Four-Body Correlations in Nuclei

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Low-energy spectra of $4n$ nuclei are described with high accuracy in terms of four-body correlated structures ("quartets"). The states of all $N \geq Z$ nuclei belonging to the $A = 24$ isobaric chain are represented as a superposition of two-quartet states, with quartets being characterized by isospin T and angular momentum J. These quartets are assumed to be those describing the lowest states in ^{20}Ne ($T_z = 0$), ^{20}F ($T_z = 1$), and ^{20}O ($T_z = 2$). We find that the spectrum of the self-conjugate nucleus ^{24}Mg can be well reproduced in terms of $T = 0$ quartets only and that, among these, the $J = 0$ quartet plays by far the leading role in the structure of the ground state. The same conclusion is drawn in the case of the three-quartet $N = Z$ nucleus ^{28}Si. As an application of the quartet formalism to nuclei not confined to the sd shell, we provide a description of the low-lying spectrum of the proton-rich ^{92}Pd. The results achieved indicate that, in $4n$ nuclei, four-body degrees of freedom are more important and more general than usually expected.

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In nuclear physics, four-body correlated structures are usually associated with α clustering. A common theoretical approach to α clustering is represented by the α-cluster model [1]. According to this model, the nucleus consists of an $N = Z$ core to which some α clusters are appended. These clusters are tightly bound and spatially localized structures of two neutrons and two protons. The α-cluster model exhibits a striking contrast with the standard shell model picture in which protons and neutrons are described instead as weakly interacting quasiparticles in a mean field. However, these two pictures are expected to coexist at low excitation energies where, due to the Pauli blocking, which acts more strongly than in the states close to the α emission threshold (e.g., see the case of the Hoyle state [2]), the α clustering is expected to manifest itself mainly as four-body correlations in the configuration space. It is thus commonly supposed that correlated four-body structures ("quartets") play a major role in the ground and excited sates of $N = Z$ nuclei. However, to our knowledge, this supposition has never been supported by compelling calculations. In this Letter, by using a simple microscopic quartet model, we will show how the low-energy spectra of $4n$ nuclei can be indeed described in terms of quartets with an accuracy comparable with state-of-the-art shell model calculations. We shall prove that this is the case not only for self-conjugate nuclei but also for $4n$ nuclei with $N \neq Z$. For the latter nuclei the low-lying states will be expressed in terms of quartets built not only by two protons and two neutrons but also by one proton and three neutrons as well as by four neutrons. This fact indicates that the four-body degrees of freedom are important also for configurations that are different from the α-like ones.

Microscopic quartet models have a long history in nuclear structure. They have been employed to treat the proton-neutron interaction, in particular the proton-neutron pairing, and to investigate the quartet condensation in $N = Z$ nuclei [3–9]. However, their complexity has always represented a hindrance to their development. Recently, a simple approach has been proposed that is able to describe accurately the ground state of the isovector pairing Hamiltonian as a condensate of quartets built by two neutrons and two protons coupled to total isospin $T = 0$ and, for spherically symmetric mean fields, to total angular momentum $J = 0$ [10]. This quartet model conserves exactly the particle number, the isospin, and the Pauli principle and can be applied for any number of quartets. Later on, a more general description of the ground state of $N = Z$ nuclei as a product of distinct quartets was proposed and applied to a description of both isovector ($T = 1$, $J = 0$) and isoscalar ($T = 0$, $J = 1$) pairing correlations [11,12].

Of course, pairing is only a part (although a crucial one) of the nuclear interaction. It is reasonable to expect that a realistic description of self-conjugate nuclei should involve not only $T = 0$, $J = 0$ quartets. In the following we will show how the approach of Refs. [11,12] has been extended to include quartets with arbitrary values of isospin and angular momentum and to treat realistic interactions of the shell model type. In the extended approach we shall identify the quartets that contribute most to the structure of the low-lying states in even-even $N = Z$ nuclei and, in particular, we shall investigate the role played by $T = 0$, $J = 0$ quartets in the ground state of these nuclei. Our analysis will be mainly confined to sd-shell nuclei.

We start by briefly illustrating our approach. We work in a spherically symmetric mean field and label the single-particle states by $i \equiv \{n_i, l_i, j_i\}$, where the standard notation for the orbital quantum numbers is used. The quartet creation operator is defined as

$$Q^{+}_{\alpha,JM,TT_z} = \sum_{i_1 j_1 J_1 T_1} \sum_{i_2 j_2 J_2 T_2} C^{(\alpha)}_{i_1 j_1 J_1 T_1, i_2 j_2 J_2 T_2} \times [[a^{+}_{i_1} a^{+}_{j_1}]^{J_1 T_1} [a^{+}_{i_2} a^{+}_{j_2}]^{J_2 T_2}]^{JT}_{MT_z},$$

where a_i^+ creates a fermion in the single particle state i and $J(T)$ and $M(T_z)$ denote, respectively, the total angular momentum (isospin) and the relative projections. No restrictions on the intermediate couplings $J_1 T_1$ and $J_2 T_2$ are introduced in the calculations. In order to generate the spectra of $4n$ nuclei we perform configuration interaction calculations in spaces built in terms of selected sets of the above quartets. After being constructed, these are kept unchanged in all calculations.

The criterion adopted for the selection of the quartets has been that of choosing, as representative of the quartets with a given isospin T, those describing the lowest levels with that isospin in nuclei with four active particles outside the inert core of reference. For applications within the sd shell, the inert core is represented by ^{16}O and the nuclei that have therefore been considered for the definition of the quartets are $^{20}_{10}\mathrm{Ne}_{10}$, $^{20}_{9}\mathrm{F}_{11}$, and $^{20}_{8}\mathrm{O}_{12}$. The lowest states of these nuclei are characterized by $T = 0$, 1, and 2, respectively. Each of these states therefore identifies a quartet with given T, J [and a projection $T_z = (N - Z)/2$]. We have carried out shell model (SM) calculations for these three nuclei and, as initial sets of quartets, we have selected those formed by the lowest six states in ^{20}Ne ($0 \leq J \leq 6$), the lowest five states in ^{20}F ($1 \leq J \leq 5$), and the lowest nine states in ^{20}O ($0 \leq J \leq 4$). The mixing amplitudes defining each collective quartet have resulted from these SM calculations. As will be seen below, we have also explored reductions of these sets to identify the most relevant quartets in the structure of the nuclei under investigation. For sd-shell nuclei we have employed the USDB interaction [13] and the SM results have been extracted from Ref. [14].

We start our analysis by examining the self-conjugate nucleus ^{24}Mg. In Fig. 1, we compare the experimental spectrum of this nucleus to that resulting from a SM calculation and to the spectrum obtained in the quartet model (QM) when all the selected $T = 0$, 1, 2 quartets are taken into account. The quartet approach is seen to reproduce well both the SM ground state correlation energy and the SM excited states up to an energy of about 9 MeV. The SM is in turn able to fit well the experimental levels.

Having verified that the selected sets of $T = 0$, 1, 2 quartets are sufficient to describe the low-energy spectrum of ^{24}Mg, it becomes of interest to investigate the role of the different quartets. We begin by focusing on isospin. In Fig. 1, on the right-hand side, we show the theoretical spectrum obtained in the quartet formalism when only $T = 0$ quartets are retained. On top of the lowest levels we also show the overlaps with the corresponding SM eigenstates. One can see that this spectrum does not exhibit

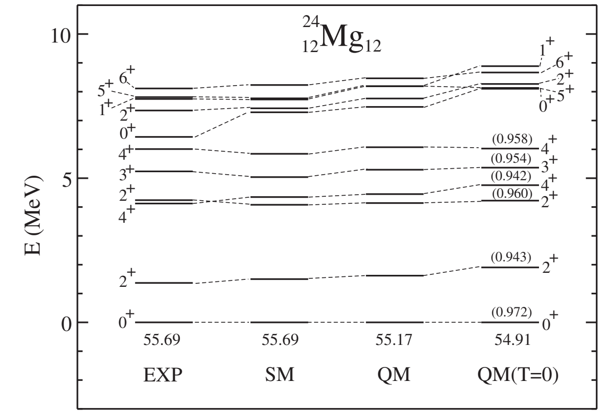

FIG. 1. Spectrum of ^{24}Mg obtained in the quartet model (QM) compared to experimental data (EXP) and shell model (SM) results [14]. QM($T = 0$) denotes the results obtained only with $T = 0$ quartets; the numbers on top of these levels are the overlaps between the QM and SM eigenfunctions. The number below each spectrum gives the ground state correlation energy, namely, the difference between the total ground state energy and the energy in the absence of interaction.

relevant differences with respect to the full QM calculation and that the above overlaps are pretty large, which confirms the good quality of the QM($T = 0$) wave functions. This result provides clear evidence of the marginal role played by the $T = 1$ and $T = 2$ quartets in the structure of these states.

As a next step, we concentrate on the ground state by employing only $T = 0$ quartets. In Fig. 2, we show how the error in the correlation energy of this state, relative to the SM result, varies by reducing, one quartet at a time and starting from the highest one in energy, the set of $T = 0$ quartets. The correlation energy remains basically unchanged up to the point where only the lowest $J = 0, 2, 4$

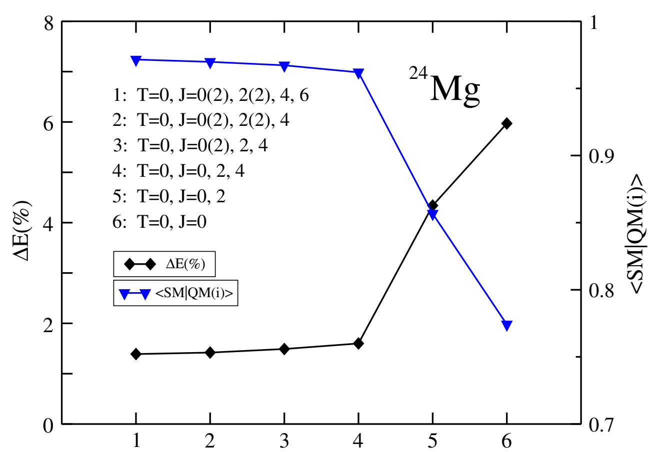

FIG. 2 (color online). Relative errors (with respect to the SM value) in the ground state correlation energy of ^{24}Mg obtained within the QM with the sets of quartets indicated in the figure. For each set we also show the overlaps between SM and QM wave functions.

quartets are left. From this point on further reductions in the set of quartets lead to significant variations in the energy. The overlaps between the SM and QM ground states, which are shown in the same figure, exhibit a behavior that is consistent with that of the energies. Thus, these calculations indicate that the $T = 0$ quartets with $J = 0, 2, 4$ play a major role in the ground state of ^{24}Mg. Among these quartets, the $T = 0$, $J = 0$ quartet is by far the one that contributes most to the correlation energy since, as can be seen in Fig. 2, an approximation in terms of only this quartet accounts for about 94% of the total energy.

We have searched for a confirmation of the results just discussed by examining another self-conjugate nucleus in the *sd* shell: ^{28}Si. We find that for this isotope the set of $T = 0$, $J = 0, 2, 4$ quartets already exhausts almost 99% of the correlation energy. Also in this case, the $T = 0$, $J = 0$ quartet is found to play a leading role, being able to account by itself for 93.4% of the total energy.

So far we have examined only self-conjugate nuclei. The range of nuclei that is accessible in terms of the set of $T = 0, 1, 2$ quartets employed for ^{24}Mg is, however, much broader. In Fig. 3, we extend the previous analysis to the whole $A = 24$ isobaric chain by comparing the low-energy spectra of $^{24}_{11}\mathrm{Na}_{13}$, $^{24}_{10}\mathrm{Ne}_{14}$, $^{24}_{9}\mathrm{F}_{15}$, and $^{24}_{8}\mathrm{O}_{16}$ obtained in the quartet model with the SM results and with the experimental data. As one can see, for all these nuclei the quartet formalism generates spectra that agree well with the SM ones. It is worth stressing that for the nuclei shown in Fig. 3 the quartets are built not only by two protons and two neutrons, as in the case of $N = Z$ nuclei, but also by one proton and three neutrons and by four neutrons. The case of quartets built by four like particles had been already discussed in Ref. [15] in relation with a treatment of the pairing Hamiltonian.

Similarly to what is observed in the self-conjugate ^{24}Mg and ^{28}Si, we have verified that a reasonable description of the low-lying states in $N > Z$ nuclei can still be achieved by significantly reducing the number of quartets involved in the QM calculations. By employing, for example, only sets of $T = 0$ and $T = 2$ quartets limited to the lowest ones with $J = 0, 2, 4$, the QM predicts ground state energies of ^{24}Ne and ^{24}O that are within 3% of the SM value. The same sets of quartets employed for ^{28}Mg and ^{28}Ne give rise to QM ground state energies that are within 2% of the SM value.

In order to verify that the results presented so far are not specific to *sd*-shell nuclei, we have carried out a similar analysis for the proton-rich nucleus $^{92}_{46}\mathrm{Pd}_{46}$. Calculations

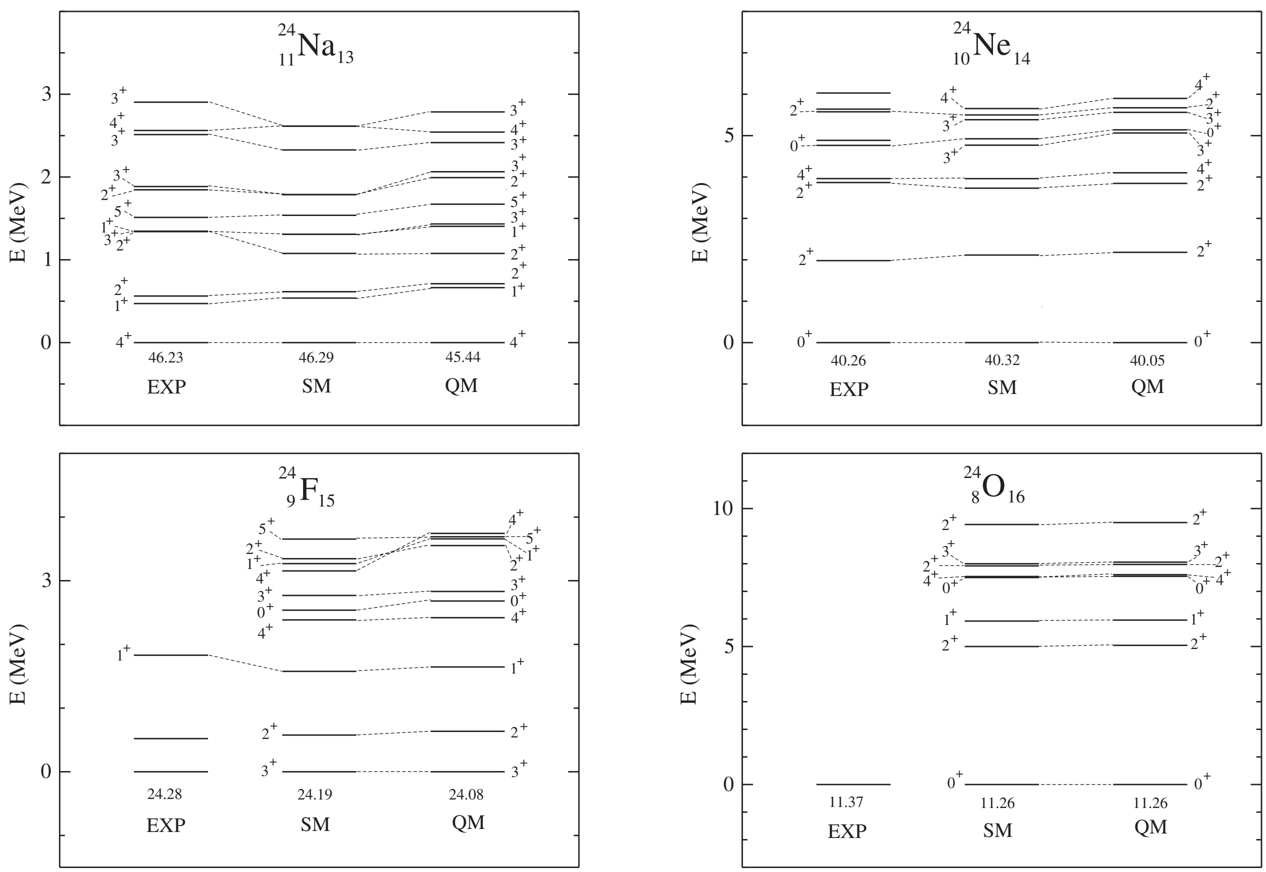

FIG. 3. The spectra of $A = 24$ nuclei generated by the QM approach compared to the experimental data and the SM predictions. See the caption of Fig. 1 for further details.

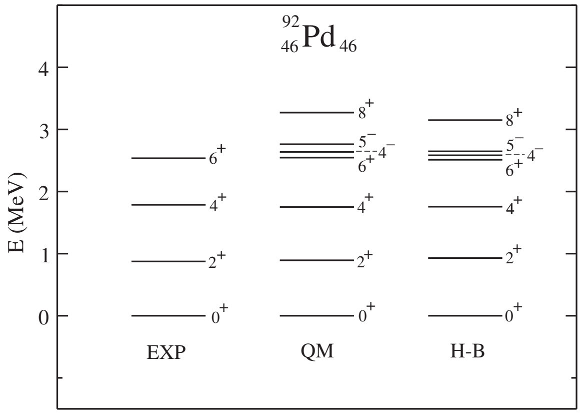

FIG. 4. The low-energy yrast spectrum of ^{92}Pd obtained in the QM approach compared to the experimental data [17] and the shell model calculation (H-B) of Ref. [18].

have been done in a space spanned by the $p_{1/2}$, $g_{9/2}$ orbitals using the F-FIT interaction of Johnstone and Skouras [16]. In Fig. 4, one sees the low-lying yrast spectrum that is obtained in the QM approach by employing only $T = 0$ quartets. These quartets are those associated with the lowest levels of $^{96}_{48}$Cd$_{48}$ (seven levels with $0 \leq J \leq 8$) and include also a negative parity level with $J = 5$. The QM yrast spectrum is seen to agree well with the experimental one [17] as well as with the SM spectrum generated within the same model space by Herndl and Brown [18]. As in the cases of the sd-shell nuclei analyzed above, the $T = 0$, $J = 0$ quartet is found to play a leading role in the ground state, accounting for almost 99% of the correlation energy. A detailed analysis of the structure of ^{92}Pd is reported in a separate study [19].

In summarizing, in this work we have presented an analysis of $4n$ nuclei in a formalism of quartets, i.e., four-body correlated structures characterized by total isospin T and total angular momentum J. The analysis has been carried out for the whole isobaric chain of $A = 24$ nuclei, ranging therefore from even-even to odd-odd nuclei as well as from self-conjugate nuclei to nuclei with only neutrons in the valence shell. For all these nuclei, as well as for the proton-rich nucleus ^{92}Pd, the quartet formalism has provided a description of the low-energy spectra comparable in accuracy to that of shell model calculations. This fact confirms the importance of quartet degrees of freedom in any type of $4n$ nuclei and validates the present quartet formalism as the appropriate tool for treating them. It is worth noticing that the matrices to be diagonalized in a QM calculation can be by orders of magnitude smaller than the corresponding SM ones. As an example, the size of the Hamiltonian matrix in the QM($T = 0$) calculation of Fig. 1 is 63 as compared to 28 503 in the SM case (both calculations being done in the m scheme). Offering a representation of the nuclear wave function in a compact and physically transparent form does constitute a crucial aspect of the QM.

As a concluding remark, we notice that the description of $4n$ self-conjugate nuclei in terms of $T = 0$ quartets of low angular momenta ($J = 0, 2, 4$) that has emerged from the present analysis exhibits a striking analogy to that of nuclear collective spectra in a formalism of S ($J = 0$), D ($J = 2$), and G ($J = 4$) pairs (e.g., see Ref. [20]; for a more general shell-model-like formalism based on collective pairs see Ref. [21]). Such an analogy encourages the extension to quartets of boson mapping techniques developed for a microscopic analysis of the interacting boson model (IBM) [22]. Thus, if the s ($J = 0$) and d ($J = 2$) IBM bosons are interpreted as the images of, respectively, the $T = 0$, $J = 0$ and $T = 0$, $J = 2$ quartets, and to the extent that the description of the fermionic Hamiltonian in the space of these quartets can be transferred onto that of a two-body Hermitian sd boson Hamiltonian, with its parameters eventually absorbing the contribution of other $T = 0$ quartets (if any), then the use of IBM-type Hamiltonians for the treatment of even-even self-conjugate nuclei finds a microscopic justification in our analysis. To our knowledge, in the literature there exists only one (old) example of such a use, which was carried out, however, on a purely phenomenological basis [23].

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