Penguin Contributions to *CP* Phases in $B_{d,s}$ Decays to Charmonium

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The precision of the *CP* phases 2β and $2\beta_s$ determined from the mixing-induced *CP* asymmetries in $B_d \to J/\psi K_S$ and $B_s \to J/\psi \phi$, respectively, is limited by the unknown long-distance contribution of a penguin diagram involving up quarks. The penguin contribution is expected to be comparable in size to the precision of the LHCb and Belle II experiments and, therefore, limits the sensitivity of the measured quantities to new physics. We analyze the infrared QCD structure of this contribution and find that all soft and collinear divergences either cancel between different diagrams or factorize into matrix elements of local four-quark operators up to terms suppressed by $\Lambda_{\rm QCD}/m_\psi$, where m_ψ denotes the J/ψ mass. Our results, which are based on an operator product expansion, allow us to calculate the penguin-to-tree ratio P/T in terms of the matrix elements of these operators and to constrain the penguin contribution to the phase 2β as $|\Delta\phi_d| \le 0.68°$. The penguin contribution to $2\beta_s$ is bounded as $|\Delta\phi_s^0| \le 0.97°$, $|\Delta\phi_s^\parallel| \le 1.22°$, and $|\Delta\phi_s^\perp| \le 0.99°$ for the case of longitudinal, parallel, and perpendicular ϕ and J/ψ polarizations, respectively. Further, we place bounds on $|\Delta\phi_d|$ for $B_d \to \psi(2S)K_S$ and the polarization amplitudes in $B_d \to J/\psi K^*$. In our approach, it is further possible to constrain P/T for decays in which P/T is Cabibbo unsuppressed, and we derive upper limits on the penguin contribution to the mixing-induced *CP* asymmetries in $B_d \to J/\psi \pi^0$, $B_d \to J/\psi \rho^0$, $B_s \to J/\psi K_S$, and $B_s \to J/\psi K^*$. For all studied decay modes, we also constrain the sizes of the direct *CP* asymmetries.

Introduction.—The mixing-induced *CP* asymmetry in the decay $B_d \to J/\psi K_S$ is the key quantity for measuring the *CP* phase of the B_d-$\bar{B}_d$ mixing amplitude. Within the standard model (SM), this *CP* asymmetry $A_{CP}^{B_d \to J/\psi K_S}(t)$ determines the angle $\beta = \arg[-V_{tb}V_{td}^*/(V_{cb}V_{cd}^*)]$ of the unitarity triangle. The *B* factories *BABAR* and Belle had been designed to measure $A_{CP}^{B_d \to J/\psi K_S}(t)$ to a high precision to probe the Kobayashi-Maskawa (KM) mechanism of *CP* violation. Within the SM, the KM phase is the only source of *CP* violation in weak transitions and, therefore, must correctly describe all *CP* asymmetries measured in weak hadron decays. The measurement of β at the *B* factories gave us sufficient confidence that the KM mechanism correctly describes *CP* violation in both *K* and B_d decays and led to the dedication of the 2008 Nobel Prize in Physics to Makoto Kobayashi and Toshihide Maskawa. Today's focus of flavor physics is the search for physics beyond the standard model which reveals itself in small deviations from the KM picture. In generic models of new physics, B-$\bar{B}$ mixing probes new physics associated with scales beyond 100 TeV; reducing the uncertainties of standard-model predictions is, therefore, of utmost importance. $B_s \to J/\psi \phi$ is the analogous key mode in the B_s-$\bar{B}_s$ system. Since the unitarity of the Cabibbo-Kobayashi-Maskawa (CKM) matrix essentially fixes $\beta_s = \arg[-V_{tb}^*V_{ts}/(V_{cb}V_{cs}^*)] = 1.0°$ to a very small value, *CP* studies of $B_s \to J/\psi \phi$ directly probe physics beyond the

SM. The decay amplitude A_f for an $\bar{b} \to \bar{c}c\bar{s}$ decay $B_q \to f$, where f is a *CP* eigenstate consisting of a charmonium state and a light meson, can be written as

$$A_f = \lambda_c^s T_f + \lambda_u^s P_f, \qquad (1)$$

with $\lambda_p^s = V_{pb}^* V_{ps}$, $p = u, c$, and

$$T_f = \frac{G_F}{\sqrt{2}} \langle f|C_1 Q_1^c + C_2 Q_2^c + \sum_j C_j Q_j|B_q\rangle, \qquad (2)$$

$$P_f = \frac{G_F}{\sqrt{2}} \langle f|C_1 Q_1^u + C_2 Q_2^u + \sum_j C_j Q_j|B_q\rangle. \qquad (3)$$

Here, $Q_1 = \bar{s}^\alpha \gamma_\mu (1 - \gamma_5) q^\beta \bar{q}^\beta \gamma^\mu (1 - \gamma_5) b^\alpha$ and $Q_2 = \bar{s}^\alpha \gamma_\mu (1 - \gamma_5) q^\alpha \bar{q}^\beta \gamma^\mu (1 - \gamma_5) b^\beta$ are the current-current operators. The index j labels the penguin operators Q_j which involve the CKM elements $\lambda_t^s = -\lambda_c^s - \lambda_u^s$. While the QCD penguin operators Q_{3-6} and Q_{8G} are important for this Letter (see Ref. [1] for their definition), electroweak penguin operators have negligible effects. The time-dependent *CP* asymmetry $A_{CP}^{B_q \to f}(t) \equiv [\Gamma(\bar{B}_q(t) \to f) - \Gamma(B_q(t) \to f)]/[\Gamma(\bar{B}_q(t) \to f) + \Gamma(B_q(t) \to f)]$ reads

$$A_{CP}^{B_q \to f}(t) = \frac{S_f \sin(\Delta M_q t) - C_f \cos(\Delta M_q t)}{\cosh(\Delta \Gamma_q t/2) + A_{\Delta \Gamma_q} \sinh(\Delta \Gamma_q t/2)}. \qquad (4)$$

Here, ΔM_q and $\Delta\Gamma_q$ are the mass and width difference, respectively, between the mass eigenstates of the B_q-$\bar{B}_q$ system. We write $S_f \approx -\eta_f \sin(\phi_q + \Delta\phi_q)$, where $CP|f\rangle = \eta_f|f\rangle$ and ϕ_q is the CP phase in the limit $P_f = 0$. The SM predictions are $\phi_d = 2\beta$ and $\phi_s = -2\beta_s$. To the first order in $\epsilon = |V_{us}V_{ub}/(V_{cs}V_{cb})| \approx 0.02$, one has

$$\tan(\Delta\phi) \simeq 2\epsilon \sin\gamma \mathrm{Re}\frac{P_f}{T_f}. \tag{5}$$

Comparing Eq. (5) [with $\gamma = (69.7 \pm 2.8)°$] with the present experimental world average $\sin\phi_d = 0.679 \pm 0.020$ [2] (meaning an error of $1.6°$ for ϕ_d) shows that the penguin contribution already matters now and will certainly do so for future measurements at LHCb and Belle II. T_f and P_f are nonperturbative multiscale matrix elements, which defy calculations from first principles of QCD.

For the prediction of the branching ratio $B(B_d \to J/\psi K_S)$, one only needs T_f, which was addressed with the method of QCD factorization [3] in Ref. [4]: in the limit of infinite charm and bottom masses, T_f can be expressed in terms of the J/ψ decay constant and the $B_d \to K_S$ form factor. The result of Ref. [4] underestimates $B(B_d \to J/\psi K_S)$ by a factor of 8. This failure, however, is not surprising, because the corrections to the infinite-mass limit are of order $\Lambda_{\mathrm{QCD}}/(m_c\alpha_s)$ and, therefore, are numerically unsuppressed for the actual value of the charm mass [3,5]. The standard approach to quantify P_f/T_f in $B_d \to J/\psi K_S$ uses the approximate $SU(3)_F$ symmetry of QCD (or its U-spin subgroup) which relates the decay of interest to $b \to c\bar{c}d$ modes like $B_s \to J/\psi K_S$ and $B_d \to J/\psi\pi^0$ [6,7]. A drawback of this method is our poor knowledge of the quality of the $SU(3)_F$ symmetry in $B_{d,s} \to J/\psi X$ (with $X = K_S, \pi^0, \dots$) decays. (Comparisons of branching ratios essentially test $SU(3)_F$ in T_f only, with little sensitivity to P_f.) Furthermore, the $b \to c\bar{c}d$ control channels have 20 times smaller statistics than their $b \to c\bar{c}s$ counterparts. $SU(3)_F$ seemingly fails in $B_s \to J/\psi\phi$, because the ϕ meson cannot be closely approximated by an $SU(3)_F$ eigenstate, but is an equal mixture of octet and singlet.

In this Letter, we present a dynamical calculation of P_f/T_f which does not assume an approximate $SU(3)_F$ symmetry. Our results also permit, for the first time, the prediction of S_f and C_f for $b \to c\bar{c}d$ decays.

Operator product expansion.—For definiteness, we first specify the discussion to $B_d \to J/\psi K_S$ and return to $B_s \to J/\psi\phi$ and other modes in the phenomenology section. For $B(B_d \to J/\psi K_S)$ we only need T_f and can neglect the penguin coefficients. It is useful to express T_f in terms of the matrix elements of

$$Q_{0V} \equiv \bar{s}\gamma_\mu(1 - \gamma_5)b\bar{c}\gamma^\mu c,$$
$$Q_{0A} \equiv \bar{s}\gamma_\mu(1 - \gamma_5)b\bar{c}\gamma^\mu\gamma_5 c,$$
$$Q_{8V} \equiv \bar{s}\gamma_\mu(1 - \gamma_5)T^a b\bar{c}\gamma^\mu T^a c,$$
$$Q_{8A} \equiv \bar{s}\gamma_\mu(1 - \gamma_5)T^a b\bar{c}\gamma^\mu\gamma_5 T^a c. \tag{6}$$

Then, T_f in Eq. (3) becomes $T_f = (G_F/\sqrt{2})\langle J/\psi K_S|C_0 (Q_{0V} - Q_{0A}) + C_8(Q_{8V} - Q_{8A})|\bar{B}_d\rangle$ with $C_0 = C_2/N_c + C_1$ and $C_8 = 2C_2$, where $N_c = 3$ is the number of colors. Using next-to-leading order (NLO) Wilson coefficients in the naive dimensional regularization (NDR) scheme [1,8] at the scale $\mu = m_\psi$, one finds $C_0 = 0.13$ and $C_8 = 2.2$. The smallness of C_0 is a well-known numerical accident entailing that the weak decay produces the $(c, \bar{c})$ pair almost in a color octet state. We normalize the matrix elements (for $j = 0, 8$) as

$$\langle Q_{jV}\rangle = V_0 v_j, \qquad \langle Q_{jA}\rangle = V_0 a_j, \tag{7}$$

to the factorized matrix element $V_0 \equiv \langle Q_{0V}\rangle_{\mathrm{fact}} = 2f_{J/\psi}m_{B_d}p_{cm}F_1^{B\to K}(m_\psi^2) = (4.26\pm0.16)\,\mathrm{GeV}^3$. The uncertainty stems from the form factor $F_1^{B\to K}(m_\psi^2) = 0.586 \pm 0.021$ [9] and the J/ψ decay constant $f_{J/\psi} = (0.405 \pm 0.005)\,\mathrm{GeV}$. $m_{B_d} = 5.28\,\mathrm{GeV}$ and $p_{cm} = 1.68\,\mathrm{GeV}$ are the B_d mass and the three-momentum of the J/ψ or K_S in the B_d rest frame. $v_{0,8}, a_{0,8}$ depend on μ in such a way that the μ-dependence of C_0, C_8 cancels from physical quantities. When we quote numerical values, we refer to the choice $\mu = m_\psi$. The large-N_c counting of our (complex) hadronic parameters is $v_0 = 1 + \mathcal{O}(1/N_c^2)$, $v_8, a_8 = \mathcal{O}(1/N_c)$, and $a_0 = \mathcal{O}(1/N_c^2)$. Normalizing the branching ratio to the experimental value, we find

$$\frac{B(B_d \to J/\psi K_S)}{B(B_d \to J/\psi K_S)_{\mathrm{exp}}} = [1 \pm 0.08]|0.47v_0 + 7.8(v_8 - a_8)|^2.$$
$$\tag{8}$$

Varying the phase of $v_8 - a_8$ between $-\pi$ and π, one finds the correct branching ratio for $0.07 \leq |v_8 - a_8| \leq 0.19$ if v_0 is set to 1. Thus, there is no mystery with the branching ratio, and the hadronic parameters obey the hierarchy expected from $1/N_c$ counting. The terms involving a_0 are negligible in view of other uncertainties and are omitted throughout this Letter.

P_f in Eq. (3) receives contributions from $Q_{1,2}^u$ and the penguin operators Q_j, $j \geq 3$. The matrix elements of the latter can be trivially expressed in terms of the operators in Eq. (6). Therefore, this contribution to P_f/T_f only depends on v_8/v_0 and a_8/v_0. Below, we will see that the magnitudes of these ratios are under control thanks to the $1/N_c$ hierarchy of v_0, v_8, a_8 and the information from $B(B_d \to J/\psi K_S)_{\mathrm{exp}}$. By varying the parameters in the allowed ranges, we can then find the maximal contribution of the penguin operators to $|\Delta\phi|$.
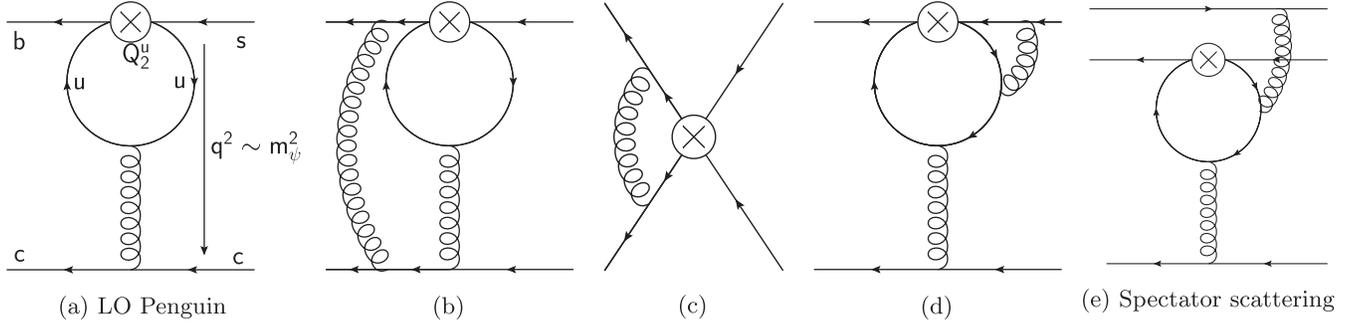
FIG. 1.   The LO diagram is shown in (a). The soft IR divergence of the diagram (b) factorizes with the corresponding diagram of the effective-theory side shown in (c). The diagram (d) is an example of a diagram with a collinear IR divergence. In (e) a spectator scattering diagram is given.

In order to apply the same strategy to $Q_{1,2}^u$, we must first express the up-quark penguin depicted in Fig. 1(a) in terms of matrix elements of the local operators in Eq. (6). In Ref. [10], it is argued that a penguin loop flown through by a hard momentum q [in our case $q^2 \sim m_\psi^2 = (3.1 \text{ GeV})^2$] can be calculated in perturbation theory ["Bander-Soni-Silverman (BSS) mechanism"]. In Ref. [11], this idea is used to find an estimate of $\langle Q_2^u \rangle$ which leads to an upper bound on $|\Delta\phi|$ which is smaller than the values found by $SU(3)_F$ arguments [7]. In this Letter, we turn the BSS idea into a rigorous field-theoretic method by proving an operator product expansion (OPE)

$$\langle J/\psi K_S | Q_j^u | B_d \rangle = \sum_k \tilde{C}_{j,k} \langle J/\psi K_S | Q_k | B_d \rangle + \cdots, \quad (9)$$

with k running over $k = 0V, 0A, 8V, 8A$. The dots represent terms suppressed by higher powers of $\Lambda_{\text{QCD}}/\sqrt{q^2}$. The Wilson coefficients $\tilde{C}_{j,k} = \tilde{C}_{j,k}^{(0)} + [\alpha_s(\mu)/(4\pi)]\tilde{C}_{j,k}^{(1)} + \cdots$ are calculated in perturbation theory to the desired order in $\alpha_s(\mu)$, with the renormalization scale $\mu = \mathcal{O}(m_\psi, m_b)$. A similar OPE has been derived to calculate charm-loop effects in the rare semileptonic decays $B \to K^{(*)}\ell^+\ell^-$ [12]. Since leptons carry no color charges, this application involves no four-quark operators like those in Eqs. (6) and (9). From Fig. 1(a), one finds $\tilde{C}_{j,k}^{(0)} = 0$ except for $\tilde{C}_{8G,8V}^{(0)} = -(m_b^2/q^2)(\alpha_s/\pi)$ and $\tilde{C}_{2,8V}^{(0)} = P(q^2)$ with the penguin function

$$P(q^2) = \frac{2}{3}\frac{\alpha_s}{4\pi}\left[\ln\left(\frac{q^2}{\mu^2}\right) - i\pi - \frac{2}{3}\right]. \quad (10)$$

Inherent to applications of the OPE, as in Ref. [12] or in this Letter, is the assumption that rescattering effects for values of q^2 far above the partonic pair-production threshold are correctly described in perturbation theory. Equation (9) captures all hadronic effects in the $(u,\bar{u}) \to (c,\bar{c})$ transition only if there is no intrinsic $(u,\bar{u})$ component in the J/ψ wave function (e.g., no J/ψ-ρ^0 mixing). A powerful check of our framework will be the confrontation of our predictions for $b \to c\bar{c}d$ transitions with data.

Proof of factorization.—In order to establish Eq. (9), we must prove that the coefficients $\tilde{C}_{j,k}$ are infrared (IR) safe. To this end, we analyze (i) the soft IR divergences of the two-loop diagrams contributing to $\langle Q_j^u \rangle$, (ii) the collinear IR divergences of these diagrams, (iii) spectator scattering diagrams, and (iv) higher-order diagrams in which the large momentum bypasses the penguin loop ("long distance penguins").

(i) An example of a diagram with a soft divergence is shown in Fig. 1(b). This soft divergence is reproduced by the corresponding diagram of the effective-theory side (i.e., rhs) of Eq. (9), depicted in Fig. 1(c), so that this divergence factorizes with $\tilde{C}_{j,k}^{(0)}$ and does not affect $\tilde{C}_{j,k}^{(1)}$. All soft divergences are from diagrams in which the additional gluon connects two external lines and cancel from $\tilde{C}_{j,k}^{(1)}$ in the same way.

(ii) Collinear divergences occur in diagrams in which a gluon is attached to the line with the strange quark, which we treat as massless. An example is shown in Fig. 1(d). If l denotes the loop momentum flowing through the gluon propagator and p_s is the momentum of the external strange quark, the collinear divergence corresponds to the region with $l^2 = 0$ and $l \propto p_s$. We can then reduce the problem to the study of one-loop diagrams with an external on-shell gluon: If we sum over all possibilities to attach this gluon to one of the lines of the LO diagram in Fig. 1(a), the collinear Ward identity of QCD ensures that this sum vanishes when the open Lorentz index of the gluon line is contracted with l^μ. This feature ensures that the collinear divergences of the sum of the two-loop diagrams vanish. (For a discussion in the context of QCD factorization see Refs. [3,13,14].) It equally holds for the effective-theory side of the OPE. The cancellation of collinear divergences is conceptually identical to the situation in typical processes in collider physics; further, it is known to be much simpler (with fewer diagrams to be discussed) if a physical gauge (with only 2 propagating gluon degrees of freedom) is adopted.

(iii) Next, we discuss the spectator scattering contributions: diagrams in which the gluon connects the b or s line

TABLE I. The maximal phase shift of ϕ_d due to penguin pollution and limits for the CP violation observables S_f and C_f in various $B_d \to f$ decays. Decays into two vector mesons involve different polarization amplitudes, indicated by 0, $\parallel$, and $\perp$ [22]. In S_f, for $f = J/\psi K^*$, $K^* \to K_s \pi^0$ is understood.

Final State	$J/\psi K_S$	$\psi(2S)K_S$	$J/\psi \pi^0$	$(J/\psi\rho)^0$	$(J/\psi\rho)^{\parallel}$	$(J/\psi\rho)^{\perp}$	$(J/\psi K^*)^0$	$(J/\psi K^*)^{\parallel}$	$(J/\psi K^*)^{\perp}$
$\max(\|\Delta\phi_d\|)$ [°]	0.68	0.74					0.85	1.13	0.93
$\max(\|\Delta S_f\|)$ [10^{-2}]	0.86	0.94	18	22	27	22	1.09	1.45	1.19
$\max(\|C_f\|)$ [10^{-2}]	1.33	1.33	29	35	41	36	1.65	2.19	1.80

with the spectator quark line trivially factorize with the corresponding diagrams on the effective side. If the gluon connects the spectator with the gluon line or a charm or up line, we have to take into account that the squared momentum in the penguin loop is $(q + l)^2$ instead of q^2. If the gluon is soft, $l^\mu \sim \Lambda_{\mathrm{QCD}}$, the expansion of the loop function P around q^2 reproduces a term which correctly factorizes with $\tilde{C}_{j,k}^{(0)}$ up to the term suppressed by $\Lambda_{\mathrm{QCD}}/\sqrt{q^2}$. If the gluon is hard collinear, with virtuality $l^2 \sim p_{\mathrm{cm}}\Lambda_{\mathrm{QCD}}$, where $p_{\mathrm{cm}} \sim 1.5$ GeV is the three-momentum of the K_S or J/ψ in the B_d rest frame, the situation is more subtle: the LO diagram is suppressed by $\Lambda_{\mathrm{QCD}}/p_{\mathrm{cm}}$, because the momentum of the spectator quark changes from zero to $\mathcal{O}(p_{\mathrm{cm}})$ in the decay, which is penalized by the light-cone distribution amplitude (LCDA) of the kaon [3]. The asymptotic form of the kaon LCDA, $\Phi(x) = 6x(1 - x)$, where x and $1 - x$ are the fractions of the kaon momentum carried by the $\bar{s}$ and d quarks, favors momentum configurations in which the kaon momentum is roughly equally shared between the two valence quarks. While the propagator of the scattered hard-collinear gluon is suppressed as $\sim 1/(\Lambda_{\mathrm{QCD}}p_{\mathrm{cm}})$, the suppression of the LO diagram is lifted because the spectator momentum is in the region $x \sim 1/2$ favored by the kaon LCDA. To identify further suppression factors, we first discuss the case that the gluon connects a charm line with the spectator: counting $q^2 \sim m_\psi^2$ and the energies of $\bar{s}$ and spectator-d quarks as $p_{\mathrm{cm}}/2$, the penguin loop gives $P[(q + l)^2] \simeq P(q^2) + (p_{\mathrm{cm}}/m_\psi)P'(q^2)$. The nonfactorizing piece involving the derivative $P'(q^2)$ comes with a factor of p_{cm}/m_ψ. The virtuality of the (anti-)charm propagator is around p_{cm} entailing a suppression factor of $\Lambda_{\mathrm{QCD}}/p_{\mathrm{cm}}$. Thus, alltogether spectator scattering from the charm lines obeys Eq. (9) up to terms of order $\Lambda_{\mathrm{QCD}}/m_\psi$. Next, we discuss spectator scattering from the up line, with a sample diagram depicted in Fig. 1(e). We find that these diagrams are power suppressed by $\Lambda_{\mathrm{QCD}}/m_\psi$. In this respect, these spectator diagrams differ from the similar photon penguins calculated in Ref. [15], which involve $P(q^2)$ for $q^2 \sim 0$ rather than $q^2 \sim m_\psi^2$.

(iv) So far, we have assumed that the underlying hard process is the penguin loop with the hard scale $\sqrt{q^2}$. But it may also be possible that the hard momentum transfer to the J/ψ occurs through a hard gluon radiated from the b or

s line, while the penguin loop is a "long-distance penguin" governed by soft QCD. Such a situation is exemplified by the diagram in Fig. 1(b) with the left gluon having virtuality $\sim m_\psi^2$. These diagrams, in which the whole weak decay process occurs with small momentum transfers, have a suppression factor $(\Lambda_{\mathrm{QCD}}/\sqrt{q^2})^3$ stemming from the hard gluon propagator and an off-shell b quark propagator (or s quark propagator).

In our power counting in (i)–(iv), we have treated p_{cm} as an intermediate scale between Λ_{QCD} and m_ψ and have found no nonfactorizable nonperturbative effects of order p_{cm}/m_ψ. While p_{cm} enters two-loop diagrams through $p_b \cdot p_s \sim m_b p_{\mathrm{cm}}$, such terms do not come with IR divergences and end up in the NLO corrections to the coefficients $\tilde{C}_{j,k}$. We find that the counting rule for p_{cm} is irrelevant, one can reproduce our results above as well for the limiting cases $p_{\mathrm{cm}} \sim \Lambda_{\mathrm{QCD}}$ and $p_{\mathrm{cm}} \sim \sqrt{q^2}$. In particular, higher orders of the OPE do not involve operators with derivatives acting on the $\bar{s}$ field. The same feature was found for $B \to K^{(*)}\ell^+\ell^-$ in the last paper of Ref. [12].

The choice $q^2 = m_\psi^2$ for $P(q^2)$ may be altered by adding a contribution of order Λ_{QCD} to $\sqrt{q^2}$. This shuffles a piece proportional to $(\Lambda_{\mathrm{QCD}}/m_\psi)P'(m_\psi^2)$ into the coefficient of the subleading operator $\bar{s}\gamma_\mu(1 - \gamma_5)T^a b[\Box - m_\psi^2]\bar{c}\gamma^\mu T^a c$, which removes the ambiguity associated with the choice of q^2. At NLO, in α_s, one also generates nonzero coefficients $\tilde{C}_{j,k}^{(1)}$ for $j = 1$ or $k = 0A, 8A$.

In conclusion, the OPE with the minimal set of operators in Eq. (7) works, the coefficients in $\tilde{C}_{j,k}$ are IR safe.

Phenomenology.—The penguin amplitude depends on the Wilson coefficients as

$$P_f = V_0(2C_4 + 2C_6 + 2C_2\tilde{C}_{2,8V}^{(0)} + C_{8G}\tilde{C}_{8G,8V}^{(0)})v_8 + \cdots, \tag{11}$$

where the dots represent the terms with v_0 and a_8 which have much smaller coefficients. The dependence of $\tilde{C}_{2,8V}^{(0)}$ [calculated from Fig. 1(a)] on the renormalization scheme cancels with the scheme dependence of $C_4 + C_6$ in Eq. (11). In the NDR scheme adopted by us, these penguin coefficients give a larger contribution to P_f than the u-penguin loop contained in $\tilde{C}_{2,8V}^{(0)}$. This is not surprising because the u-penguin loop enters at NLO, while $C_4 + C_6$

TABLE II.　Same as Table I for $B_s \to f$ decays.

Final State	$J/\psi K_S$	$(J/\psi\phi)^0$	$(J/\psi\phi)^\parallel$	$(J/\psi\phi)^\perp$	$(J/\psi K^*)^0$	$(J/\psi K^*)^\parallel$	$(J/\psi K^*)^\perp$
$\max(\lvert\Delta\phi_s\rvert)$ [°]		0.97	1.22	0.99			
$\max(\lvert\Delta S_f\rvert)$ $[10^{-2}]$	26	1.70	2.13	1.73	40	58	35
$\max(\lvert C_f\rvert)$ $[10^{-2}]$	27	1.89	2.35	1.92	43	64	37

already contributes at LO. The omission of this dominant LO piece explains the smallness of the result in Ref. [11].

For the prediction of P_f/T_f, we implement the constraint from $B(B \to f)$ exemplified for $f = J/\psi K_S$ in Eq. (8) in the following way: adapting a phase convention in which A_f in Eq. (1) is real and positive, we can determine a_8 in terms of V_0, v_0, v_8, and the measured $B(B \to f)$ [16,17]. Then, we use this to eliminate a_8 from P_f/T_f. For example, we find

$$\frac{P_{J/\psi K_S}}{T_{J/\psi K_S}} = 0.01 - 0.02v_0 - (0.71 + 0.33i)v_8, \quad (12)$$

for the central value of V_0 [quoted after Eq. (7)] and $B(B_d \to J/\psi K_S)$. We vary v_8 and v_0 in their allowed ranges $|v_8| < 1/3$ and $|v_0| = 1 \pm 0.15$ with the constraint that $|a_8| \leq 1/3$ must be obeyed. The allowed ranges for $\Delta\phi$, C_f, and $\Delta S_f \equiv S_f + \eta_f \sin\phi_q$ are almost symmetric around zero. We list the upper bounds on their magnitudes for several decay modes in Tables I and II. The results include the uncertainties from V_0, the branching ratios, CKM parameters [18], and higher-order terms in our OPE. For the $b \to \bar{c}cd$ decay modes with Cabibbo-unsuppressed P_f/T_f, the expansion in Eq. (5) has been replaced by the exact formula (see, e.g., Refs. [6,7]). Our bounds are conservative, as the considered ranges for v_8 and a_8 are wide [permitting even sizable cancellations in Eq. (8)]. From Eqs. (8) and (12), one verifies that any additional information on magnitude or phase of one of these parameters will substantially reduce the ranges quoted in Tables I and II. Our results for $B_d \to J/\psi\pi^0$ favor the Belle measurement $C_{J/\psi\pi^0} = -0.08 \pm 0.17$, $S_{J/\psi\pi^0} = -0.65 \pm 0.22$ [19] over the BABAR result $C_{J/\psi\pi^0} = -0.20 \pm 0.19$, $S_{J/\psi\pi^0} = -1.23 \pm 0.21$ [20]. [In the absence of penguin pollution, $C_{J/\psi\pi^0} = 0$ and $S_{J/\psi\pi^0} = -\sin(2\beta) = -0.69 \pm 0.02$.] In the case of a more precise and nonvanishing measurement of $C_{J/\psi\pi^0}$, for example, $C_{J/\psi\pi^0} = -0.10 \pm 0.01$, which corresponds to the current world average with a ten times smaller error, we can also put stronger restrictions on the shift of the mixing-induced CP violation $|\Delta S_{J/\psi\pi^0}| \leq 0.13$. A measurement of $C_{J/\psi\pi^0}$ that is consistent with zero, however, does not improve the bound. This feature occurs in all decay modes with Cabibbo-unsuppressed P_f/T_f. The measurements of S_f and C_f for the $B_d \to J/\psi\rho^0$ polarization amplitudes [21] comply with the ranges in Table I.

Conclusions.—We have established a factorization formula (to leading power in Λ_{QCD}/m_ψ) for the penguin contribution to the CP-violating coefficients S_f and C_f in $A_{CP}^{B_q \to f}(t)$ for final states f containing charmonium and the related shift $\Delta\phi_q$ of the corresponding CP phase. As a crucial result, the penguin contributions involve the same hadronic matrix elements as the tree amplitude. This allows us to constrain P_f/T_f, which determines S_f and C_f, and to find, e.g., $|\Delta\phi_d| \leq 0.68°$ for $B_d \to J/\psi K_S$ and $|\Delta\phi_s^\perp| \leq 0.99°$ for $B_d \to J/\psi\phi$, representing bounds that were thought to be incalculable from first principles. Novel territory are our predictions for S_f and C_f in $b \to \bar{c}cd$ decays, in which P_f/T_f is Cabibbo unsuppressed. Future experimental probes of these predictions will constitute a powerful test of our theoretical framework, whose key ingredient is an OPE for the up-quark penguin loop. There are no similar consistency checks for the standard predictions of P_f/T_f based on $SU(3)_F$ symmetry, which, moreover, cannot be used for $B_s \to J/\psi\phi$. Further, we remark that our results do not depend on any properties of the charmonium LCDA.

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