Ref. [44] for a simple effective field theory where the scalar glueball field is identified with the dilaton of QCD (a scalar field with a potential matched to the QCD trace anomaly). With $\alpha = 1$ the nonchiral enhancement is canceled by the kinematical suppression to order m_P^2/M^2, thus restoring approximate flavor symmetry, while for larger values of m_P the net effect is a (slight) reduction of the decay rate.

TABLE I. Flavor-asymmetric deviation of branching ratios of glueball candidate $f_0(1710)$ compared to the nonchiral enhancement in the Witten-Sakai-Sugimoto model resulting from (18) and (19) with $m_P = m_{\pi,K,\eta} = \{140, 497, 548\}$ MeV.

$f_0(1710)$	Expt. (PDG)	WSS massive
$4/3 \times \Gamma(\pi\pi)/\Gamma(K\bar{K})$	$0.55^{+0.15}_{-0.23}$	0.463
$4 \times \Gamma(\eta\eta)/\Gamma(K\bar{K})$	1.92 ± 0.60	1.12

In Table I we compare the deviations from flavor symmetry as they are reported by the Particle Data Group (PDG) [11] with the modifications resulting from (18) and (19). Remarkably, the experimental ratio $\Gamma(\pi\pi)/\Gamma(K\bar{K})$ is reproduced within the experimental error bar, whereas the prediction for $\Gamma(\eta\eta)/\Gamma(K\bar{K})$ remains within 1.33 standard deviations.

In Table II we compare our complete set of predictions for the decay rates for a scalar glueball with mass corresponding to either 1505 MeV [$f_0(1500)$] or 1722 MeV [$f_0(1710)$], with and without the inclusion of masses for the pseudoscalar mesons, to experiment. In the case of $f_0(1500)$, the (experimentally well-known) decay pattern is matched neither qualitatively nor quantitatively [28]. The inclusion of the pseudoscalar masses helps for the total width, but modifies the decay pattern adversely. For $f_0(1710)$, branching ratios are less accurately known. The prediction for the total width is increased slightly above the experimental value when masses are included, but the decay pattern into two pseudoscalar mesons is improved markedly, as we have already shown in Table I. In fact, the experimental results quoted for the partial widths should be considered as upper values, as they ignore the possibility of decay into four or more pions; decay of $f_0(1710)$ into two ω mesons and further to six pions has been seen [11] in $J/\psi \to \gamma f_0(1710)$ (in fact this has been seen at the level of 75% of the rate into two pions [11], so the holographic prediction may not be very far off) [45]. The only remaining major mismatch between existing experimental data for $f_0(1710)$ [48] and the prediction of the Witten-Sakai-Sugimoto model thus appears to be the rather high rate for decay into four pions, which is predicted by the latter to proceed through 2ρ and $\rho\pi\pi$ at the level of about twice the rate for decay into two ω mesons [28]; this prediction is not modified significantly by the introduction of quark masses, since the corresponding vertices are unchanged at the level of our approximation.

TABLE II. Experimental data for the decay pattern of the glueball candidates $f_0(1500)$ and $f_0(1710)$ from Ref. [11] [except for those marked by a star, which are from Ref. [49], where the total width of $f_0(1710)$ was split under the assumption of a negligible branching ratio to four or more pions, using data from the BES [50] (upper entry) and WA102 [51] (lower entry) Collaborations, respectively], compared to the prediction obtained in Ref. [28] from the mode G_D in the chiral Witten-Sakai-Sugimoto model, and, finally, to the extrapolation of the massive case proposed in this Letter.

Decay	Γ/M		
	(Expt. [11])	(WSS chiral [28])	(WSS massive)
$f_0(1500)$ (total)	0.072(5)	0.027–0.037	0.057–0.077
$f_0(1500) \to 4\pi$	0.036(3)	0.003–0.005	0.003–0.005
$f_0(1500) \to 2\pi$	0.025(2)	0.009–0.012	0.010–0.014
$f_0(1500) \to 2K$	0.006(1)	0.012–0.016	0.034–0.045
$f_0(1500) \to 2\eta$	0.004(1)	0.003–0.004	0.010–0.013
$f_0(1710)$ (total)	0.078(4)	0.059–0.076	0.083–0.106
$f_0(1710) \to 2K$	$*\left\{\begin{matrix} 0.041(2) \\ 0.047(17) \end{matrix}\right.$	0.012–0.016	0.029–0.038
$f_0(1710) \to 2\eta$	$*\left\{\begin{matrix} 0.020(10) \\ 0.022(11) \end{matrix}\right.$	0.003–0.004	0.009–0.011
$f_0(1710) \to 2\pi$	$*\left\{\begin{matrix} 0.017(4) \\ 0.009(2) \end{matrix}\right.$	0.009–0.012	0.010–0.013
$f_0(1710) \to 2\rho$, $\rho\pi\pi \to 4\pi$	...	0.024–0.030	0.024–0.030
$f_0(1710) \to 2\omega \to 6\pi$	Seen	0.011–0.014	0.011–0.014

To summarize, by extrapolating the exactly calculable coupling of scalar glueballs to the mass term of the isosinglet pseudoscalar meson in the originally chiral Witten-Sakai-Sugimoto model, we found a significantly enhanced decay of scalar glueballs into kaons and η mesons compared to flavor-symmetric ratios. This is in line with the previously proposed mechanism of chiral suppression of scalar glueball decay, which has been posited as an explanation of how the isoscalar meson $f_0(1710)$, with its preferred decay into two kaons, could be predominantly gluonic rather than an $s\bar{s}$ state [15,16]. From this we conclude that the top-down holographic Witten-Sakai-Sugimoto model may well be consistent with a glueball interpretation of $f_0(1710)$, while disfavoring the other popular glueball candidate $f_0(1500)$. In this case, the successful reproduction of the branching ratios given in Table I is correlated to a sufficiently small rate for the decay $G \to \eta\eta'$, for which only upper limits exist so far [52]. Moreover, the Witten-Sakai-Sugimoto model predicts significant partial widths for the decay of $f_0(1710)$ into four and six pions; according to [11], only the latter have been so far confirmed experimentally.

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