New Exotic Meson and Baryon Resonances from Doubly Heavy Hadronic Molecules

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We predict several new exotic doubly heavy hadronic resonances, inferring from the observed exotic bottomoniumlike and charmoniumlike narrow states $X(3872)$, $Z_b(10610)$, $Z_b(10650)$, $Z_c(3900)$, and $Z_c(4020/4025)$. We interpret the binding mechanism as mostly molecularlike isospin-exchange attraction between two heavy-light mesons in a relative S-wave state. We then generalize it to other systems containing two heavy hadrons which can couple through isospin exchange. The new predicted states include resonances in meson-meson, meson-baryon, baryon-baryon, and baryon-antibaryon channels. These include those giving rise to final states involving a heavy quark $Q = c, b$ and antiquark $\bar{Q}' = \bar{c}, \bar{b}$, namely, $D\bar{D}^*$, $D^*\bar{D}^*$, D^*B^*, $\bar{B}B^*$, $\bar{B}^*B^*$, $\Sigma_c\bar{D}^*$, Σ_cB^*, $\Sigma_b\bar{D}^*$, Σ_bB^*, $\Sigma_c\bar{\Sigma}_c$, $\Sigma_c\bar{\Lambda}_c$, $\Sigma_c\bar{\Lambda}_b$, $\Sigma_b\bar{\Sigma}_b$, $\Sigma_b\bar{\Lambda}_b$, and $\Sigma_b\bar{\Lambda}_c$, as well as corresponding S-wave states giving rise to QQ' or $\bar{Q}\bar{Q}'$.

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During the last few years there have been several experimental discoveries of bottomoniumlike and charmoniumlike charged manifestly exotic narrow isovector resonances $Z_b(10610)$, $Z_b(10650)$ [1–7], $Z_c(3900)$ [8–12], and $Z_c(4020/4025)$ [13–17]. All four resonances lie very close to two heavy meson thresholds: $\bar{B}B^*$, $\bar{B}^*B^*$, $\bar{D}D^*$, and $\bar{D}^*D^*$, respectively. The discoveries of these states were preceded by the observation of the now well-established $X(3872)$ [18] extremely narrow resonance right at the $\bar{D}D^*$ threshold. Conspicuously absent from this list are resonances at the $\bar{D}D$ and $\bar{B}B$ thresholds.

In all the states where J^P has been unambiguously measured it is 1^+. The charged states decay into heavy quarkonia (e.g., J/ψ, ψ', Υ, or h_b) and a charged pion. (Some of the states also decay to two heavy mesons, typically with much larger branching ratios, despite much smaller phase space.) So they are manifestly exotic and their minimal quark content is $\bar{Q}Qq\bar{q}$, i.e., that of a tetraquark, where $Q = c, b$ and $q = u, d$.

Yet, despite large phase space (hundreds of MeV) for decay into $\bar{Q}Q$ and pion(s), these resonances have narrow widths, indicating a very small overlap of their wave functions with the corresponding quarkonia. This provides strong circumstantial evidence in favor of molecular interpretation, namely, that rather than containing all four quarks in a single confinement volume, the resonances are loosely bound S-wave states of heavy-light mesons, $Q\bar{q} - \bar{Q}q$.

Such "molecular" states, $\bar{D}D^*$, etc., were introduced in Refs. [19] and [20]. They were later extensively discussed [21–25] in analogy with the deuteron that binds via exchange of pions and other light mesons.

A crucial element of the binding mechanism proposed in Ref. [21] is that pions are expected to play a major role in generating the attractive potential. (There must also be a shorter range repulsive force to stabilize the interaction.) From today's perspective one may generalize this to exchange of light quarks in their lowest-mass configuration, i.e., a pseudoscalar carrying one unit of isospin. Such a binding mechanism immediately explains the conspicuous absence of $\bar{D}D$ and $\bar{B}B$ among the observed resonances. A resonance in a channel containing two heavy pseudoscalar mesons cannot form through exchange of a pseudoscalar pion, because such an exchange would require a three-pseudoscalar vertex, e.g., $DD\pi$, which is forbidden in QCD by parity conservation.

On the other hand, $\bar{D}D^*$ and $\bar{D}^*D^*$ (and their bottomonium counterparts) can bind through pion exchange. In the $\bar{D}D^*$ case $\bar{D}$ emits a pion and turns into $\bar{D}^*$, while D^* absorbs a pion and turns into D, so $\bar{D}D^* \to \bar{D}^*D$, etc. The physical state is $(\bar{D}D^* + \bar{D}^*D)/\sqrt{2}$, so it turns into itself.

In the $\bar{D}^*D^*$ case, a D^* can emit a pion and remain a D^*. D^* has negative parity, so the emitted pion must be emitted in a P wave. The orbital angular momentum can couple with $S = 1$ from D^* spin to give a total $J = 1$. The same argument applies to $\bar{D}^*$, so $\bar{D}^*D^*$ turns into itself after pion emission.

Thus the conditions for existence of the resonance are as follows: (a) The state contains two heavy hadrons. They have to be heavy, as the repulsive kinetic energy is inversely proportional to the reduced mass (see, e.g., Ref. [26]). (For a more recent discussion see Ref. [27].) (b) The two hadrons carry isospin, so that they can couple to pions. Channels like $\Sigma_c\bar{\Lambda}_c$, in which one of the particles has zero isospin, can exchange a pion to become the equal-mass channel $\Lambda_c\bar{\Sigma}_c$. (c) The spin and parity of the two hadrons have to be such that they can bind through single pion exchange. (d) The hadrons making up the molecule have to be sufficiently narrow, as the molecule's width cannot be smaller than the sum of its constituents' widths [28–31].

Methods have been proposed [32] to distinguish molecular states of two heavy mesons from alternative models. Our discussion is confined to molecular states, but includes meson-baryon, baryon-baryon, and baryon-antibaryon channels. Exotic baryon-antibaryon resonances were proposed earlier [33], but without the additional binding conferred by a heavy quark-antiquark pair they would probably be too broad to detect. The binding mechanism can apply to two heavy baryons leading to a prediction of a doubly heavy $\Sigma_b^+\Sigma_b^-$ dibaryon [34–36]. We emphasize that the pion-exchange binding mechanism can, in principle, apply to any two heavy hadrons which carry isospin and satisfy condition (c) above, be they mesons or baryons.

For pion exchange between states 1 and 2 with isospins $I_{1,2}$ and spins $S_{1,2}$, the effective potential is proportional to [21]

$$V \sim \pm(I_1 \cdot I_2)(S_1 \cdot S_2) \quad \text{for } (qq, q\bar{q}) \text{ interactions}, \tag{1}$$

where q or $\bar{q}$ stands for the light quark(s) or antiquark(s) in hadrons 1 and 2, as long as the total spins S_i are correlated with the direction of the light-quark spins. (This is true for D^*, B^*, Σ_c, and Σ_b.)

The new states we are discussing are expected to be narrow, just like the Z_c and Z_b states. They are either below threshold or slightly above threshold with regard to the two body channels in which pion exchange occurs. So there will be little or no phase space for decay into such channels. On the other hand, even though they will have plenty of phase space for decay into quarkonium and states made from light quarks, their wave functions will have small overlap with such final states. This is because they are loosely bound and therefore in the initial wave function the heavy quarks spend most of their time far from each other. Resonances are possible also in states with higher isospin, but their masses are expected to be higher, and their widths are expected to be larger, e.g.,

$$\begin{aligned} M[Z_c(4020/4025)] &> M[X(3872)], \\ \Gamma[Z_c(4020/4025)] &\gg \Gamma[X(3872)], \end{aligned} \tag{2}$$

the latter because of the larger phase space for the "fall-apart" mode into two heavy mesons.

A quick inspection leads to the following most likely candidates containing a heavy quark $Q = c$ or b and a heavy antiquark $\bar{Q}' = \bar{c}$ or $\bar{b}$: $D\bar{D}^*$, $D^*\bar{D}^*$, D^*B^*, $\bar{B}B^*$, $\bar{B}^*B^*$, $\Sigma_c\bar{D}^*$, Σ_cB^*, $\Sigma_b\bar{D}^*$, Σ_bB^*, $\Sigma_c\bar{\Sigma}_c$, $\Sigma_c\bar{\Lambda}_c$, $\Sigma_c\bar{\Lambda}_b$, $\Sigma_b\bar{\Sigma}_b$, $\Sigma_b\bar{\Lambda}_b$, and $\Sigma_b\bar{\Lambda}_c$. As noted above, these are the states whose heavy-quark content is $c\bar{c}$, $b\bar{b}$, $b\bar{c}$, or $c\bar{b}$. The first two types of states can decay strongly to charmonium or bottomonium plus pion(s), while the latter two involve a $B_c^\pm$ in the final state. (This could provide a distinctive signature at the LHC [27].) There will also be corresponding states (such as the $\Sigma_b\Sigma_b$ dibaryon proposed in Refs. [34–36]) whose heavy-quark content is QQ' or $\bar{Q}\bar{Q}'$. The thresholds and some sample decay modes for the states with heavy-quark content $Q\bar{Q}'$ are displayed in Table I.

We have listed in Table I channels that can undergo transitions either to themselves or to equal-mass channels via pion exchange. The channels $\Sigma_c\bar{\Lambda}_b$ and $\Lambda_c\bar{\Sigma}_b$ are the sole exception, which we have listed for the purpose of discussion. Pion exchange permits the transitions $\Sigma_c\bar{\Lambda}_b \leftrightarrow \Lambda_c\bar{\Sigma}_b$ and $\Sigma_b\bar{\Lambda}_c \leftrightarrow \Lambda_b\bar{\Sigma}_c$, channels whose thresholds differ by 27.6 MeV from one another. Another pair (not listed in the Table) whose thresholds differ by only 26 MeV are $\Sigma_c\bar{D}$ (threshold 4321 MeV) and $\Lambda_c\bar{D}^*$ (threshold 4295 MeV). It will be interesting to see if such nearby thresholds have any role in fostering pion exchange.

A detailed analysis such as that of Refs. [21–25], is needed to determine whether pion exchange is sufficient to bind two hadrons in each of the channels listed in Table I or the corresponding states with QQ' or $\bar{Q}\bar{Q}'$. For the case of $D\bar{D}^*$ and $B\bar{B}^*$ we include here a detailed example of mixing between channels that have equal mass in the isospin symmetry limit. For the former, the four channels are

$$[D^0\bar{D}^{*0}, D^{*0}\bar{D}^0, D^+D^{*-}, D^{*+}D^-]. \tag{3}$$

In this basis, pion exchange leads to a potential proportional to the matrix

$$V \sim \begin{bmatrix} 0 & -1 & 0 & -2 \\ -1 & 0 & -2 & 0 \\ 0 & -2 & 0 & -1 \\ -2 & 0 & -1 & 0 \end{bmatrix}. \tag{4}$$

The eigenvalues and eigenvectors are

$$1\colon\ [1, 1, -1, -1] \quad C = +, I = 1, \tag{5}$$

$$-1\colon\ [1, -1, -1, 1] \quad C = -, I = 1, \tag{6}$$

$$3\colon\ [1, -1, 1, -1] \quad C = -, I = 0. \tag{7}$$

$$-3\colon\ [1, 1, 1, 1] \quad C = +, I = 0. \tag{8}$$

The last corresponds to the most deeply bound state $X(3872)$. The state with eigenvalue -1, negative C, and $I = 1$ can be identified with the $Z_c(3900)$.

In this particular case the mixing matrix is 4 × 4. In the real world the mass difference between $D^0\bar{D}^{*0}$ and D^+D^{*-} is much larger than the binding energy, so the physical $X(3872)$ is reduced to a two-channel mixture of $D^0\bar{D}^{*0}$ and $D^{*0}\bar{D}^0$. In the $B\bar{B}^*$ case, isospin breaking is much smaller and the binding is expected to be stronger, because of the larger reduced mass, leading to smaller kinetic energy.

TABLE I. Thresholds for molecular states consisting of a hadron with a heavy quark $Q = c$ or b and an antiquark $\bar{Q}' = \bar{c}$ or $\bar{b}$. Similar thresholds hold for states with QQ' or $\bar{Q}\bar{Q}'$. For non-self-conjugate cases, charge-conjugate channels are also implied. Here q represents a light quark u or d. Only states which can undergo transitions to equal-mass channels via pion exchange are shown. Isospin violation in hadron masses is ignored. Charge-conjugate baryonic states have opposite parity.

Channel	Minimum isospin	Minimal quark content[a,b]	Threshold (MeV)[c]	S-wave J^P	Example of decay mode
$D\bar{D}^*$	0	$c\bar{c}q\bar{q}$	3875.8	1^+	$J/\psi\pi\pi$
$D^*\bar{D}^*$	0	$c\bar{c}q\bar{q}$	4017.2	$0^+, 1^+, 2^+$	$J/\psi\pi\pi$
D^*B^*	0	$c\bar{b}q\bar{q}$	7333.8	$0^+, 1^+, 2^+$	B_c+ pion(s)
$\bar{B}B^*$	0	$b\bar{b}q\bar{q}$	10 604.6	1^+	$\Upsilon(ns)\omega$
$\bar{B}^*B^*$	0	$b\bar{b}q\bar{q}$	10 650.4	$0^+, 1^+, 2^+$	$\Upsilon(ns)\omega$
$\Sigma_c\bar{D}^*$	1/2	$c\bar{c}qqq'$	4462.4	$1/2^-, 3/2^-$	$J/\psi p$
Σ_cB^*	1/2	$c\bar{b}qqq'$	7779.5	$1/2^-, 3/2^-$	$B_c^+ p$
$\Sigma_b\bar{D}^*$	1/2	$b\bar{c}qqq'$	7823.0	$1/2^-, 3/2^-$	$B_c^- p$
Σ_bB^*	1/2	$b\bar{b}qqq'$	11 139.6	$1/2^-, 3/2^-$	$\Upsilon(nS)p$
$\Sigma_c\bar{\Lambda}_c$	1	$c\bar{c}qq'\bar{u}\,\bar{d}$	4740.3	$0^-, 1^-$	$J/\psi\pi$
$\Sigma_c\bar{\Sigma}_c$	0	$c\bar{c}qq'\bar{q}\bar{q}'$	4907.6	$0^-, 1^-$	$J/\psi\pi\pi$
$\Sigma_c\bar{\Lambda}_b$	1	$c\bar{b}qq'\bar{u}\,\bar{d}$	8073.3[d]	$0^-, 1^-$	$B_c^+\pi$
$\Sigma_b\bar{\Lambda}_c$	1	$b\bar{c}qq'\bar{u}\,\bar{d}$	8100.9[d]	$0^-, 1^-$	$B_c^-\pi$
$\Sigma_b\bar{\Lambda}_b$	1	$b\bar{b}qq'\bar{u}\,\bar{d}$	11 433.9	$0^-, 1^-$	$\Upsilon(nS)\pi$
$\Sigma_b\bar{\Sigma}_b$	0	$b\bar{b}qq'\bar{q}\bar{q}'$	11 628.8	$0^-, 1^-$	$\Upsilon(nS)\pi\pi$

[a] Ignoring annihilation of quarks.
[b] Plus other charge states when $I \neq 0$.
[c] Based on isospin-averaged masses.
[d] Thresholds differ by 27.6 MeV.

Therefore, in the case of X_b, the bottomonium analogue of $X(3872)$, one expects a full four-channel mixing.

Analogous mixing is expected in other S-wave meson-meson, meson-baryon, and baryon-(anti)baryon channels. The channels $D^{*0}\bar{D}^{*0}$ and $D^{*+}D^{*-}$ constitute three separate two-channel problems for states of total spin $J = 0$, 1, and 2, with eigenchannels corresponding to isospin $I = 0$ and 1, while the channels $\Sigma_c\bar{D}^*$ constitute two separate two-channel problems for $J = 1/2$ and $3/2$ with eigenchannels corresponding to $I = 1/2$ and $3/2$.

The expression (1) predicts the most attractive $D^*\bar{D}^*$ channel to have $I = S = 0$. The only state discovered so far near that threshold is $Z_c(4020/4025)$ with $I = 1$. In addition to $Z_c(4020/4025)$ there could be another state near the $D^*\bar{D}^*$ threshold, with lower mass and $I = 0$.

The next threshold above $D^*\bar{D}^*$ in Table I is that of $\Sigma_c\bar{D}^*$, at 4462 MeV. Application of Eq. (1) with a $+$ sign for qq interaction predicts two lowest levels with the same binding energy: $S = 1/2, I = 3/2$ and $S = 3/2, I = 1/2$. The $J/\psi p$ mode listed in the Table can only come from the latter (spin-3/2) state.

As little is known about pion couplings to most of the states in Table I, and as exchanges other than pions and configurations other than S waves (as discussed, e.g., in Ref. [21]) may play a role, it is too early to calculate the binding in most cases. Our purpose here is to call attention to some interesting possible thresholds whose effects could show up in final states consisting of heavy quarkonium plus light-quark mesons or baryons. Such final states are accessible in several current experiments.

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The LHCb Collaboration has just posted results [39] on a new narrow exotic resonance $P_c(4450)$ in the $J/\psi p$ channel, with a mass of $4449.8 \pm 1.7 \pm 2.5$ MeV, a width of $39 \pm 5 \pm 8$ MeV, and statistical significance 12σ. Its mass and spin are consistent with the $\Sigma_c\bar{D}^*$ $I = 1/2$, $S = 3/2$ resonance that we predict based on Table I.

In the same Letter LHCb reports discovering another, lighter and wider Breit-Wigner structure $P_c(4380)$, also in the $J/\psi p$ channel, with a mass of $4380 \pm 8 \pm 29$ MeV and a width of $205 \pm 18 \pm 86$ MeV. This structure is *not* predicted by our approach. At this point it isn't clear if $P_c(4380)$ is a regular resonance, because of the unusual shape of its Argand plot in Fig. 9(b) of Ref. [39], in contradistinction with the pristine plot for $P_c(4450)$ in

Fig. 9(a), though this could just be due to smaller statistics. If it is not a *bona fide* resonance, it is possible that $P_c(4380)$ results from the vicinity of the threshold, e.g., along the lines discussed in Ref. [40].

If $P_c(4380)$ does turn out to be a genuine resonance after all, it is very unlikely to be of molecular nature. This is because of the deep binding, about 80 MeV below the $\Sigma_c \bar{D}^*$ threshold and the rather large width. Instead, it would likely be some kind of a "genuine" P-wave pentaquark, for which both the large binding and the large width are much more natural. A P wave would then be essential in order for $P_c(4380)$ and $P_c(4450)$ to have opposite parities, as reported by LHCb. The true nature of $P_c(4380)$ is an intriguing issue, which is an outstanding challenge for future experiments. In particular, three recent papers [41–43] propose photoproduction off a proton target as a test for the resonant nature of the enhancements at 4380 and 4450 MeV.

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[1] K. F. Chen *et al.* (Belle Collaboration), Phys. Rev. Lett. **100**, 112001 (2008).

[2] M. Karliner and H. J. Lipkin, arXiv:0802.0649.

[3] A. Bondar *et al.* (Belle Collaboration), Phys. Rev. Lett. **108**, 122001 (2012).

[4] I. Adachi *et al.* (Belle Collaboration), arXiv:1209.6450.

[5] P. Krokovny *et al.* (Belle Collaboration), Phys. Rev. D **88**, 052016 (2013).

[6] A. Garmash *et al.* (Belle Collaboration), Phys. Rev. D **91**, 072003 (2015).

[7] For a recent review, see P. Krokovny, *Proceedings of the International Workshop on QCD Exotics*, 7–12 June 2015, Shandong University, Jinan, China, http://indico.ihep.ac.cn/event/4827/session/0/contribution/1/material/slides/0.pdf.

[8] M. Ablikim *et al.* (BESIII Collaboration), Phys. Rev. Lett. **110**, 252001 (2013).

[9] Z. Q. Liu *et al.* (Belle Collaboration), Phys. Rev. Lett. **110**, 252002 (2013).

[10] T. Xiao, S. Dobbs, A. Tomaradze, and K. K. Seth, Phys. Lett. B **727**, 366 (2013).

[11] M. Ablikim *et al.* (BESIII Collaboration), Phys. Rev. Lett. **112**, 022001 (2014).

[12] M. Ablikim *et al.* (BESIII Collaboration), arXiv:1506.06018 [Phys. Rev. Lett. (to be published)].

[13] M. Ablikim *et al.* (BESIII Collaboration), Phys. Rev. Lett. **112**, 132001 (2014).

[14] M. Ablikim *et al.* (BESIII Collaboration), Phys. Rev. Lett. **111**, 242001 (2013).

[15] K. Chilikin *et al.* (Belle Collaboration), Phys. Rev. D **90**, 112009 (2014).

[16] M. Ablikim *et al.* (BESIII Collaboration), Phys. Rev. Lett. **113**, 212002 (2014).

[17] For a recent review, see Y. Guo, *Proceedings of the International Workshop on QCD Exotics*, 7–12 June 2015, Shandong University, Jinan, China, http://indico.ihep.ac.cn/event/4827/session/0/contribution/7/material/slides/0.pdf.

[18] S. K. Choi *et al.* (Belle Collaboration), Phys. Rev. Lett. **91**, 262001 (2003); D. Acosta *et al.* (CDF Collaboration), Phys. Rev. Lett. **93**, 072001 (2004); B. Aubert *et al.* (*BABAR* Collaboration), Phys. Rev. D **71**, 071103 (2005); V. M. Abazov *et al.* (D0 Collaboration), Phys. Rev. Lett. **93**, 162002 (2004).

[19] M. B. Voloshin and L. B. Okun, Pis'ma Zh. Eksp. Teor. Fiz. **23**, 369 (1976) [JETP Lett. **23**, 333 (1976)].

[20] A. De Rújula, H. Georgi, and S. L. Glashow, Phys. Rev. Lett. **38**, 317 (1977).

[21] N. A. Törnqvist, Phys. Rev. Lett. **67**, 556 (1991); N. A. Törnqvist, Z. Phys. C **61**, 525 (1994).

[22] N. A. Törnqvist, Phys. Lett. B **590**, 209 (2004).

[23] C. E. Thomas and F. E. Close, Phys. Rev. D **78**, 034007 (2008).

[24] M. Suzuki, Phys. Rev. D **72**, 114013 (2005).

[25] S. Fleming, M. Kusunoki, T. Mehen, and U. van Kolck, Phys. Rev. D **76**, 034006 (2007).

[26] T. E. O. Ericson and G. Karl, Phys. Lett. B **309**, 426 (1993).

[27] X. Q. Li and X. Liu, Eur. Phys. J. C **74**, 3198 (2014).

[28] C. Hanhart, Y. S. Kalashnikova, and A. V. Nefediev, Phys. Rev. D **81**, 094028 (2010).

[29] A. A. Filin, A. Romanov, V. Baru, C. Hanhart, Y. S. Kalashnikova, A. E. Kudryavtsev, U.-G. Meißner, and A. V. Nefediev, Phys. Rev. Lett. **105**, 019101 (2010).

[30] F.-K. Guo and Ulf.-G. Meißner, Phys. Rev. D **84**, 014013 (2011).

[31] M. Karliner (unpublished).

[32] M. Cleven, F. K. Guo, C. Hanhart, Q. Wang, and Q. Zhao, Phys. Rev. D **92**, 014005 (2015).

[33] J. L. Rosner, Phys. Rev. Lett. **21**, 950 (1968).

[34] M. Karliner, H. J. Lipkin, and N. A. Tornqvist, arXiv:1109.3472.

[35] M. Karliner, H. J. Lipkin, and N. A. Tornqvist, Nucl. Phys. B, Proc. Suppl. **225–227**, 102 (2012).

[36] M. Karliner and S. Nussinov, J. High Energy Phys. 07 (2013) 153.

[37] Z. C. Yang, Z. F. Sun, J. He, X. Liu, and S. L. Zhu, Chin. Phys. C **36**, 6 (2012).

[38] Y. D. Chen, C. F. Qiao, P. N. Shen, and Z. Q. Zeng, Phys. Rev. D **88**, 114007 (2013).

[39] R. Aaij *et al.* (LHCb Collaboration), Phys. Rev. Lett. **115**, 072001 (2015).

[40] M. B. Voloshin, arXiv:1507.02639.

[41] Q. Wang, X. H. Liu, and Q. Zhao, arXiv:1508.00339.

[42] V. Kubarovsky and M. B. Voloshin, arXiv:1508.00888.

[43] M. Karliner and J. L. Rosner, arXiv:1508.01496.