

Vacuum Friction on a Rotating Pair of Atoms

Hervé Bercegol^{1,*} and Roland Lehoucq²

¹*Service de Physique de l'Etat Condensé, DSM/IRAMIS/SPEC/SPHYNX, CNRS UMR 3680, CEA Saclay, F-91191 Gif-sur-Yvette, France*

²*Laboratoire AIM-Paris-Saclay, DSM/Irfu, CNRS UMR 7158, Université Paris Diderot, CEA Saclay, F-91191 Gif-sur-Yvette, France*

(Received 18 June 2015; published 28 August 2015)

Zero-point quantum fluctuations of the electromagnetic vacuum create the widely known London–van der Waals attractive force between two atoms. Recently, there has been a revived interest in the interaction of rotating matter with the quantum vacuum. Here, we consider a rotating pair of atoms maintained by London–van der Waals forces and calculate the frictional torque they experience due to zero-point radiation. Using a semiclassical framework derived from the fluctuation dissipation theorem, we take into account the full electrostatic coupling between induced dipoles. Considering the case of zero temperature only, we find a braking torque proportional to the angular velocity and to the third power of the fine structure constant. Although very small compared to London–van der Waals attraction, the torque is strong enough to induce the formation of dimers in binary collisions. This new friction phenomenon at the atomic level should induce a paradigm change in the explanation of irreversibility.

DOI: [10.1103/PhysRevLett.115.090402](https://doi.org/10.1103/PhysRevLett.115.090402)

PACS numbers: 03.65.Sq, 12.20.-m, 34.20.-b, 45.20.dc

We would like to answer a still vivid question: “What are the various routes of energy conversion between matter in relative motion and propagating electromagnetic (EM) fields?” Scientists have an extensive knowledge on the interactions between internal energy levels of atomic matter and EM waves [1]. However, it has long been thought that nonionized gases (atomic or molecular) have negligible interactions with radiation. Furthermore, Boltzmann’s equation (the cornerstone of 20th-century macroscopic physics) relies on the hypothetical ground of elastic, energy-conserving atomic collisions [2]. Later, collision-induced absorption and emission [3] studies, especially in atomic mixtures, evidenced the nonuniversality of Boltzmann’s assumption. Meanwhile, the particular case of a pure atomic gas was long discarded due to a lack of quantitatively relevant interactions [3]. In contrast to this standard assumption, we find that the quantum vacuum exerts a non-negligible braking torque on two atoms rotating one around the other [Eq. (9) below].

Atomic matter in its ground state is neutral and does not carry any permanent dipolar or higher-order electrostatic moment. However, London [4] showed with quantum mechanics that the fluctuating electrostatic atomic dipoles give way to the universal van der Waals forces discovered decades before. Forces induced by quantum fluctuations were later shown to be affected by EM propagation over long distances [5] (i.e., greater than atomic wavelengths) and to be macroscopically detectable [6]. After accurately measuring this Casimir force [7], research turned to the dynamical Casimir effect, also called Casimir friction, viz. on EM radiation emitted by macroscopic neutral bodies in relative motion [8].

Efforts were directed towards the calculation of dissipative components [9–12] of van der Waals forces between

macroscopic bodies. The question of emissive collisions between atoms was also raised: Are these forces conservative [13]? In this research, translational motion was mainly considered, with a general conclusion that the dissipated energy was negligible [14,15], especially in the zero-temperature case. On the contrary, we focus on rotational motion and find at zero temperature a non-negligible friction.

Zel’dovich [16] suggests that a rotating body could amplify an incoming EM radiation, thus losing its rotational energy and angular momentum at the expense of the field. The specific case of the scattering of a zero-point EM field by matter in rotation was studied decades later [17–19]. These authors treated the case of a rotating macroscopic body (dielectric or metallic) interacting with an EM field of variable temperature. Although nanoscopic materials could be considered, their EM characteristics were always represented by a dielectric constant, describing a linear first-order interaction within the material. The same is true with the usual treatment of fluctuation-induced interaction between atoms: The equations mainly deal with the perturbative term. Only recently, the self-consistent nonperturbative coupling was evoked [20].

Our approach combines both ways and considers two identical, neutral atoms rotating one around the other. The oscillator’s dipoles and the vacuum field [21] are fluctuating quantum quantities. Their combined dynamics is treated herein via a semiclassical framework derived from the fluctuation dissipation theorem (FDT) [22]. A similar approach was taken previously [18] and shown to be an alternative to quantum treatments [23]. Research herein concentrates on a zero-temperature field only for which Milonni [21] thoroughly discussed the fluctuation-dissipation

relation linking the vacuum field and an atomic dipole, already noticed by Callen and Welton [22].

In the frequency space, the FDT connects the self-correlations of a fluctuating physical quantity to the imaginary part of its response function. At zero temperature, the FDT for the components of the field at one location simply yields the self-correlation relation of the quantum vacuum:

$$\langle E_n[\omega]E_m[\omega'] \rangle = \frac{1}{3}\delta[\omega + \omega']\delta_{nm}\frac{\hbar|\omega|^3}{4\pi\epsilon_0\pi c^3}, \quad (1)$$

where the brackets $\langle \dots \rangle$ represent the average over the fluctuations, $\delta[\dots]$ is Dirac's function, and δ_{nm} is the Kronecker symbol. Herein, two atoms interacting with the vacuum field are put into a nonequilibrium situation: mutual attraction and symmetry-breaking rotation. The FDT permits calculating the nonequilibrium behavior of the system from its equilibrium fluctuations. We will be using Eq. (1) to obtain the torque exerted by the field on the rotating pair of atoms. Before that, we need to express the polarizability of two rotating atoms.

A harmonic oscillator of natural angular frequency ω_0 damped by the radiation reaction describes an atom in its ground state. Applying [24,25] Newton's second law in the instantaneous inertial frame of the oscillator gives the Abraham-Lorentz equation, which is Fourier transformed to obtain the atom's polarizability $\alpha[\omega]$:

$$\alpha[\omega] = \frac{q^2}{\mu} \frac{1}{\omega_0^2 - \omega^2 - i\tau\omega^3}, \quad (2)$$

where q is the charge in SI units, μ the reduced mass of the electron-nucleus system, and τ the radiation reaction time [24,26] defined by $\tau = \{[2(q^2/4\pi\epsilon_0)]/3\mu c^3\}$, where c is the speed of light in vacuum. The polarizability [Eq. (2)] relates the Fourier component of an atomic dipole to that of the local field that it experiences in an inertial frame. Long-standing research [27,28] has tackled the well-known pathologies of the Abraham-Lorentz equation, related to the impossibility of a point electron in classical physics. Corrections to the Abraham-Lorentz approximation were developed in order to obtain a viable classical equation [28]. Contrary to this point of view, our semiclassical calculation treats dipoles immersed into a fluctuating vacuum field. This method allows approaching numerous properties of atoms interacting with the quantum vacuum [21]. Importantly, the FDT (1) is obtained [22] by considering equilibrium between a fluctuating field and atomic dipoles, the polarizability of which follows Eq. (2).

Now, two atoms rotate around their common center of mass. The distance separating them is r , and their angular velocity is Ω , which is very small compared to ω_0 . Applying Newton's law to each atom in the inertial laboratory frame (in uppercase letters in Fig. 1) yields two relations:

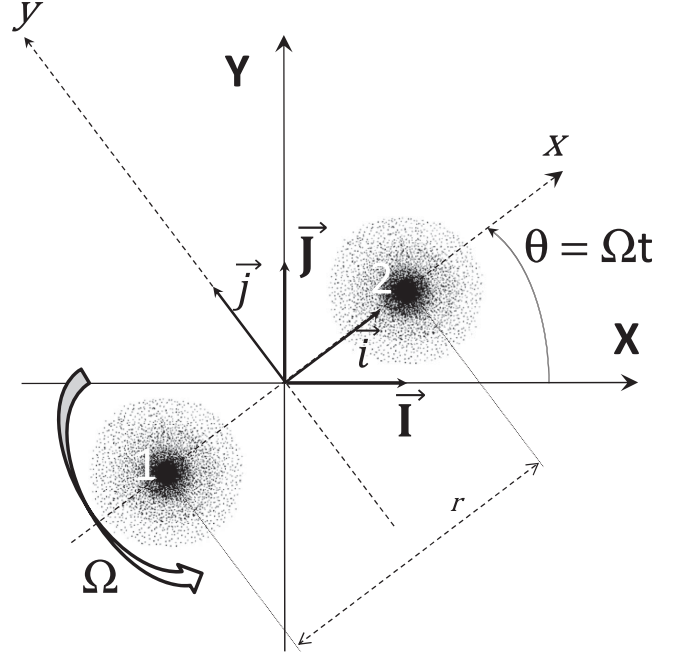


FIG. 1. A rotating pair of atoms. The atomic pair represented in its rotation plane. In the inertial frame $\vec{I}, \vec{J}, \vec{K}$ (upper case letters), the segment linking the two oscillators turns with angular velocity Ω ; its unit vector, fixed in the rotating frame $\vec{i}, \vec{j}, \vec{k}$ (italic lowercase letters), is $\vec{i} = \cos\theta\vec{I} + \sin\theta\vec{J}$ with $\theta = \Omega t$.

$$\ddot{\vec{P}}_1 - \tau\ddot{\vec{P}}_1 + \omega_0^2\vec{P}_1 = \frac{q^2}{\mu} \left(\vec{E}_1 + \frac{1}{4\pi\epsilon_0} \frac{[3\vec{i}(\vec{i} \cdot \vec{P}_2) - \vec{P}_2]}{r^3} \right), \quad (3a)$$

$$\ddot{\vec{P}}_2 - \tau\ddot{\vec{P}}_2 + \omega_0^2\vec{P}_2 = \frac{q^2}{\mu} \left(\vec{E}_2 + \frac{1}{4\pi\epsilon_0} \frac{[3\vec{i}(\vec{i} \cdot \vec{P}_1) - \vec{P}_1]}{r^3} \right). \quad (3b)$$

Each atom j feels a total field which is the sum of the fluctuating zero-point vacuum field \vec{E}_j at its position and the electrostatic dipolar field caused by the companion dipole. These equations include the well-known London-van der Waals attraction [20], whether the atoms rotate or not. In the following, the attraction causes the centripetal acceleration. Distances between atoms remain short (i.e., $\omega_0 r/c \ll 1$) in order to neglect the propagation of EM fields and to simplify calculations. Yet, r remains significantly greater than a_0 (the Bohr radius) in order to neglect atomic repulsion. This hypothesis implies $\vec{E}_1 = \vec{E}_2 \equiv \vec{E}$.

Adding Eqs. (3a) and (3b) gives the total dipole $\vec{P} = \vec{P}_1 + \vec{P}_2$ which obeys

$$\begin{aligned} \ddot{\vec{P}} - \tau\ddot{\vec{P}} + \omega_0^2\vec{P} &= \frac{2q^2}{\mu} \vec{E} + \frac{q^2}{4\pi\epsilon_0\mu r^3} [3(\cos\theta\vec{I} + \sin\theta\vec{J})(P_x\cos\theta + P_y\sin\theta) \\ &\quad - (P_x\vec{I} + P_y\vec{J} + P_z\vec{K})]. \end{aligned} \quad (4)$$

After Fourier transform of Eq. (4) and some tedious algebra (cf. Ref. [24], Sec. 2), the components of \vec{P} result:

$$P_x[\omega] = \alpha_{xy}[\omega] \left(\frac{G_+[\omega] + \frac{3}{2} \frac{\alpha_{xy}[\omega+2\Omega]}{4\pi\epsilon_0 r^3} G_-[\omega+2\Omega]}{1 - \frac{9}{4} \frac{\alpha_{xy}[\omega+2\Omega]\alpha_{xy}[\omega]}{(4\pi\epsilon_0 r^3)^2}} + \frac{G_-[\omega] + \frac{3}{2} \frac{\alpha_{xy}[\omega-2\Omega]}{4\pi\epsilon_0 r^3} G_+[\omega-2\Omega]}{1 - \frac{9}{4} \frac{\alpha_{xy}[\omega-2\Omega]\alpha_{xy}[\omega]}{(4\pi\epsilon_0 r^3)^2}} \right), \quad (5a)$$

$$P_y[\omega] = -i\alpha_{xy}[\omega] \left(\frac{G_+[\omega] + \frac{3}{2} \frac{\alpha_{xy}[\omega+2\Omega]}{4\pi\epsilon_0 r^3} G_-[\omega+2\Omega]}{1 - \frac{9}{4} \frac{\alpha_{xy}[\omega+2\Omega]\alpha_{xy}[\omega]}{(4\pi\epsilon_0 r^3)^2}} - \frac{G_-[\omega] + \frac{3}{2} \frac{\alpha_{xy}[\omega-2\Omega]}{4\pi\epsilon_0 r^3} G_+[\omega-2\Omega]}{1 - \frac{9}{4} \frac{\alpha_{xy}[\omega-2\Omega]\alpha_{xy}[\omega]}{(4\pi\epsilon_0 r^3)^2}} \right), \quad (5b)$$

$$P_z[\omega] = 2 \frac{q^2}{\mu} \frac{1}{\omega_z^2 - \omega^2 - i\tau\omega^3} E_z[\omega], \quad (5c)$$

defining $G_+[\omega] = E_x[\omega] + iE_y[\omega]$, $G_-[\omega] = E_x[\omega] - iE_y[\omega]$, $\alpha_{xy}[\omega] = \{q^2/[\mu(\omega_{xy}^2 - \omega^2 - i\tau\omega^3)]\}$, $\omega_{xy}^2 = \omega_0^2(1 - [q^2/2\mu\omega_0^2 4\pi\epsilon_0 r^3]) = \omega_0^2(1 - [\alpha_0/2r^3])$, and $\omega_z^2 = \omega_0^2(1 + [\alpha_0/r^3])$ with the usual definition $\alpha_0 = [(q^2/4\pi\epsilon_0)/\mu\omega_0^2]$. By letting ω_0 be the ionization frequency of the hydrogen atom and the volume α_0 be $4a_0^3$, values of r larger than a_0 result. For example, if $r > 5a_0$, then $(\alpha_0/r^3) \ll 1$ results. Equations (5a) and (5b) express the generalized susceptibility of \vec{P} , which mixes both field components at three different frequencies ($\omega, \omega \pm 2\Omega$).

The total electric field exerts a torque on the oscillators' dipole, the fluctuation-averaged value of which $\vec{\Gamma}^{\text{SC}}$ is given by

$$\begin{aligned} \vec{\Gamma}^{\text{SC}} &= \langle \vec{P}[t] \wedge \vec{E}^{\text{SC}}[t] \rangle \\ &= \int_{\omega=-\infty}^{+\infty} \int_{\omega'=-\infty}^{+\infty} \langle \vec{P}[\omega] \wedge \vec{E}^{\text{SC}}[\omega'] \rangle e^{-i\omega t} e^{-i\omega' t} d\omega d\omega', \end{aligned} \quad (6)$$

where \vec{E}^{SC} is the total (self-consistent) field seen by the atom, viz. the sum of the vacuum field and the induced field:

$$\vec{E}^{\text{SC}}[\omega'] = \frac{\vec{P}[\omega']}{2\alpha[\omega']}. \quad (7)$$

We can now use (5a), (5b), (7), and (1) to express [24] the integrand of Eq. (6) as the sum of two integrals on all modes of the vacuum field:

$$\vec{\Gamma}^{\text{SC}} = (\Gamma_1^{\text{SC}} + \Gamma_2^{\text{SC}}) \vec{K}, \quad (8a)$$

$$\Gamma_1^{\text{SC}} = \int_{-\infty}^{+\infty} i \frac{|\alpha_{xy}[\omega]|^2}{\alpha^*[\omega]} \frac{2}{3} \frac{\hbar}{4\pi\epsilon_0 \pi c^3} |\omega|^3 \left(\frac{1}{|1 - \frac{9}{4} \frac{\alpha_{xy}[\omega+2\Omega]\alpha_{xy}[\omega]}{(4\pi\epsilon_0 r^3)^2}|^2} - \frac{1}{|1 - \frac{9}{4} \frac{\alpha_{xy}[\omega-2\Omega]\alpha_{xy}[\omega]}{(4\pi\epsilon_0 r^3)^2}|^2} \right) d\omega, \quad (8b)$$

$$\Gamma_2^{\text{SC}} = \int_{-\infty}^{+\infty} i \frac{|\alpha_{xy}[\omega]|^2}{\alpha^*[\omega]} \frac{2}{3} \frac{\hbar}{4\pi\epsilon_0 \pi c^3} \left(\frac{|\frac{3}{2} \frac{\alpha_{xy}[\omega+2\Omega]}{4\pi\epsilon_0 r^3}|^2 |\omega+2\Omega|^3}{|1 - \frac{9}{4} \frac{\alpha_{xy}[\omega+2\Omega]\alpha_{xy}[\omega]}{(4\pi\epsilon_0 r^3)^2}|^2} - \frac{|\frac{3}{2} \frac{\alpha_{xy}[\omega-2\Omega]}{4\pi\epsilon_0 r^3}|^2 |\omega-2\Omega|^3}{|1 - \frac{9}{4} \frac{\alpha_{xy}[\omega-2\Omega]\alpha_{xy}[\omega]}{(4\pi\epsilon_0 r^3)^2}|^2} \right) d\omega, \quad (8c)$$

where we have been using the isotropy of the vacuum field, with the * referring to the complex conjugate. Γ_2^{SC} is a negligibly small quantity [24] which happens to be the only torque remaining in a first-order perturbative treatment of Eqs. (3a) and (3b). Γ_1^{SC} depends entirely on the self-consistent interaction.

Using a few changes of variable and developing in Taylor series (cf. Ref. [24], Sec. 5) with respect to the small parameters Ω/ω_0 and $\beta = \tau\omega_0$, one obtains

$$\Gamma_1^{\text{SC}} \cong -\frac{9}{8} \tau\omega_0 \left(\frac{\alpha_0}{r^3} \right)^2 \hbar\Omega, \quad (9)$$

where the factor $\tau\omega_0$ equals nearly the third power of the fine structure constant [24], numerically about 10^{-7} . Although Eq. (9) occurs through a Taylor expansion of the integrand of Eq. (8b), its validity is much wider than the quality of the

expansion could suggest. The numerical integration of (8b) yields nearly exactly (9) for numerous tested values in the range $(\Omega/\omega_0) \leq 10^{-3}$ and $(\alpha_0/r^3) \ll 10^{-1}$. The torque (9) was obtained as the effect of the total (vacuum + induced) field on the self-consistent dipoles. The same result occurs when one considers the effect of the vacuum field only on those dipoles, within first order in Ω/ω_0 .

The braking torque given by Eq. (9) is the main result of this Letter. The equivalent tangential braking force is very small compared to the van der Waals attractive force, their ratio being of the order of $\tau\Omega$: For example, $\tau\Omega \approx 10^{-11}$ if $(\Omega/\omega_0) \approx 10^{-4}$. Nevertheless, this torque decreases the kinetic energy and the angular momentum of the atoms. A linear torque gives rise to a temporally exponential attenuation of the angular momentum with a characteristic time T , depending on r :

$$T[r] = \frac{\frac{1}{2}Mr^2}{\frac{9}{8}\left(\frac{\alpha_0}{r}\right)^2\tau\omega_0\hbar} \quad (10)$$

with M the mass of one atom. T is on the order of 10^{-2} s for the numerical case already considered, with $r \approx 5a_0$ and M the mass of the hydrogen atom. Before discussing the physical results, two comments on the order of magnitude are warranted. On the one hand, the braking time given by Eq. (10) is rather long compared to the duration of most atomic collisions, generally on the order of 10^{-10} s or shorter, but it could be relevant when macroscopic processes are at stake. In particular, this quantum friction effect should be considered as a noticeable contribution to energy dissipation and entropy growth in gaseous systems. Within our theoretical development, the friction phenomenon shall be present in any atomic, or molecular, interaction. Thus, the standard explanation of irreversibility should be revised: Instead of resting on probability considerations, it could be derived from the universal existence of dynamical friction forces between atomic structures. This attractive task is nevertheless secondary compared to the experimental testing of Eq. (9), which is briefly discussed below.

On the other hand, the braking torque (9) might be relevant macroscopically, but it is small enough to have stayed unnoticed and to have been overlooked in the past. Let us now compare this theoretical result with other published work and discuss its consequences and testability.

As recalled above, Casimir friction at zero temperature was previously considered for rectilinear motion mainly. In the case of a metallic plate sliding at a fixed distance from a second similar plate [11,29] with a relative velocity v , the friction force was found scaling as v^3 . A cubic power law in velocity was also obtained recently [30,31] for the atom-surface drag force at zero temperature, in contrast to several different results previously published on this Casimir-Polder configuration. The material and geometric hypotheses of Refs. [11] and [29–31] differ from the case of two rotating atoms for which we find a friction torque (9) linear in the azimuthal velocity $r\Omega$. The discrepancy is thus not surprising, but more work would be necessary to explain it. In a different perspective, also considering a pure translational motion, Høye and Brevik [14] and independently Barton [15] calculated the energy loss in an atomic collision. They neglected the effect of van der Waals attraction on the trajectory, consequently forbidding any rotation of the atoms. They found a very small friction at zero temperature, which varied as $\exp[-v]$ and was totally negligible for nonrelativistic velocities. By taking the effect of van der Waals forces into account and integrating the full electrostatic coupling, we find a very different result, linear in $r\Omega$.

Previous calculations also considered rotating media [18,19]. They found the effect of vacuum friction to be negligible on isolated dielectrics in rotation. The interaction between the atoms in those materials was considered to be at equilibrium, giving rise to a polarizability, or dispersion

relation, insensitive to thermal and mechanical parameters. Herein, on the contrary, the dissipative torque results from the strong dependence of the two atoms' polarizability on the interatomic distance and from the self-consistent treatment of their interactions. It is fair to note that a different polarizability function would result in a different velocity dependence. In another configuration, a conductive sphere rotating near a surface [32] experiences a frictional torque scaling as $(r\Omega)^3$ at zero temperature and as $r\Omega$ at high temperatures.

Further work is needed to give a comprehensive description of quantum friction in all these diverse configurations where the physics of momentum transfer is similar: Virtual photons are exchanged by the atoms, resulting on average in momentum loss by the material system. A related concern of former work is of interest to our result. According to Ref. [19], a rotating body would drag along nearby objects and share its angular momentum with them, through the vacuum field. Herein, the question is, “How and how much can a rotating pair of atoms influence the motion of another pair in the vicinity?” This drives attention to the physical ways by which the energy is radiated away.

The radiation reaction term leads to energy and angular momentum loss, and it involves the emission of an outgoing wave. This radiation takes place via photon emission, which cannot be tackled within the present semiclassical framework. The emission phenomenon should be the subject of further work. Nevertheless, we can still deduce two properties of this EM emission. First, it is characterized by its energy and angular momentum outflow, the ratio of which is the average frequency Ω . Second, due to the symmetry of the system, the emission process shall not carry any linear momentum.

Apart from experimental tests by detection of EM emission, the consequences of (9) should be studied in the mechanism for dimer formation [33]. With strictly conservative interactions, a third body is needed to induce the capture of one atom by another. Figure 2 illustrates how Eq. (9) can change the situation of a binary collision. Because of the $1/r^6$ attractive potential, the two atoms will classically experience a so-called “centrifugal barrier” of height depending on their angular momentum (cf. Ref. [33], Sec. 4.2). At or near the barrier, the two atoms orbit extensively, thus resembling the case of Fig. 1. Slowed down by (9), the atoms can “fall” into one of the bound dimer states.

A barrier towards the experimental test of the torque (9) is its smallness, due in part to the atomic polarizability α_0 of the order of the atomic volume a_0^3 . However, the effective size of the atom depends on its excitation level. For example, very recent experiments [34] provided the first direct measurement of the attractive van der Waals force between atoms in Rydberg states, the size of which is much larger than a_0 . The same kind of system could be used to detect a potential dissipative torque.

The vacuum friction expressed in Eq. (9) is strong enough to induce a paradigm change in the explanation of

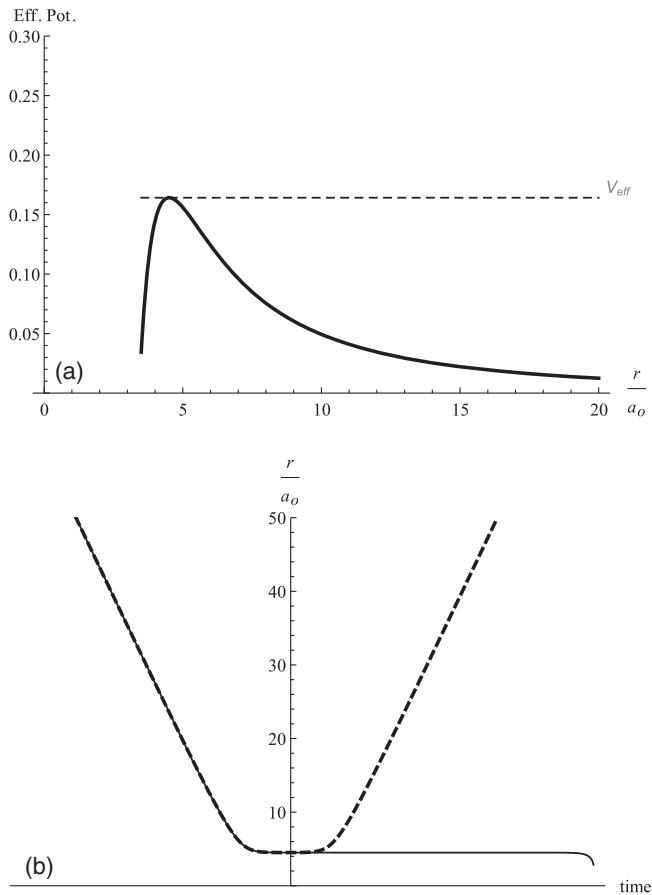


FIG. 2. Effect of the dissipative torque on a binary collision. (a) The effective potential (van der Waals + centrifugal) seen by the two-atom system depends on the angular momentum L . Effective energy for $L^2 = 10\hbar^2$ (solid line); the apex is at $(r/a_0) \sim 4.5$ for an effective energy V_{eff} (dashed line). (b) Numerical integration of the classical dynamics of the two atoms with $L^2 = 10\hbar^2$ and effective energy slightly under V_{eff} . The plain line takes into account the dissipative torque (9), while the dashed line does not. In both cases, the system is “orbiting” at $(r/a_0) \sim 4.5$. If no energy is lost to the field, the dynamics is reversible and the atoms finally separate. Vacuum friction prevents the separation and finally induces the two atoms to fall one towards the other.

irreversibility. But any attempt to reach such a goal should be aware of two other pending jobs. On the one hand, experimental testing awaits the design of dedicated experiments. On the other hand, further steps on the theoretical side should include the extension of this semiclassical calculation with the tools of quantum electrodynamics.

We benefited from interesting discussions with F. Ladieu, F. Daviaud, Y. Pomeau, M. Le Berre, H. Chaté, H. Desvaux, L. Desvilletes, B. Dubrulle, G. Ferrando, F. Graner, A. Lambrecht, K. Mallick, J.-M. Mestdagh, S. Reynaud, and C. Rountree. We very much appreciated a helpful reading and calculation by an anonymous referee. CEA provided its support through the DSM-Énergie program.

*herve.bercegol@cea.fr

- [1] C. Cohen-Tannoudji, J. Dupont-Roc, and G. Grynberg, *Atom-Photon Interactions: Basic Processes and Applications* (Wiley, New York, 1992).
- [2] L. D. Landau and E. M. Lifshitz, *Statistical Physics, Part I. Course of Theoretical Physics* (Pergamon, Oxford, 1980), Vol. 5.
- [3] L. Frommhold, *Collision-Induced Absorption in Gases* (Cambridge University Press, Cambridge, England, 1993).
- [4] F. London, *Z. Phys.* **63**, 245 (1930).
- [5] H. B. G. Casimir and D. Polder, *Phys. Rev.* **73**, 360 (1948).
- [6] H. B. G. Casimir, *Proc. K. Ned. Akad. Wet.* **51**, 150 (1948).
- [7] S. K. Lamoreaux, *Phys. Rev. Lett.* **78**, 5 (1997).
- [8] M. Kardar and R. Golestanian, *Rev. Mod. Phys.* **71**, 1233 (1999).
- [9] J. Mahanty, *J. Phys. B* **13**, 4391 (1980).
- [10] L. Levitov, *Europhys. Lett.* **8**, 499 (1989).
- [11] J. B. Pendry, *J. Phys. Condens. Matter* **9**, 10301 (1997).
- [12] A. I. Volokitin and B. N. Persson, *Rev. Mod. Phys.* **79**, 1291 (2007).
- [13] P. O. Westlund and H. Wennerström, *Phys. Rev. A* **71**, 062106 (2005).
- [14] J. S. Høye and I. Brevik, *Europhys. Lett.* **91**, 60003 (2010).
- [15] G. Barton, *New J. Phys.* **12**, 113044 (2010).
- [16] Ya. B. Zel’Dovich, *JETP Lett.* **14**, 180 (1971).
- [17] Y. Pomeau, *Europhys. Lett.* **74**, 951 (2006).
- [18] A. Manjavacas and F. J. García de Abajo, *Phys. Rev. Lett.* **105**, 113601 (2010).
- [19] M. F. Maghrebi, R. L. Jaffe, and M. Kardar, *Phys. Rev. Lett.* **108**, 230403 (2012).
- [20] P. R. Berman, G. W. Ford, and P. W. Milonni, *Phys. Rev. A* **89**, 022127 (2014).
- [21] P. W. Milonni, *The Quantum Vacuum: An Introduction to Quantum Electrodynamics* (Academic, San Diego, 1994).
- [22] H. B. Callen and T. A. Welton, *Phys. Rev.* **83**, 34 (1951).
- [23] A. Manjavacas and F. J. García de Abajo, *Phys. Rev. A* **82**, 063827 (2010).
- [24] See Supplemental Material at <http://link.aps.org/supplemental/10.1103/PhysRevLett.115.090402> for detailed calculations.
- [25] J. D. Jackson, *Classical Electrodynamics* (Wiley, New York, 1999).
- [26] R. P. Feynman, R. B. Leighton, and M. Sands, *The Feynman Lectures on Physics, Vol. 2: Mainly Electromagnetism and Matter* (Addison-Wesley, Reading, MA, 1964).
- [27] G. W. Ford and R. F. O’Connell, *Phys. Lett. A* **157**, 217 (1991).
- [28] A. M. Steane, *Am. J. Phys.* **83**, 256 (2015).
- [29] J. S. Høye and I. Brevik, *Eur. Phys. J. D* **68**, 61 (2014).
- [30] F. Intravaia, R. O. Behunin, and D. A. R. Dalvit, *Phys. Rev. A* **89**, 050101 (2014).
- [31] F. Intravaia, V. E. Mkrtchian, S. Y. Buhmann, S. Scheel, D. A. Dalvit, and C. Henkel, *J. Phys. Condens. Matter* **27**, 214020 (2015).
- [32] R. Zhao, A. Manjavacas, F. J. García de Abajo, and J. B. Pendry, *Phys. Rev. Lett.* **109**, 123604 (2012).
- [33] G. C. Maitland, M. Rigby, E. B. Smith, and W. A. Wakeham, *Intermolecular Forces* (Clarendon, Oxford, 1981).
- [34] L. Béguin, A. Vernier, R. Chicireanu, T. Lahaye, and A. Browaeys, *Phys. Rev. Lett.* **110**, 263201 (2013).