Charge-Induced Spin Torque in Anomalous Hall Ferromagnets

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We demonstrate that spin-orbit coupled electrons in a magnetically doped system exert a spin torque on the local magnetization, without a flowing current, when the chemical potential is modulated in a magnetic field. The spin torque is proportional to the anomalous Hall conductivity, and its effective field strength may overcome the Zeeman field. Using this effect, the direction of the local magnetization is switched by gate control in a thin film. This charge-induced spin torque is essentially an equilibrium effect, in contrast to the conventional current-induced spin-orbit torque, and, thus, devices using this operating principle possibly have higher efficiency than the conventional ones. In addition to a comprehensive phenomenological derivation, we present a physical understanding based on a model of a Dirac-Weyl semimetal, possibly realized in a magnetically doped topological insulator. The effect might be realized also in nanoscale transition materials, complex oxide ferromagnets, and dilute magnetic semiconductors.

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Introduction.—The electric control of spin magnetization aims to be used in next-generation magnetic devices, allowing information to be written electronically. Spin-transfer torque random-access memory has emerged as a potential candidate for such versatile devices: a spin-polarized current exerts a spin-transfer torque on the magnetization and switches the direction via the exchange interaction [1]. It is known that the driving spin-polarized current needs to exceed a threshold current, and a noncollinear magnetization structure, such as spin valves, tunnel junctions, or domain walls, is required. These might be central issues for low-power-consumption magnetic-recording devices. The spin-orbit torque has been recently proposed to control the magnetization direction without noncollinear configurations. The threshold current density is $\sim 10^6$ A/cm^2 for a number of magnetic materials [2], so much effort has been made to search for materials having high efficiency [3–5].

In this Letter, we shall propose an alternative mechanism to switch the magnetization by electrical means in anomalous Hall ferromagnets consisting of local spins and itinerant band electrons. The anomalous Hall effect (AHE) occurs in solids with broken time-reversal symmetry, typically in a ferromagnetic phase, as a consequence of spin-orbit coupling [6]. In particular, the intrinsic AHE originates to the spin-orbit coupled band structure and can be described in terms of Berry curvatures [7]. In many cases, the intrinsic effect appears to be the dominant contribution to the AHE in the low-temperature clean limit of metallic ferromagnets [6].

We derive a generic expression of the spin torque term induced by the chemical potential modulation and a magnetic field in anomalous Hall ferromagnets based on a comprehensive phenomenological argument. This torque is proportional to the anomalous Hall conductivity. When this torque effect overcomes the Zeeman effect, the magnetization can be controlled locally, pointing parallel and antiparallel to the external magnetic field, depending on the sign of the chemical potential modulation. Devices using this operating principle are free from joule heating and, thus, possibly have a much higher efficiency than conventional ones. As an example of spin-orbit coupled band electrons, we consider a Weyl semimetal [8–12] realized in magnetically doped topological insulator materials [13], where a physical understanding and an estimation of the effective field are given.

Charge-induced spin torque.—In isotropic ferromagnets, the off-diagonal conductivity tensor σ_{ij} $(i \neq j)$ may be expressed in the form [6]

$$\sigma_{ij} = \epsilon_{ijk}\sigma_{\mathrm{AHE}}\hat{M}_k \tag{1}$$

in a vanishing magnetic field, where $\hat{\boldsymbol{M}}$ is the normalized directional vector of the magnetization, and σ_{AHE} is the magnitude of the anomalous Hall conductivity. In the following, we ignore the disorder effects and consider only the intrinsic contribution at zero temperature.

According to the Středa formula [7,14], the intrinsic Hall conductivity controls the charge density n induced when a uniform magnetic field $\boldsymbol{B}$ is applied:

$$\sigma_{ij} = -ec\epsilon_{ijk}\frac{\partial n}{\partial B_k}, \tag{2}$$

where c is speed of light, and e is the electron charge. Combining these two relations, we obtain the relation between the electron density and the magnetization,

$$n_{\mathrm{ind}} = -\frac{\sigma_{\mathrm{AHE}}}{ec}\hat{\boldsymbol{M}}\cdot\boldsymbol{B}. \tag{3}$$

In a magnetic field, the right-hand side of Eq. (1) is replaced by the $\sigma_{\mathrm{AHE}}\hat{M}_k + \sigma_{\mathrm{OHE}}\hat{B}_k$. However, the second term, the ordinary contribution $\sigma_{\mathrm{OHE}} \propto |\boldsymbol{B}|$, appears to be of second order in the field in Eq. (3) and is ignored since we focus on the linear response regime. These relations are derived in uniform systems, but when the magnetic field and magnetization vary slowly in space and time, it is natural to assume that these are locally applicable. In the following, σ_{AHE} denotes the magnitude of the anomalous Hall conductivity in the ideal uniform case.

In thermodynamics, the number of particles is conjugate to the chemical potential described by the thermodynamic potential: $-\int d^3x n_{\mathrm{ind}}\delta\mu_F$. By substituting Eq. (3) into this relation, we derive a generic thermodynamic potential for charge and spin coupling in anomalous Hall ferromagnets as

$$\Omega_{\mathrm{CS}} = \int d^3x \frac{\sigma_{\mathrm{AHE}}}{ec}\delta\mu_F \hat{\boldsymbol{M}}\cdot\boldsymbol{B}, \tag{4}$$

where $\delta\mu_F$ is a local chemical potential, which is defined as a deviation from the Fermi energy. When the magnetization is uniform while the chemical potential varies in space, Eq. (4) can be rewritten as $\Omega_{\mathrm{CS}} = -(1/c)\int d^3x \boldsymbol{A}\cdot\boldsymbol{j}_{\mathrm{AHE}}$, where $\boldsymbol{j}_{\mathrm{AHE}} = \sigma_{\mathrm{AHE}}\boldsymbol{E}\times\hat{\boldsymbol{M}}$ is the anomalous Hall current, $\boldsymbol{E} = \boldsymbol{\nabla}\delta\mu_F/e$, and $\boldsymbol{A}$ is the vector potential. Therefore, Eq. (4) describes the anomalous Hall effect. From a microscopic model, Eq. (4) may be obtained by integration over the fermionic degrees of freedom.

Coupling between the modulation of the chemical potential and the magnetization direction described by Eq. (4) indicates the possibility of the magnetization switching by gate tuning in a ferromagnetic thin film. The coupling energy of the magnetization and an applied magnetic field is $E_{\mathrm{field}} = E_{\mathrm{Zeeman}} + \Omega_{\mathrm{CS}}$. Here, $E_{\mathrm{Zeeman}} = -\int d^3x \rho_s g\mu_B S\hat{\boldsymbol{M}}\cdot\boldsymbol{B}$ is the Zeeman term, and ρ_s, g, and S being the density, the Lande factor, and spin of the magnetic moments, respectively. In addition to the Zeeman torque $\boldsymbol{T}_{\mathrm{Zeeman}} = g\mu_B S\boldsymbol{B}\times\hat{\boldsymbol{M}}$, there exists an induced torque term given by

$$\begin{aligned}\boldsymbol{T}_{\mathrm{CS}} &= -\frac{\delta\Omega_{\mathrm{CS}}}{\delta(\rho_s\hat{\boldsymbol{M}})}\times\hat{\boldsymbol{M}} \\ &= -\frac{\sigma_{\mathrm{AHE}}}{ec\rho_s}\delta\mu_F(\boldsymbol{x})\boldsymbol{B}\times\hat{\boldsymbol{M}}\end{aligned} \tag{5}$$

in a magnetic field. This is the main finding of the present work.

In a thin film, the chemical potential can be locally tuned by gating. When σ_{AHE} and $\delta\mu_F$ are large enough, depending on the sign of $\delta\mu_F$, the local magnetization points parallel or antiparallel to the external magnetic field. This torque is contrasted to the current-induced spin-transfer torque [1], which can be expressed as $\boldsymbol{T}_{\mathrm{STT}} = (\hbar S/e)(\boldsymbol{j}\cdot\boldsymbol{\nabla})\hat{\boldsymbol{M}}$ in the adiabatic limit. For $\boldsymbol{T}_{\mathrm{STT}}$, a constant electric current $\boldsymbol{j}$ is needed which generates the joule heating, while $\boldsymbol{T}_{\mathrm{CS}}$ requires only the chemical potential modulation; the effect is essentially dissipationless.

Microscopic derivation in Weyl semimetals.—In the rest of the Letter, we consider the torque term Eq. (5) from a microscopic point of view, where the system consists of spin-orbit coupled itinerant electrons and local spins interacting via exchange coupling

$$\mathcal{H}_{\mathrm{exc}} = JSx_s\hat{\boldsymbol{M}}(\boldsymbol{x},t)\cdot\boldsymbol{\sigma}. \tag{6}$$

Here, J is the exchange coupling constant, $x_s = \rho_s a^3$ is the ratio of the magnetic dopants, a^3 being the volume of the unit cell, and $\boldsymbol{\sigma} = (\sigma_x, \sigma_y, \sigma_z)$ are Pauli matrices describing the electron spin degrees of freedom. The torque induced by exchange coupling is given by $-JSa^3\langle\boldsymbol{\sigma}\rangle\times\hat{\boldsymbol{M}}$. It has been proposed that in the presence of spin-orbit coupling, a flowing current produces a nonequilibrium spin density $\langle\boldsymbol{\sigma}\rangle_{\mathrm{neq}}$ and, thus, the spin-orbit torque: $\boldsymbol{T}_{\mathrm{SOT}} = -JSa^3\langle\boldsymbol{\sigma}\rangle_{\mathrm{neq}}\times\hat{\boldsymbol{M}}$ [2–4]. By contrast, Eq. (5) is induced by the modulation of the chemical potential and an external magnetic field, where the spin density $\langle\boldsymbol{\sigma}\rangle$ is finite in equilibrium.

As a concrete example of spin-orbit coupled ferromagnets, we consider a Weyl semimetal and show that a finite spin density is generated by chemical potential tuning. A simplified model consists of a Dirac semimetal (DSM) and local spins of magnetic dopants [13]. This can be related to magnetically doped topological insulators [15] such as chromium-doped Bi_2Se_3 [16–18] or chromium-doped $(Bi,Sb)_2Te_3$ [19], where by doping Cr the strength of spin-orbit coupling is reduced and the original band gap may collapse at a certain range of the doping ratio [13,20]. The low-energy effective Hamiltonian is given as $\mathcal{H}_{\mathrm{WSM}} = \mathcal{H}_{\mathrm{DSM}} + \mathcal{H}_{\mathrm{exc}}$. Here,

$$\mathcal{H}_{\mathrm{DSM}} = v_F\tau_z\boldsymbol{\sigma}\cdot\left(-i\hbar\boldsymbol{\nabla} + \frac{e}{c}A(\boldsymbol{x},t)\right) + e\phi(\boldsymbol{x},t) \tag{7}$$

describes massless Dirac fermions in three dimensions [15,21–24], where $(\boldsymbol{A},\phi)$ is the electromagnetic potential, v_F is the velocity, and the chirality $\tau_z = \pm$labels the two-degenerate Weyl nodes. We note that this model differs from the Weyl semimetal phase proposed in a topological insulator multilayer [9]. In the latter system, the magnetization needs to point perpendicularly to the layers, and, thus, the off-diagonal conductivity tensor cannot be expressed in the form of Eq. (1).

The proposed phenomena can be understood from the energetic point of view. Here we consider Dirac fermions in a uniform magnetic field pointing in the $+z$ direction. Defining the ladder operator $a = \sqrt{c/2\hbar eB_z}(\pi_x - i\pi_y)$ satisfying $[a,a^\dagger] = 1$, where $\boldsymbol{\pi} = -i\hbar\boldsymbol{\nabla} + (e/c)\boldsymbol{A} + \tau_z(JSx_s/v_F)\hat{\boldsymbol{M}}$, the Hamiltonian for a Weyl semimetal can be written as [10,25,26]

$$\mathcal{H}_{\mathrm{WSM}} = \tau_z \hbar v_F \begin{pmatrix} k_z + \tau_z \frac{JSx_s}{\hbar v_F}\hat{M}_z & \sqrt{\frac{2eB_z}{\hbar c}}a \\ \sqrt{\frac{2eB_z}{\hbar c}}a^\dagger & -k_z - \tau_z \frac{JSx_s}{\hbar v_F}\hat{M}_z \end{pmatrix}. \quad (8)$$

The zeroth Landau level states are obtained as $(0, |0\rangle)^t$ in the spinor representation, where the $|n\rangle$'s are the eigenstates of the number operator $a^\dagger a$. The energy dispersion is given by

$$E_0(k_z) = -\tau_z \hbar v_F k_z - JSx_s \hat{M}_z. \quad (9)$$

Typical situations are illustrated in Fig. 1(a) $\hat{M}_z < 0$ (antiparallel to $\boldsymbol{B}$) and 1(b) $\hat{M}_z > 0$ (parallel to $\boldsymbol{B}$). As represented by solid lines, the energies of the zeroth Landau level depend on the sign of $\hat{M}_z$. By contrast, nonzero Landau levels

$$E_n(k_z) = \pm\hbar v_F \sqrt{\left(k_z + \frac{JSx_s}{\hbar v_F}\tau_z \hat{M}_z\right)^2 + \frac{2eB_z}{\hbar c}|n|} \quad (10)$$

represented by the dashed lines in Fig. 1 are particle-hole symmetric, and the spectra do not differ for the opposite sign of $\hat{M}_z$.

When the modulation of the chemical potential $\delta\mu_F$ is introduced, electrons move from the high-potential region $[\delta\mu_F/(-e) > 0]$ to the low-potential region $[\delta\mu_F/(-e) < 0]$. In the absence of the exchange interaction, all the Landau levels shift equally in energy by $\delta\mu_F$. In the presence of the exchange interaction, on the other hand, the density of electrons depends on the value of $\hat{M}_z$ as expected from Eq. (9), and, thus, there is a correlation

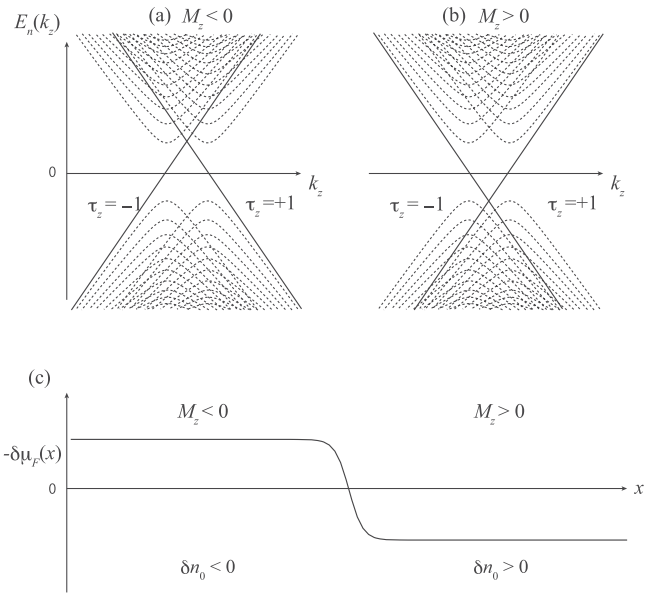

FIG. 1. The energy dispersion of the Landau levels as a function of k_z for (a) $\hat{M}_z < 0$ and (b) $\hat{M}_z > 0$. (c) The modulation of the chemical potential as a function of the position.

between $\hat{M}_z$ and $\delta\mu_F$. To see this quantitatively, we count the number of electrons changed by $\hat{M}_z$ from the case of $\hat{M}_z = 0$, fixing the magnetic field B_z (> 0). Only the zeroth Landau level depends on the sign of $\hat{M}_z$ and, thus, modifies the density of electrons given by

$$n_{\mathrm{ind}} = \frac{eB_z}{hc}\rho_F^{(1\mathrm{D})}\Delta E, \quad (11)$$

where $\Delta E = JSx_s\hat{M}_z$ is the energy shift of the zeroth Landau level. Here, eB_z/hc is the degeneracy of the zeroth Landau level per area, and $\rho_F^{(1\mathrm{D})} = 2/2\pi\hbar v_F$ is the density of states of one-dimensional fermions, the product of them being the density of states in three dimensions. Equation (11) is consistent with Eq. (3) and the anomalous Hall conductivity

$$\sigma_{\mathrm{AHE}} = -\frac{e^2 JSx_s}{2\pi^2\hbar^2 v_F} \quad (12)$$

obtained from $\mathcal{H}_{\mathrm{WSM}}$ [13]. To minimize the total energy, $\hat{\boldsymbol{M}}$ points in the direction of $-\boldsymbol{B}$ in the high-potential region $[\delta\mu_F/(-e) > 0]$, while $\hat{\boldsymbol{M}}$ points in the direction of $+\boldsymbol{B}$ in the low-potential region $[\delta\mu_F/(-e) < 0]$, as depicted in Fig. 1(c). The energy gained corresponds to Ω_{CS}.

The thermodynamic potential Ω_{CS} can be also derived from microscopic field theory. The effective action for the electromagnetic response in a Weyl semimetal has been derived as [10,26–28] $S_\theta = (e^2/4\pi^2\hbar c)\int dtd^3x\theta(\boldsymbol{x},t)\boldsymbol{E}(\boldsymbol{x},t)\cdot\boldsymbol{B}(\boldsymbol{x},t)$. Here, $\theta(\boldsymbol{x},t)$ is the axion field, which is related to the magnetization direction in our model as

$$\frac{1}{2}\boldsymbol{\nabla}\theta(\boldsymbol{x},t) = \frac{JSx_s}{\hbar v_F}\hat{\boldsymbol{M}}(\boldsymbol{x},t) = \frac{1}{\hbar v_F}\frac{\partial\mathcal{H}_{\mathrm{exc}}}{\partial\boldsymbol{\sigma}}. \quad (13)$$

The procedure to obtain the axion term S_θ is as follows. First, in the Lagrangian formalism

$$S_{\mathrm{WSM}} = \int dtd^3x\psi^\dagger[i\hbar\partial_t - (\mathcal{H}_{\mathrm{DSM}} + \mathcal{H}_{\mathrm{exc}})]\psi, \quad (14)$$

we remove the exchange term $\mathcal{H}_{\mathrm{exc}}$ by the chiral gauge transformation; $\psi \to e^{i\tau_z\theta/2}\psi$ where θ satisfies the condition Eq. (13). In the Grassman functional theory, the Jacobian J_θ is introduced by this transformation. After proper regularization [29], the axion term is given by $S_\theta = S_{\mathrm{WSM}} - S_{\mathrm{DSM}} = -i\ln J_\theta$, where S_{DSM} is S_{WSM} at $J = 0$. The charge current is derived from S_θ as $\boldsymbol{j} = c(\delta S_\theta/\delta\boldsymbol{A}) = (e^2/4\pi^2\hbar)\boldsymbol{\nabla}\theta\times\boldsymbol{E}$. With Eq. (13), we obtain the anomalous Hall conductivity Eq. (12). By substituting Eq. (12) for Eq. (4), the thermodynamical potential Ω_{CS} for Weyl semimetals can be obtained, which is in agreement with S_θ when the electric field is written as $\boldsymbol{E} = \boldsymbol{\nabla}\delta\mu_F/e$.

With typical material parameters for Cr-doped Bi_2Se_3, we quantitatively estimate the ratio of the effective magnetic field to the external magnetic field ($x_s = 0.1$, $J = 2.0$ eV, $\hbar v_F = 2.2$ eV Å^{-1} and $\rho_s = 1.1 \times 10^{-4}$ Å^{-3}) [16,17,19,21,22]

$$\boldsymbol{B}_{\rm eff} \equiv \frac{1}{g\mu_B S\rho_s}\frac{\delta}{\delta \hat{\boldsymbol{M}}}(E_{\rm Zeeman} + \Omega_{\rm CS}) \approx \left[1 - 6.5 \times \left(\frac{\delta\mu_F}{[\rm eV]}\right)\right]\boldsymbol{B}. \quad (15)$$

If the chemical potential can be shifted by ~0.2 eV, the direction of the effective field $\boldsymbol{B}_{\rm eff}$ and, thus, the magnetization are reversed. In the above arguments, we have neglected the Zeeman interaction for band electrons. In strongly spin-orbit coupled systems, it is known that the Lande factor of itinerant electrons can be larger than that in a vacuum. Nevertheless, the typical energy scale of the Zeeman effect in $(Bi, Sb)_2Te_3$ is $g^*\mu_B \approx 1$ meV/T [21,22], which is negligibly small compered to the exchange interaction $JSx_s \approx 500$ meV in Eq. (6) [16].

In the above argument, we consider only the intrinsic contribution. In the presence of disorder, there exists the extrinsic contributions known as the skew-scattering and side-jump effects. Generally, the Hall conductivity consists of two parts $\sigma^{\rm I}_{ij} = (\hbar e^2/4\pi){\rm Tr}[v_i G^+ v_j (G^+ - G^-) - v_j G^- v_i (G^+ - G^-)]$ and $\sigma^{\rm II}_{ij} = (ie^2/4\pi){\rm Tr}[(x_i v_j - x_j v_i)(G^+ - G^-)]$ [14], where $G^{\pm} = (E_F \pm i0 - \mathcal{H})^{-1}$ and E_F is the Fermi energy. $\sigma^{\rm I}_{ij}$ is associated with states on the Fermi surface. $\sigma^{\rm II}_{ij}$, on the other hand, is the contribution of all states below the Fermi energy and is a thermodynamic equilibrium property of the ferromagnet. In most cases, $\sigma^{\rm I}_{ij}$ corresponds to the extrinsic contribution [6], while $\sigma^{\rm II}_{xy}$ the intrinsic contribution. The left-hand side of Eq. (2) is $\sigma^{\rm II}_{xy}$. The anomalous Hall conductivities $\sigma^{\rm I}_{ij}$ and $\sigma^{\rm II}_{xy}$ of a disordered Weyl metal have been computed in the model of topological insulator multilayers in Ref. [30]. It was found that the extrinsic contribution to the anomalous Hall effect is absent as long as the Fermi level is sufficiently close to zero (Weyl nodes). This indicates that the charge-induced spin torque $\boldsymbol{T}_{\rm CS}$ of a Weyl semimetal is robust against disorder.

The mechanism of the induced effective field Eq. (15) differs qualitatively from that proposed in a ferromagnet deposited on a topological insulator [31]. On the surface of a topological insulator, where Dirac-Weyl fermions demonstrate the quantum anomalous Hall effect [15], a dissipationless Hall current $\boldsymbol{j}$ produces a spin density $\langle\boldsymbol{\sigma}\rangle = -(1/ev_F)\hat{\mathbf{z}} \times \boldsymbol{j}$ due to spin-momentum locking [15], which gives an effective field and a torque [31–33], as in the case of spin-orbit torque $\boldsymbol{T}_{\rm SOT}$ [34–38]. The effective field $\boldsymbol{B}_{\rm eff}$ generated by a current is pointing in the in-plane direction, while the easy axis of the local magnetization is perpendicular to the surface. To switch the magnetization, therefore, the current needs to exceed the threshold current, which might be challenging because a large current destroys the quantum Hall regime at the surface [31].

Conclusion.—In this Letter, we derived a generic thermodynamic potential which describes coupling of the local spin magnetization and the charge density of itinerant band electrons in the three-dimensional anomalous Hall ferromagnets. This indicates that a spin torque is locally induced by gate control without a flowing current. As an example, a Weyl semimetal was analyzed, and the strength of the effect was estimated. The torque term Eq. (5) overcomes the Zeeman effect when the shift of the chemical potential is large enough, and, thus, the direction of the magnetization can be controlled by gate tuning. The spin torque Eq. (5) can be generated in ordinary ferromagnets with large anomalous Hall conductivity such as transition materials, complex oxide ferromagnets, and magnetic semiconductors. In practice, the region of switched magnetizations should be smaller than the Thomas-Fermi screening length as experimentally feasible and required for nanoscale devices. The proposed mechanism of the induced spin torque in this work potentially has great advantage in application to low-energy-consumption nonvolatile memory devices.

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