Landau Theory of Helical Fermi Liquids

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We construct a phenomenological Landau theory for the two-dimensional helical Fermi liquid found on the surface of a three-dimensional time-reversal invariant topological insulator. In the presence of rotation symmetry, interactions between quasiparticles are described by ten independent Landau parameters per angular momentum channel, by contrast with the two (symmetric and antisymmetric) Landau parameters for a conventional spin-degenerate Fermi liquid. We project quasiparticle states onto the Fermi surface and obtain an effectively spinless, projected Landau theory with a single projected Landau parameter per angular momentum channel that captures the spin-momentum locking or nontrivial Berry phase of the Fermi surface. As a result of this nontrivial Berry phase, projection to the Fermi surface can increase or lower the angular momentum of the quasiparticle interactions. We derive equilibrium properties, criteria for Fermi surface instabilities, and collective mode dispersions in terms of the projected Landau parameters. We briefly discuss experimental means of measuring projected Landau parameters.

The Landau theory of Fermi liquids (FLs) [1], or FL theory for short, is the cornerstone of our understanding of weakly correlated, gapless Fermi systems at low temperatures, such as ^{3}He atoms in the normal liquid state and itinerant electrons in metals. FL theory explains the puzzling observation that despite strong interactions between the constituent fermions, many Fermi systems behave essentially as free Fermi gases, except for the renormalization of their physical properties, which is captured by dimensionless quantities known as Landau parameters. These Landau parameters describe how the elementary excitations of the FL—the quasiparticles and quasiholes—interact with one another.

Topological insulators [2] provide new types of gapless Fermi systems: topological surface or edge states. In the absence of interparticle interactions, electrons propagating on the edge of a two-dimensional (2D) topological insulator [3] form a 1D helical Fermi gas [4]. In the presence of interactions, the 1D helical Fermi gas becomes a 1D helical Luttinger liquid [5] with no sharply defined Fermi points. In 3D topological insulators, surface electrons form a 2D helical Fermi gas [6], which is expected to evolve adiabatically into a 2D helical FL in the presence of electron-electron interactions.

This Letter presents a FL theory for the interacting 2D surface states of the 3D topological insulator. To our knowledge, such a helical FL theory has been missing in the literature despite the recent surge of interest in the effects of electron-electron interactions in topological insulators [7]. In the spirit of standard FL theory [1], we focus on systems with a discrete time-reversal symmetry, the protecting symmetry of topological insulators, as well as continuous translation and spatial rotation symmetries. We further consider the simplest case of a single surface Fermi surface—denoted simply as the Fermi surface in the following—which by rotation symmetry must be circular. This does not apply to certain topological insulators whose Fermi surface is strongly anisotropic, such as Bi_2Te_3 with 0.67% Sn doping [8] where there are large hexagonal warping effects due to the rhombohedral crystal structure of the bulk material [9]. However, in several other topological insulators such as Bi_2Se_3 [10], Bi_2Te_2Se [11], $Sb_xBi_{2-x}Se_2Te$ [11], $Bi_{1.5}Sb_{0.5}Te_{1.7}Se_{1.3}$ [12], $Tl_{1-x}Bi_{1+x}Se_{2-\delta}$ [13], strained α-Sn on InSb(001) [14], and strained HgTe [15], the Fermi surface as observed in angle-resolved photoemission spectroscopy (ARPES) is very nearly circular. However, due to spin-momentum locking in the topological surface states [6]—a consequence of strong spin-orbit coupling, rotation symmetry in a helical FL must necessarily involve spin degrees of freedom, which leads to a theory rather different from that of the conventional spin-degenerate FL. Moreover, the existence of a single nondegenerate Fermi surface—a consequence of the topological character of the bulk—eventually leads, via the application of the general principles of FL theory, to an effectively spinless FL theory. The physical properties of the resulting helical FL are nevertheless distinct from those of a truly spinless FL, due to a nontrivial mapping between physical, spinful quasiparticles and the effective, spinless quasiparticles. For the same reason, our helical FL theory is also qualitatively different from recently constructed FL theories of nontopological spin-orbit coupled systems such as the Rashba 2D electron gas [16] and 3D spin-orbit coupled metals [17], which are characterized by two (spin-split) Fermi surfaces.

FL theory views the many-fermion system as a gas of elementary excitations above the ground state, the quasiparticles. Because translation symmetry is assumed, the momentum $\boldsymbol{p} = (p_x, p_y)$ of the quasiparticles is well defined and a configuration of quasiparticles is specified by a distribution function $n_{\boldsymbol{p}}$. In a conventional FL, spin is conserved and the distribution function is diagonal in spin space, $n_{\boldsymbol{p}\sigma} = \langle c_{\boldsymbol{p}\sigma}^{\dagger} c_{\boldsymbol{p}\sigma} \rangle$, where $c_{\boldsymbol{p}\sigma}^{\dagger}$ ($c_{\boldsymbol{p}\sigma}$) is a creation (annihilation) operator for a fermion with momentum $\boldsymbol{p}$ and spin $\sigma = \uparrow, \downarrow$, but in systems with spin-orbit coupling such as the helical FL the distribution function is generally a matrix in spin space, $n_{\boldsymbol{p}}^{\alpha\beta} = \langle c_{\boldsymbol{p}\alpha}^{\dagger} c_{\boldsymbol{p}\beta} \rangle$ [17]. The central quantity in FL theory is the energy δE of the gas of interacting quasiparticles relative to the ground-state energy, expressed as a functional of the deviation $\delta n_{\boldsymbol{p}}^{\alpha\beta} \equiv n_{\boldsymbol{p}}^{\alpha\beta} - n_{\boldsymbol{p}}^{(0)\alpha\beta}$ of the distribution function from its value in the ground state

$$\delta E[\delta n_{\boldsymbol{p}}] = \int đ p \, h_{\alpha\beta}(\boldsymbol{p}) \delta n_{\boldsymbol{p}}^{\alpha\beta}$$
$$+ \frac{1}{2} \int đ p \, đ p' V_{\alpha\beta;\gamma\delta}(\hat{\boldsymbol{p}}, \hat{\boldsymbol{p}}') \delta n_{\boldsymbol{p}}^{\alpha\beta} \delta n_{\boldsymbol{p}'}^{\gamma\delta}, \quad (1)$$

where (working in units such that $\hbar = 1$)

$$h(\boldsymbol{p}) = v_F \hat{z} \cdot (\boldsymbol{\sigma} \times \boldsymbol{p}) \quad (2)$$

is the single-particle Dirac Hamiltonian of the topological surface state [2] with v_F the Fermi velocity [18]; $V_{\alpha\beta;\gamma\delta}(\hat{\boldsymbol{p}}, \hat{\boldsymbol{p}}')$ is a reduced two-body interaction that depends only on the unit vector $\hat{\boldsymbol{p}} \equiv \boldsymbol{p}/|\boldsymbol{p}|$ parametrizing the Fermi surface, and we denote the integration measure by $\int đ p \equiv \int [(d^2 p)/(2\pi)^2]$. The form of Eq. (1) can be obtained from a generic, translationally invariant interaction $V_{\alpha\beta;\gamma\delta}(\boldsymbol{k}, \boldsymbol{k}', \boldsymbol{q})$ by requiring that all fermionic momenta lie on the Fermi surface [19].

Our first goal is to derive the most general form of the two-body interaction $V_{\alpha\beta;\gamma\delta}(\hat{\boldsymbol{p}}, \hat{\boldsymbol{p}}')$ consistent with the general principles of quantum mechanics and the symmetries of the problem. This goal is most easily achieved by expanding the two-body interaction as

$$V_{\alpha\beta;\gamma\delta}(\hat{\boldsymbol{p}}, \hat{\boldsymbol{p}}') = \sum_{\mu,\nu=0}^{3} \sum_{l,l'=-\infty}^{\infty} V_{\mu\nu}^{ll'} e^{i(l\theta_{\boldsymbol{p}} + l'\theta_{\boldsymbol{p}'})} \sigma_{\alpha\beta}^{\mu} \sigma_{\gamma\delta}^{\nu}, \quad (3)$$

where $\hat{\boldsymbol{p}} = (\cos\theta_{\boldsymbol{p}}, \sin\theta_{\boldsymbol{p}})$, l, l' are angular momentum quantum numbers, and the set of four 2×2 Hermitian matrices $\sigma^{\mu} = (1, \boldsymbol{\sigma})$, where 1 denotes the identity matrix, allows us to construct the quasiparticle charge $\delta\rho_{\boldsymbol{p}}$ and spin $\delta s_{\boldsymbol{p}}^i$ densities ($i = x, y, z$)

$$\delta\rho_{\boldsymbol{p}} = \sigma_{\alpha\beta}^{0} \delta n_{\boldsymbol{p}}^{\alpha\beta} = \delta_{\alpha\beta} \delta n_{\boldsymbol{p}}^{\alpha\beta}, \qquad \delta s_{\boldsymbol{p}}^i = \frac{1}{2}\sigma_{\alpha\beta}^i \delta n_{\boldsymbol{p}}^{\alpha\beta}. \quad (4)$$

Upon substituting Eq. (3) in Eq. (1), one obtains three classes of terms: charge-charge interactions proportional to $V_{00}^{ll'}$, spin-spin interactions proportional to $V_{ij}^{ll'}$, and spin-charge interactions proportional to $V_{0i}^{ll'} = V_{i0}^{l'l}$. Time-reversal symmetry implies that the angular momenta l and l' must differ by an even integer for charge-charge and spin-spin interactions and by an odd integer for spin-charge interactions [19].

The main difference between a conventional FL and a spin-orbit coupled FL such as the helical FL lies in the consequences of rotation symmetry. The single-particle Hamiltonian (2) is neither invariant under a spatial rotation nor invariant under a spin rotation, but is invariant under a simultaneous rotation of spatial and spin coordinates: $[J_z, h(\boldsymbol{p})] = 0$, where $J_z = -i(\partial/\partial\theta_{\boldsymbol{p}}) + \frac{1}{2}\sigma^z$ is the total (orbital plus spin) angular momentum in the z direction. Requiring that the interaction term in Eq. (1) be also invariant under such rotations, we find that it can be written as the sum of three terms δV_{cc}, δV_{sc}, and δV_{ss}, where [19]

$$\delta V_{\text{cc}} = \frac{1}{2} \sum_{l=0}^{\infty} \int đ p \, đ p' f_l^{\text{cc}} \cos l\theta_{\boldsymbol{p}\boldsymbol{p}'} \delta\rho_{\boldsymbol{p}} \delta\rho_{\boldsymbol{p}'} \quad (5)$$

is the charge-charge interaction,

$$\delta V_{\text{sc}} = \sum_{l=0}^{\infty} \int đ p \, đ p'$$
$$\times [(f_l^{\text{sc},1} \cos l\theta_{\boldsymbol{p}\boldsymbol{p}'} + f_l^{\text{sc},2} \sin l\theta_{\boldsymbol{p}\boldsymbol{p}'}) \delta\rho_{\boldsymbol{p}} \hat{\boldsymbol{p}}' \cdot \delta s_{\boldsymbol{p}'}$$
$$+ (f_l^{\text{sc},3} \cos l\theta_{\boldsymbol{p}\boldsymbol{p}'} + f_l^{\text{sc},4} \sin l\theta_{\boldsymbol{p}\boldsymbol{p}'}) \delta\rho_{\boldsymbol{p}} \hat{\boldsymbol{p}}' \times \delta s_{\boldsymbol{p}'}] \quad (6)$$

is the spin-charge interaction, and

$$\delta V_{\text{ss}} = \frac{1}{2} \sum_{l=0}^{\infty} \int đ p \, đ p' \{\cos l\theta_{\boldsymbol{p}\boldsymbol{p}'}(f_l^{\text{ss},1}(\delta s_{\boldsymbol{p}}^x \delta s_{\boldsymbol{p}'}^x + \delta s_{\boldsymbol{p}}^y \delta s_{\boldsymbol{p}'}^y) + f_l^{\text{ss},2} \delta s_{\boldsymbol{p}}^z \delta s_{\boldsymbol{p}'}^z) + f_l^{\text{ss},3} \sin l\theta_{\boldsymbol{p}\boldsymbol{p}'} \delta s_{\boldsymbol{p}} \times \delta s_{\boldsymbol{p}'}$$
$$+ \cos l\theta_{\boldsymbol{p}\boldsymbol{p}'}(f_l^{\text{ss},4}[(\hat{\boldsymbol{p}} \cdot \delta s_{\boldsymbol{p}})(\hat{\boldsymbol{p}}' \times \delta s_{\boldsymbol{p}'}) + (\hat{\boldsymbol{p}} \times \delta s_{\boldsymbol{p}})(\hat{\boldsymbol{p}}' \cdot \delta s_{\boldsymbol{p}'})] + f_l^{\text{ss},5}[(\hat{\boldsymbol{p}} \cdot \delta s_{\boldsymbol{p}})(\hat{\boldsymbol{p}}' \cdot \delta s_{\boldsymbol{p}'}) - (\hat{\boldsymbol{p}} \times \delta s_{\boldsymbol{p}})(\hat{\boldsymbol{p}}' \times \delta s_{\boldsymbol{p}'})])\} \quad (7)$$

is the spin-spin interaction. We denote by $\theta_{\boldsymbol{p}\boldsymbol{p}'} \equiv \theta_{\boldsymbol{p}'} - \theta_{\boldsymbol{p}}$ the relative angle between $\hat{\boldsymbol{p}}$ and $\hat{\boldsymbol{p}}'$, and write $\boldsymbol{a} \times \boldsymbol{b} \equiv \hat{z} \cdot (\boldsymbol{a} \times \boldsymbol{b})$ for the cross product of two in-plane vectors.

Equations (5)–(7), the first main result of this work, represent the most general short-range two-body interaction in a helical FL consistent with translation, rotation, and time-reversal symmetries. The interaction is specified by

ten real Landau parameters for each value of the relative angular momentum $l = 0, 1, 2, \ldots$: one charge-charge parameter f_l^{cc}, four spin-charge parameters $f_l^{sc,1}, \ldots, f_l^{sc,4}$, and five spin-spin parameters $f_l^{ss,1}, \ldots, f_l^{ss,5}$. This stands in contrast to the two Landau parameters f_l^s (spin symmetric) and f_l^a (spin antisymmetric) in a conventional FL [1], which would correspond to $f_l^s = f_l^{cc}$ and $f_l^a = \frac{1}{4} f_l^{ss,1} = \frac{1}{4} f_l^{ss,2}$ in the absence of spin-orbit coupling. In particular, spin-orbit coupling allows for a nonzero spin-charge interaction (6), which would be forbidden by separate spatial and spin rotation symmetries in a conventional FL. The spin-spin interaction (7) also exhibits novel features: $f_l^{ss,1} \neq f_l^{ss,2}$ in general, which corresponds to an XXZ interaction with Ising anisotropy rather than the conventional SU(2)-symmetric Heisenberg interaction, $f_l^{ss,3}$ is a Dzyaloshinskii-Moriya interaction, and $f_l^{ss,4}$, $f_l^{ss,5}$ are anisotropic spin-spin interactions similar to those found in compass models [22], but with a continuous rather than discrete spin-orbit rotation symmetry.

While Eqs. (5)–(7) in conjunction with Eq. (1) correctly describe the helical FL, in the spirit of FL theory one can go one step further and only retain electron states on the Fermi surface. Because of the strong spin-orbit coupling present in the Dirac Hamiltonian (2), such electrons are annihilated by the operator $\psi_{p\pm} = \frac{1}{\sqrt{2}}(ie^{-i\theta_p} c_{p\uparrow} \pm c_{p\downarrow})$, where positive (+) helicity corresponds to a positive Fermi energy $\epsilon_F > 0$ above the Dirac point, and negative (−) helicity corresponds to a negative Fermi energy $\epsilon_F < 0$. Inverting this relation, one can express the spin eigenoperators $c_{p\sigma}$ in terms of the helicity eigenoperators $\psi_{p\pm}$ as $c_{p\uparrow} = (ie^{-i\theta_p}/\sqrt{2})(\psi_{p+} + \psi_{p-})$ and $c_{p\downarrow} = \frac{1}{\sqrt{2}}(\psi_{p+} - \psi_{p-})$. Choosing $\epsilon_F > 0$ for definiteness, the Fermi surface consists exclusively of electron states of positive helicity, such that one may wish to drop the negative helicity eigenoperators ψ_{p-} entirely from these expressions for $c_{p\uparrow}$ and $c_{p\downarrow}$. Applying this procedure to Eq. (1) yields a Landau functional for an effectively spinless FL theory

$$\delta \bar{E}[\delta \bar{n}_p] = \int \mathrm{d}p\, \epsilon_p^0 \delta \bar{n}_p + \frac{1}{2} \sum_{l=0}^{\infty} \int \mathrm{d}p\, \mathrm{d}p'\, \bar{f}_l \cos l\theta_{pp'} \delta \bar{n}_p \delta \bar{n}_{p'},$$

(8)

where $\epsilon_p^0 = v_F |p|$ is the dispersion relation of positive helicity quasiparticles, $\delta \bar{n}_p = \bar{n}_p - \bar{n}_p^{(0)}$ with $\bar{n}_p \equiv \langle \psi_{p+}^\dagger \psi_{p+} \rangle$ is the distribution function for these quasiparticles, and $\bar{f}_l$ are effectively spinless, projected Landau parameters related to the ten unprojected Landau parameters previously discussed by

$$\bar{f}_l = f_l^{cc} - f_l^{sc,3} - \frac{1}{4} f_l^{ss,5} + \frac{1}{8}(f_{l-1}^{ss,1} - f_{l-1}^{ss,3} + f_{l+1}^{ss,1} + f_{l+1}^{ss,3}),$$

(9)

for $l = 0, 1, 2, \ldots$, with the definition $f_{-1}^{ss,1} = f_{-1}^{ss,3} \equiv 0$. The quasiparticle charge and spin densities (4) are given in terms of $\delta \bar{n}_p$ by

$$\delta \rho_p = \delta \bar{n}_p, \qquad \delta s_p^i = \frac{1}{2} \epsilon_{ij} \hat{p}_j \delta \bar{n}_p, \qquad i = x, y, \qquad \delta s_p^z = 0,$$

(10)

where the last two equalities express spin-momentum locking in the xy plane. Equations (8)–(10), together with the definitions of the unprojected Landau parameters in Eqs. (5)–(7), are the second main result of this work.

Before deriving the physical properties of the helical FL from the projected Landau functional (8), we pause to discuss a number of interesting features of the relationship (9) between projected and unprojected Landau parameters. The unprojected Landau parameters $f_l^{sc,1}$, $f_l^{sc,2}$, and $f_l^{ss,4}$ do not enter the projected interaction because spin and momentum are perpendicular on the Fermi surface ($\hat{p} \cdot \delta s_p = 0$) due to spin-momentum locking. The parameter $f_l^{sc,4}$ does not enter either because it produces a projected interaction that is odd under $p \leftrightarrow p'$, which is inconsistent with particle indistinguishability. The last term on the right-hand side of Eq. (9) shows that projection to the Fermi surface can effectively raise or lower the angular momentum of the unprojected interaction. For example, for $l = 1$ one has

$$\bar{f}_1 = f_1^{cc} - f_1^{sc,3} - \frac{1}{4} f_1^{ss,5} + \frac{1}{8}(f_0^{ss,1} - f_0^{ss,3} + f_2^{ss,1} + f_2^{ss,3});$$

(11)

that is, an isotropic, s-wave ($l = 0$) microscopic interaction can produce an anisotropic, p-wave ($l = 1$) effective interaction in the projected theory. This can be seen as the particle-hole counterpart to the effective p-wave interaction in the Bardeen-Cooper-Schrieffer (BCS) channel produced on the doped surface of a 3D topological insulator by a microscopic s-wave BCS interaction [23].

As in standard FL theory, many physical properties of the helical FL can be derived from the projected Landau functional (8). The simplest property is Luttinger's theorem [24], i.e., the relation $p_F = \sqrt{4\pi n}$ between Fermi momentum p_F and total density n of quasiparticles, which is also equal to the total density of electrons (defining a system with $p_F = 0$ as the vacuum). That Luttinger's theorem holds in its original form despite the presence of strong spin-orbit coupling is a consequence of the existence of a single helical Fermi surface, which is only possible on the surface of a 3D topological phase. Interactions in topologically trivial spin-orbit coupled systems such as the Rashba 2D electron gas can individually renormalize the Fermi momenta of the two spin-split Fermi surfaces [16]. Other equilibrium properties of the helical FL can be calculated from the quasiparticle energy ϵ_p, defined as the

functional derivative of the Landau functional with respect to the distribution function,

$$\epsilon_{\boldsymbol{p}} = \frac{\delta \bar{E}}{\delta \bar{n}_{\boldsymbol{p}}} = \epsilon_{\boldsymbol{p}}^0 + \sum_{l=0}^{\infty} \int đp' \bar{f}_l \cos l\theta_{\boldsymbol{pp'}} \delta \bar{n}_{\boldsymbol{p'}}. \quad (12)$$

From Eq. (12) one can follow the standard FL approach [19] to derive the electronic specific heat coefficient $\gamma \equiv c_v/T$ and electronic compressibility κ of the helical FL at zero temperature

$$\gamma = \frac{1}{3}\pi^2 k_B^2 \rho(\epsilon_F), \qquad \kappa = \frac{\rho(\epsilon_F)}{n^2} \frac{1}{1+\bar{F}_0}, \quad (13)$$

where we define dimensionless Landau parameters $\bar{F}_0 \equiv \rho(\epsilon_F)\bar{f}_0$ and $\bar{F}_l \equiv \frac{1}{2}\rho(\epsilon_F)\bar{f}_l$, $l = 1, 2, 3, \ldots$, with $\rho(\epsilon_F) = \epsilon_F/2\pi v_F^2$ the density of states of the helical FL at the Fermi energy $\epsilon_F = v_F p_F$. The compressibility becomes negative for $\bar{F}_0 < -1$, signaling an instability towards phase separation [25]. Unlike in a standard FL, here this condition can be reached not only for attractive density-density interactions, but also as a result of spin-charge or even purely spin-spin interactions, given the relation (9) between the projected and unprojected Landau parameters.

The renormalized Fermi velocity v_F differs in general from the Fermi velocity of noninteracting electrons v_F^0. This is similar in spirit to the renormalization of the quasiparticle mass in a standard FL. The derivation of the latter relies on Galilean invariance, while in the helical FL, Galilean invariance is broken by spin-orbit coupling. However, adiabatic continuity still implies that the total flux of quasiparticles is equal to the total flux of electrons [1]. The latter is calculated from the quantum-mechanical velocity operator for electrons $\mathbf{v}_e = v_F^0(\hat{z} \times \boldsymbol{\sigma})$, which, for momentum-independent microscopic interactions [26], is the same as in the absence of interactions [28]: it is a function of the noninteracting Fermi velocity, rather than the renormalized one. The total quasiparticle flux is a function of the quasiparticle velocity $\mathbf{v}_{\mathrm{qp}} = \nabla_{\boldsymbol{p}} \epsilon_{\boldsymbol{p}}$. Equating the two fluxes yields a relation between the two Fermi velocities [19]

$$\frac{v_F^0}{v_F} = 1 + \bar{F}_1, \quad (14)$$

which is the helical FL analog of the relation $(m^*/m) = 1 + \frac{1}{3}F_1^s$ between renormalized m^* and noninteracting m quasiparticle masses in a standard FL [1].

The spin susceptibility introduces some added subtleties: unlike in a standard FL, it is not, strictly speaking, a Fermi surface property. Indeed, it depends explicitly on a high-energy cutoff Λ already in the noninteracting limit [29,30]. In a standard FL, one can always choose the spin quantization axis to be parallel to the applied magnetic

field $\boldsymbol{B}$, such that the quasiparticle energy shift $\delta\epsilon_{\boldsymbol{p}\sigma} = \frac{1}{2}g\mu_B B\sigma$ due to Zeeman coupling (g is the g factor, μ_B is the Bohr magneton) is diagonal in the spin basis $\sigma = \pm 1$. The resulting change in occupation numbers is localized to the Fermi surface in the zero-field limit, causing the spin susceptibility to be a Fermi surface property. In the helical FL, there is no freedom to choose the spin quantization axis due to spin-momentum locking, and the Zeeman coupling contains off-diagonal terms in the helicity basis. The projected FL theory (8), which projects out negative helicity states, cannot take these off-diagonal terms into account and thus should not be expected to yield exact results for the spin susceptibility. Nevertheless, one can calculate the Fermi surface contribution to the spin susceptibility using Eq. (8) and compare it in the noninteracting limit to an exact calculation that takes both helicities into account. The spin susceptibility tensor χ_{ij} is found to be diagonal with in-plane $\chi_{xx} = \chi_{yy}$ and out-of-plane χ_{zz} components given by

$$\chi_{xx} = \frac{1}{8}g^2\mu_B^2\rho(\epsilon_F)\frac{1}{1+\bar{F}_1}, \qquad \chi_{zz} = 0, \quad (15)$$

in the projected FL theory, and

$$\chi_{xx} = \frac{1}{8}g^2\mu_B^2\rho(\Lambda), \qquad \chi_{zz} = \frac{1}{4}g^2\mu_B^2[\rho(\Lambda) - \rho(\epsilon_F)], \quad (16)$$

for the noninteracting Dirac surface state, including both helicities [19]. Thus, in the noninteracting limit, Eqs. (15) and (16) agree in the formal limit of large Fermi energy $\epsilon_F \to \Lambda$. By contrast with the spin susceptibility of the standard FL, which is renormalized by the spin-antisymmetric $l = 0$ Landau parameter F_0^a, here it is renormalized by a $l = 1$ Landau parameter due to spin-momentum locking on the Fermi surface.

Pomeranchuk instabilities [31] are instabilities of the Fermi surface towards spontaneous, static distortions of its shape. To study such instabilities in the helical FL, one characterizes distortions of the Fermi surface by an angle-dependent Fermi momentum, expanded in angular momentum components,

$$p_F(\theta) - p_F = \sum_{l=-\infty}^{\infty} A_l e^{il\theta}, \quad (17)$$

where $A_{-l} = A_l^*$ because $p_F(\theta)$ is real. Substituting this expression into the Landau functional (8), one finds that the energy $\delta\bar{E}$ remains positive, and thus the helical FL remains stable, if and only if [19]

$$\bar{F}_l > -1, \quad (18)$$

for all $l = 0, 1, 2, \ldots$. This is the same as Pomeranchuk's original criterion in 2D, but applied this time to the projected Landau parameters, which are nontrivial

functions of the unprojected ones. It contains as special cases the instability towards phase separation, already seen, as well as an instability towards in-plane magnetic order [32] for $\bar{F}_1 \to -1$ that is signaled by divergences of the in-plane spin susceptibility (15) and the renormalized Fermi velocity (14). The latter divergence also accompanies the $l = 1$ spin-symmetric Pomeranchuk instability of the standard FL [33]. The $l = 2$ instability is towards quadrupolar distortions of the helical Fermi surface, characterized in the projected FL theory by a nonzero value of the traceless, symmetric nematic order parameter $\bar{Q}_{ij} = \int \mathrm{d}p\, \bar{Q}_{ij}(\boldsymbol{p})$, where $\bar{Q}_{ij}(\boldsymbol{p}) = (2\hat{p}_i\hat{p}_j - \delta_{ij})\delta\bar{n}_{\boldsymbol{p}}$. This effectively spinless order parameter is identical to the one that describes nematic order in a standard spin-degenerate FL [34]. In the original unprojected theory, however, this translates into a nonzero value of $Q_{ij} = \int \mathrm{d}p\, Q_{ij}(\boldsymbol{p})$, where

$$Q_{ij}(\boldsymbol{p}) = \hat{p}_i \delta s_{\boldsymbol{p}}^j + \hat{p}_j \delta s_{\boldsymbol{p}}^i - \delta_{ij}\hat{\boldsymbol{p}} \cdot \delta\boldsymbol{s}_{\boldsymbol{p}} \qquad (19)$$

is a quadrupolar order parameter involving both spatial and spin degrees of freedom that was recently discussed in the context of possible instabilities of surface Majorana fermions in the topological superfluid ^{3}He-B [35] and 3D spin-orbit coupled metals [17,36]. Thus the quadrupolar distortion of a helical Fermi surface is necessarily accompanied by a time-reversal invariant form of magnetic order similar in spirit to spin nematic order [37].

Nonequilibrium properties of the helical FL such as collective modes can also be studied using the projected FL theory, assuming that the relaxation-time approximation is valid such that scattering between states of different helicities can be neglected. In the hydrodynamic regime $\omega\tau \ll 1$, where τ is the quasiparticle collision time, the helical FL supports ordinary sound waves (first sound) with velocity [19]

$$c_1 = v_F \sqrt{\frac{1}{2}(1 + \bar{F}_0)(1 + \bar{F}_1)}, \qquad (20)$$

while in the collisionless regime $\omega\tau \gg 1$ a zero sound mode may exist under certain conditions [28]. If $\bar{F}_0 > 0$ only is nonzero, the zero sound velocity is given in the limits of strong and weak interactions by [19]

$$c_0 \approx v_F \sqrt{\frac{1}{2}\bar{F}_0}, \qquad \bar{F}_0 \to \infty, \qquad (21)$$

$$c_0 \approx v_F \left(1 + \frac{1}{2}\bar{F}_0^2\right), \qquad \bar{F}_0 \to 0. \qquad (22)$$

We conclude by discussing prospects for the experimental determination of the projected Landau parameters $\bar{F}_l$. ARPES can determine p_F, which, via Luttinger's theorem, yields the density n. Using Eq. (13), $\bar{F}_0$ could then be inferred from measurements of the heat capacity and electronic compressibility of the surface states. The latter

can in principle be determined directly from the ARPES data or via single electron transistor microscopy [38]. To determine $\bar{F}_1$, one could perform a transient spin grating experiment [28] to generate a spin-density wave with momentum $\boldsymbol{q}$ and transverse amplitude $s_{\boldsymbol{q}}^T$. Because of spin-momentum locking, this will induce a density wave at the same momentum with amplitude $n_{\boldsymbol{q}}$. Assuming that Coulomb interactions are screened, the existence of an undamped sound mode at frequency $\omega = c_s q$ implies a relation between the two amplitudes [19]

$$\frac{s_{\boldsymbol{q}}^T}{n_{\boldsymbol{q}}} = \frac{1}{1 + \bar{F}_1}\frac{c_s}{v_F}, \qquad (23)$$

where c_s is either c_1 or c_0 depending on whether one is in the hydrodynamic or collisionless regime. Using Eqs. (20)–(22) one can extract $\bar{F}_1$ from a measurement of the amplitude ratio $s_{\boldsymbol{q}}^T/n_{\boldsymbol{q}}$ and previous knowledge of $\bar{F}_0$.

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