Effect of Orbital Angular Momentum on Nondiffracting Ultrashort Optical Pulses

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We introduce a new class of nondiffracting optical pulses possessing orbital angular momentum. By generalizing the X-wave solution of the Maxwell equation, we discover the coupling between angular momentum and the temporal degrees of freedom of ultrashort pulses. The spatial twist of propagation invariant light pulse turns out to be directly related to the number of optical cycles. Our results may trigger the development of novel multilevel classical and quantum transmission channels free of dispersion and diffraction. They may also find application in the manipulation of nanostructured objects by ultrashort pulses and for novel approaches to the spatiotemporal measurements in ultrafast photonics.

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Since the development of the laser, and in particular of mode locking [1] and Q-switching [2] techniques, optical pulses have attracted a lot of interest, and their development has influenced many fields of fundamental and applied research such as atomic physics, spectroscopy, communications, material processing, and medicine, to name a few [3,4]. As they are essentially suitable superpositions of plane waves that travel with different frequencies, optical pulses tend to diffract in both space and time during propagation. This feature constitutes a limiting factor for some applications like lithography [5], where the spatial broadening affects the quality of the generated mask, or communication science, where temporal broadening can induce additional noise between adjacent channels [6]. To solve these issues, so-called localized waves [7], i.e., nondiffracting electromagnetic fields in both space and time, have been proposed in the last decades, with their most famous representatives being the X waves. First introduced in acoustics [8,9], X waves were subsequently studied in different areas of physics, such as nonlinear optics [10,11], Bose-Einstein condensates [12], quantum optics [13], and waveguide arrays [14,15], to name a few. Recently, they have also been proposed as a possible solution to efficient free space communication [16].

X waves are polychromatic superpositions of Bessel waves, and the related vast literature mostly considers superpositions of Bessel beams of order zero, and neglects their possible orbital angular momentum (OAM) content [17]. OAM is indeed related to the twisted wave front of higher order Bessel beam solutions of the Maxwell equations [18]. Seemingly, despite some published papers [19–22], OAM is still seldom associated to ultrashort pulses, and only very recently have some experimental results reported femtosecond vortex beams [23,24] and Laguerre-Gauss supercontinuum [25].

The fact that light possesses both linear and angular momentum has been known since the pioneering works by Poynting [26] and Darwin [27]. However, it is only thanks to the seminal works of Berry and Nye in 1974 [28] and Allen and Woerdman in 1992 [29] that the OAM concept was brought into the field of optical beams, where it has been extensively studied both from a fundamental [30–33] and experimental point of view, leading to striking applications such as optical tweezers [34] and spanners [35]. Very recently, OAM has also been proposed as a new degree of freedom for encoding information in a superdense manner, and both its classical [36] and quantum [37] features have been investigated.

A challenging issue at the moment is generating ultrashort pulses with variable OAM; this would open unprecedented possibilities in terms of light wave transmission systems unaffected by dispersion and diffractions. In these terms, a very promising direction is combining the nondiffracting character of X waves with the superdense coding by OAM. The well-known resilience of X waves against the interaction with object and randomness, in fact, could result in protecting the information encoded into the OAM of the X wave against perturbations. In particular, for free space communication, the self-healing character of X waves may result in a minimization of the turbulence-driven cross talk between different OAM channels. Moreover, the extension of the concept of OAM to untrashort optical pulses might give new insights on light-matter interaction, as recent works suggest [38].

Following the recent experimental realization of higher order Bessel beams by holographic techniques [39], in this Letter we propose a new class of OAM-carrying nondiffracting pulses. We consider superpositions of mth order Bessel beams with an exponentially decaying spectrum, and generalize the well known fundamental X waves [7]. This new class of few-cycles optical pulses is not only an exact model for the connection between OAM and the untrafast photonics, but they are a new tool for OAM-based free-space quantum and classical communications by localized waves. We first introduce the general solution for X waves with OAM. We start from a Bessel beam [40], namely

$$\psi(R,\theta,z) = J_m\left(\frac{\omega}{c}\sin\vartheta_0 R\right)e^{im\theta}e^{i(\omega/c)z\cos\vartheta_0}, \quad (1)$$

where $\omega = ck$ and ϑ_0 is the Bessel cone angle, i.e., the beam's characteristic parameter. Bessel beams carry OAM because they are eigenstates of the z components of the angular momentum operator $\hat{L}_z = -i\partial/\partial\theta$ with eigenvalue m [41]. It is well known that Eq. (1) well describes a monochromatic field. It is not difficult, however, to generalize this result to the nonmonochromatic case, where, following Ref. [7], the scalar field

$$\phi(\mathbf{r},t) = e^{im\theta}\int d\omega f(\omega) J_m\left(\frac{\omega}{c}\sin\vartheta_0 R\right)e^{-i\omega[t-(z/c)\cos\vartheta_0]}, \quad (2)$$

is an exact solution of the full wave equation, $f(\omega)$ being an arbitrary spectrum. Equation (2) is known in literature to describe localized waves, namely field configurations that are well localized both in space and time. If the following exponentially decaying spectrum is used,

$$f(\omega) = \omega^n e^{-\alpha\omega}\Theta(\omega), \quad (3)$$

where α is a parameter with the dimensions of a time that controls the width of the spectrum and $\Theta(\omega)$ is the Heaviside step function [42], Eq. (2) with $m = 0$ describes the well-known *fundamental X waves* [7]. For $m \neq 0$, however, Eq. (2) admits the following analytical solution [43]:

$$\phi_m(\mathbf{r},t) = \frac{C_{n,m}\xi^m e^{im\theta}}{(\alpha + i\zeta)^{n+1}}\,{}_2F_1(a,b;m+1;-\xi^2), \quad (4)$$

where $C_{n,m} = (n+m)!/(2^m m!)$, $a = (n+m+1)/2$, $b = (n+m+2)/2$, $\xi = R\sin\vartheta_0/[c(\alpha+i\zeta)]$, $\zeta = (z/c)\cos\vartheta_0 - t$ is the comoving reference frame attached to the X wave and ${}_2F_1(a,b;c,z)$ is the Gauss hypergeometric function [42]. Equation (4) represents a scalar X wave carrying m units of OAM. Before proceeding further with our analysis, it is worth noticing that OAM does not affect the propagation invariant (in time and space) features of the X waves. Moreover, as ultrashort pulses are often modeled through an exponentially decaying spectrum [3], Eq. (4) can be therefore taken as a scalar ultrashort nondiffracting wave that carries OAM. To correctly describe an optical ultrashort pulse, however, the scalar theory is no longer sufficient, and a full vector theory is required. An exact vectorial solution of Maxwell's equation can be built from a scalar function by means of the so-called Hertz vector potentials [44]. We choose $\Pi(\mathbf{r},t) = \hat{\mathbf{z}}\phi_m(\mathbf{r},t)$ as the Hertz potential, and the electric and magnetic vector fields are given by [44]

$$\mathbf{E}(\mathbf{r},t) = \nabla\times\nabla\times\Pi(\mathbf{r},t), \quad (5a)$$

$$\mathbf{B}(\mathbf{r},t) = \frac{\partial}{\partial t}[\nabla\times\Pi(\mathbf{r},t)]. \quad (5b)$$

The explicit expression of the vector electric and magnetic fields generated with the above equations are available in the Supplemental Material [45]. The real and imaginary parts of the x component of the electric field $E_x(\mathbf{r},t)$ are depicted in Fig. 1 as a function of the comoving coordinate ζ. As can be seen, the real part is an odd function that crosses the axis $\zeta = 0$ three times, while the imaginary part of the field is an even function, with only two crossings. This corresponds to single-cycle optical pulses following the definition in Ref. [46].

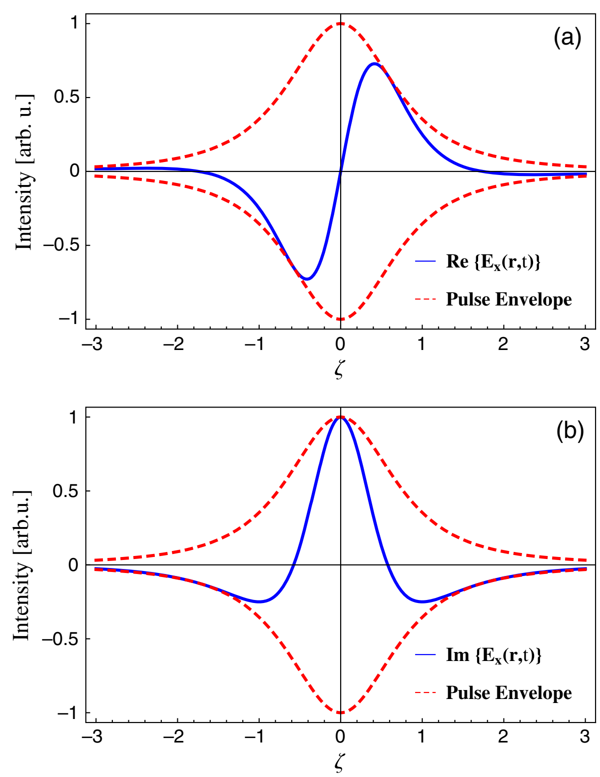

FIG. 1 (color online). Real (a) and imaginary (b) part of the electric field component $E_x(\mathbf{r},t)$ as a function of the comoving coordinate $\zeta = (z/c)\cos\vartheta_0 - t$, for the case $m = 1$. The real part is an odd function that crosses the line $\zeta = 0$ three times, while the imaginary part is an even function, with two crossings. Parameters: $\vartheta_0 = 0.01$ (corresponding to the paraxial case) and $\alpha = 1$; ζ is dimensionless.

This is the first result of this Letter: Eq. (4) and its vector counterpart, defined through Eqs. (5), represent a new class of optical pulses, namely fundamental X waves with OAM. With this result we can investigate the effects of OAM on optical pulses.

We consider, in particular, the case $m = 1$; i.e., we assume to have a single-cycled optical pulse with one unit of OAM, and we study the connection between OAM and its temporal properties. We focus our attention on the $E_x(\mathbf{r}, t)$ component; a similar discussion also holds for the other components. Figure 2(a) shows the real part of the field for various values of the OAM parameter m. As can be seen, OAM affects the pulse's temporal properties by changing its carrier frequency. As m grows, in fact, the pulse's carrier frequency ω_c also increases. Correspondingly, the field oscillates more rapidly, and the number of the optical cycles changes. To estabilish the relation between the carrier frequency ω_c and the OAM parameter m, we recall that an optical pulse is written as the product of an envelope function $A(\mathbf{r}, t)$ and a harmonic term, i.e., $A(\mathbf{r}, t)\exp(-i\omega_c t)$. As detailed in the Supplemental Material [45], for a general field, the carrier frequency can be calculated as the derivative of the phase ψ of the field in $t = 0$.

Here, $\psi(\mathbf{r}, t) = \arg[E_x(\mathbf{r}, t)]$ and we obtain

$$\omega_c = \left|\frac{\partial\psi(\mathbf{r}, \zeta)}{\partial\zeta}\right|_{\zeta=0} = \frac{m+2}{\alpha}, \qquad (6)$$

where α is the spectral width of the pulse and the derivative has been taken with respect to the comoving coordinate ζ. It is worth noticing that the result of Eq. (6) is exact only in the paraxial regime, where $\vartheta_0 \ll 1$. However, although for the nonparaxial case the expression of ω_c is much more complicated, it can be demonstrated that the variation of ω_c with m can still be well described by a slightly modified version of Eq. (6), namely $\omega_c = (m+2)/\alpha + \sigma_m$, where σ_m accounts for the nonparaxial corrections. Although σ_m actually depends on m, this dependence is very weak, and it can be treated, at the leading order in m, like a constant shift. Equation (6) is therefore valid independently of the value of ϑ_0.

Figure 2(b) shows the oscillatory term $\cos\psi(\mathbf{r}, \zeta)$ (which, apart from an unimportant multiplicative factor, represents the real part of the field $E_x(\mathbf{r}, t)$ [47]) for various values of the OAM parameter m. This term increases by one optical cycle every three units of OAM. This result can be interpreted in a simple intuitive way: as the amount of OAM carried by the pulse grows, the pulse itself adapts to it by increasing the number of optical cycles that it is able to perform under its envelope.

The additional effect of OAM on the pulse is reported in Fig. 3(a), where the envelope of the field is plotted for various values of the OAM parameter m. As the amount of OAM increases, the pulse duration given by its full width at

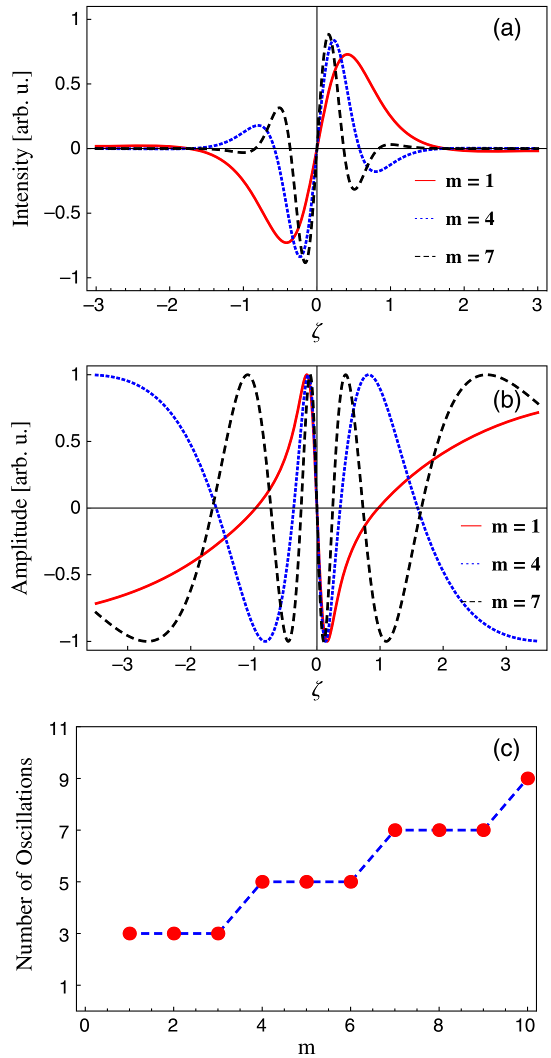

FIG. 2 (color online). (a) Real part of the x component of the electric field $E_x(\mathbf{r}, t)$ for different values of the OAM parameter m. (b) Oscillatory function $\cos\psi(\mathbf{r}, \zeta)$ for different values of the OAM parameter. The choice of these values corresponds to $\omega_c = 3/\alpha$, $\omega_c = 6/\alpha = 2(\omega_c)|_{m=1}$, and $\omega_c = 9/\alpha = 3(\omega_c)|_{m=1}$, respectively. When the OAM increases, the carrier frequency blueshifts, with an increase of the number of optical cycles, but the envelope FWHM decreases, thus resulting in a shorter pulse. (c) Calculated (red dots) number of oscillations performed by $\cos\psi(\mathbf{r}, \zeta)$ as a function of m. With every three units of m added, the number of cycles of $\cos\psi(\mathbf{r}, \zeta)$ grows by one unity. The number of oscillations is calculated by counting the number of zero crossing N_0 of each cosine term and using $N_0 = 2N_{\mathrm{osc}} + 1$. Here, $\vartheta_0 = 0.01$ (corresponding to the paraxial case) and $\alpha = 1$ has been chosen. In particular, $\alpha = 1$ means that ζ is a dimensionless quantity.

half maximum (FWHM) $\Delta\tau$, becomes smaller. To quantify this OAM-driven compression, we have numerically estimated $\Delta\tau$ for various m, and we show the results in Fig. 3(b): $\Delta\tau$ decreases exponentially with m. This is the second result of our Letter. The amount of OAM carried by a nondiffracting optical pulse affects its temporal

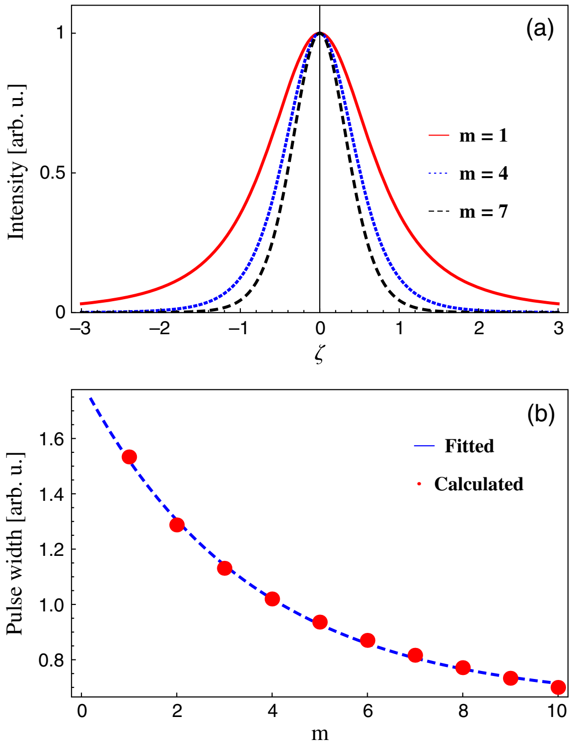

FIG. 3 (color online). (a) Pulse envelope of the x component of the electric field $E_x(\mathbf{r}, t)$ for different values of the OAM parameter m. (b) Calculated (red dots) and fitted (blue dashed line) pulse width $\Delta\tau$ as a function of the OAM m. The fit predicts an exponential shortening of the pulse width with respect to m, according to $Ae^{-Bm} + C$. We have $A = 1.16$, $B = 0.28$, and $C = 0.64$. Here, $\vartheta_0 = 0.01$ (corresponding to the paraxial case) and $\alpha = 1$ has been chosen. In particular, $\alpha = 1$ means that ζ is a dimensionless quantity.

properties; namely, it varies its time duration $\Delta\tau$ (making it smaller as m increases) and also changes its carrier frequency in such a way that the pulse gains an extra optical cycle every three units of OAM it carries (after the first one). In other words, in order to have high values of OAM in propagation-invariant pulses one needs to resort to higher frequencies and shorter temporal duration. At a fixed carrier frequency the maximal angular momentum is given by Eq. (6).

Equation (6), moreover, constitutes a fundamental physical limit to the amount of OAM that a *single cycle* pulse can carry. If, for example, $m = 4$ units of OAM are assigned to a single-cycled pulse, then, according to Eq. (6), its electric field will be able to perform two full cycles (instead of one), and the pulse will lose its single-cycle character. Although it may appear that this result limits the possibilities offered by X waves carrying OAM, one does not have to forget that this fundamental limit applies only to *single-cycled* pulses. If one, instead, uses few- (or many-) cycle pulses, then the result of Eq. (6) does not constitute a limitation anymore, but it simply states that the carrier frequency of such an optical pulse is being shifted by an amount proportional to the OAM the pulse itself carries. In this latter case, in fact, the shortening of the pulse duration induced by OAM has surely a higher impact than the shift of its carrier frequency. For the case of optical communications, for example, the OAM-induced pulse shortening has a double effect. On one hand, being able to use OAM allows one to encode a higher amount of information per pulse. On the other hand, however, the OAM-induced pulse shortening allows one to send more pulses in the same time window, thus automatically giving the possibility for a denser coding.

In conclusion, we have introduced a new class of optical pulses possessing OAM by generalizing the well-known fundamental X waves. We have theoretically investigated the effects of OAM onto such pulses and shown that, as the amount of OAM carried by the pulse increases, the pulse's carrier frequency increases accordingly, thus resulting in a shortening of the pulse width and the appearance (at certain discrete threshold of OAM) of extra field oscillations. Given the enormous interest that OAM has generated in the past years, we believe that its extension to the domain of optical pulses presented here with OAM-carrying X waves will open the way for new and intriguing applications. For example, besides optical communications, optical pulses possessing OAM might be interesting for spectroscopy, where the presence of OAM could give easy access to the rotational spectrum of molecules. In addition, as it has been recently pointed out in Ref. [48], the nonlinear interaction of optical pulses propagating in a medium has interesting consequences from the optomechanical point of view, as it contributes significantly to the radiation pressure. The introduction of OAM in this picture will surely offer new ways to control optomechanical effects and to allow for the investigation of novel regimes of interaction between light and matter.

As a last observation, the results presented here also set a fundamental limit to the amount of OAM that a *single cycle* nondiffracting optical pulse can carry. This could have interesting consequences for the case of superdense free space communication protocols with X waves.

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