Resolving the Tevatron Top Quark Forward-Backward Asymmetry Puzzle: Fully Differential Next-to-Next-to-Leading-Order Calculation

Michal Czakon,[1] Paul Fiedler,[1] and Alexander Mitov[2]
[1]Institut für Theoretische Teilchenphysik und Kosmologie, RWTH Aachen University, D-52056 Aachen, Germany
[2]Cavendish Laboratory, University of Cambridge, Cambridge CB3 0HE, United Kingdom
(Received 13 January 2015; revised manuscript received 28 May 2015; published 28 July 2015)

We determine the dominant missing standard model (SM) contribution to the top quark pair forward-backward asymmetry at the Tevatron. Contrary to past expectations, we find a large, around 27%, shift relative to the well-known value of the inclusive asymmetry in next-to-leading order QCD. Combining all known standard model corrections, we find that $A_{FB}^{SM} = 0.095 \pm 0.007$. This value is in agreement with the latest DØ measurement [V. M. Abazov *et al.* (D0 Collaboration), Phys. Rev. D 90, 072011 (2014)] $A_{\mathrm{FB}}^{\mathrm{DØ}} = 0.106 \pm 0.03$ and about 1.5σ below that of CDF [T. Aaltonen *et al.* (CDF Collaboration), Phys. Rev. D 87, 092002 (2013)] $A_{\mathrm{FB}}^{\mathrm{CDF}} = 0.164 \pm 0.047$. Our result is derived from a fully differential calculation of the next-to-next-to leading order (NNLO) QCD corrections to inclusive top pair production at hadron colliders and includes—without any approximation—all partonic channels contributing to this process. This is the first complete fully differential calculation in NNLO QCD of a two–to–two scattering process with all colored partons.

DOI: 10.1103/PhysRevLett.115.052001 PACS numbers: 12.38.Bx, 14.65.Ha

Introduction.—At the Tevatron $p\bar{p}$ collider top quarks are produced predominantly in the hemisphere defined by the direction of the proton beam [1,2]. Such a production rate difference is often referred to as forward-backward asymmetry (A_{FB}). The Tevatron collider is uniquely positioned for the measurement of this asymmetry since A_{FB} is not present at pp colliders, e.g., the LHC (although a related, albeit strongly diluted asymmetry can be measured at the LHC; see for example Ref. [3] for more details).

This unique Tevatron capability, coupled with the persistent discrepancy [4] between the measured and predicted A_{FB}, have turned this observable into one of the most influential measurements performed at the Tevatron. Indeed, the A_{FB}-related publications by the CDF [4–9] and DØ [10–15] Collaborations have initiated major research activity both in explaining the discrepancy with beyond the standard model (BSM) physics (see e.g., Refs. [16,17]) and in estimating A_{FB} within the standard model [1,2,18–28] (see Ref. [29] for an in-depth review).

The effort to reconcile this discrepancy within the SM has so far been hampered because of the lack of a convincing estimate of the missing SM corrections. In this work we calculate the dominant missing correction and provide a realistic uncertainty estimate for A_{FB} in the SM. Our conclusion is that the SM prediction is under good theoretical control and agrees very well with the latest measurement—both inclusive and differential—from the DØ [15] Collaboration. For inclusive A_{FB}, we find reasonable agreement with the latest measurement from the CDF Collaboration [6].

A_{FB}: brief history and current status.—The focus of this work is A_{FB} for stable top quarks. For lepton-level A_{FB}, we refer the reader to Refs. [8,9,12–14,25,28,30].

A nonvanishing A_{FB} is predicted at next-to-leading order (NLO) in QCD. It was originally evaluated by Kühn and Rodrigo [1,2] long before the first measurements became available. The early measurements of A_{FB} showed [4] a very large discrepancy with respect to the SM prediction, especially at large $t\bar{t}$ invariant mass $M_{t\bar{t}} > 450$ GeV. Subsequent refinements of the measurements established [6] a less-pronounced A_{FB} at large $M_{t\bar{t}}$, which was still 2σ to 3σ above the SM prediction. Earlier this year, the DØ Collaboration published [15] an A_{FB} measurement at full data set, which turned out to be significantly lower than that of the CDF Collaboration [6] and thus much closer to the SM predictions.

The significance of the discrepancy between measurement and the SM theory prediction for A_{FB} has always critically hinged on the size of missing higher-order corrections. Here, we recall the calculation of the NLO QCD corrections [31] to A_{FB} in the related process $t\bar{t}j$, where a nearly -100% correction was found. Such a very large correction, if it were to also appear in $t\bar{t}$, would have had the potential of removing the discrepancy. Still, a careful analysis performed by Melnikov and Schulze [32] suggests that A_{FB} in $t\bar{t}$ is unlikely to receive very large corrections in the next order in QCD (i.e., in NNLO QCD) and is "most likely stable against yet higher order corrections." Our calculation of the NNLO QCD correction to A_{FB} is in line with their findings. (We equate "large" with "important, but not spoiling perturbative convergence," while "very large" might imply spoiling of perturbative convergence).

In a series of papers [23,24,28] it was found that, unexpectedly, electroweak (EW) corrections to $A_{\rm FB}$ are quite large. For example, for inclusive $A_{\rm FB}$, they are around 25% of the NLO QCD term. Contributions from Sudakov EW corrections have also been computed [19].

So far, the only source of information about higher-order QCD corrections to $A_{\rm FB}$ has been soft-gluon resummation. It was first applied at next-to-leading logarithmic accuracy (NLL) in Ref. [20] and later extended to NNLL in Ref. [21,22]. Further understanding of the nature of such soft emissions came in the context of parton showers and from probing them down to a single gluon emission [27]. From Refs. [20,22,27] one concludes that, beyond NLO QCD, soft-gluon emission generates negligible corrections to inclusive $A_{\rm FB}$. The natural interpretation of this result, especially when augmented with the conclusions of Ref. [32], was that the missing NNLO QCD contributions to $A_{\rm FB}$ in $t\bar{t}$ may be small and may not significantly affect the SM $A_{\rm FB}$ prediction. Contrary to the above expectations we find that the NNLO QCD corrections are large and originate mostly from emissions that are not controlled by soft-gluon resummation.

An alternative approach to computing $A_{\rm FB}$, based on the Principle of Maximum Conformality [33] scale setting, was used in Ref. [26]. The authors derive a value for $A_{\rm FB}$, which is significantly higher than the usual NLO QCD correction, in agreement with the CDF measurement. While the related Brodsky-Lepage-Mackenzie (BLM) [34] scale setting procedure is known [35] to work well even beyond fully inclusive observables, its applicability in top production at hadron colliders is not as established. For example, the NNLO results [36–39] for the terms quadratic in the number of massless quarks (N_F) in the total $t\bar{t}$ cross section differ from those predicted within the BLM approach. (In particular, the term $\propto N_F^2$ in $q\bar{q}\to t\bar{t}+X$ is known analytically [39]. The difference with respect to the BLM prediction is $\propto \pi^2\sigma_{\rm Born}$, and can be thought of as due to an analytical continuation to spacelike kinematics).

Finally, we recall the impact on $A_{\rm FB}$ from asymmetries in the subtracted $t\bar{t}$ backgrounds [40], as well as the possibility [15,41] that final state $t\bar{t}$-spectator interactions could contribute to $A_{\rm FB}$. The latter problem has been addressed in Ref. [42], where it was shown that such interactions are strongly suppressed for single-inclusive top (or $\bar{t}$) observables but need not be for double-inclusive observables (like the ones we study in this Letter) in the presence of strong jet vetoes. (The agreement between single- and double-inclusive measurements of $A_{\rm FB}$ [4] might be an indication that such a mechanism for generating $A_{\rm FB}$ in inclusive $t\bar{t}$ production may not be playing a significant role. Improved modeling of the so-called gap fraction [43] may help in clarifying this issue).

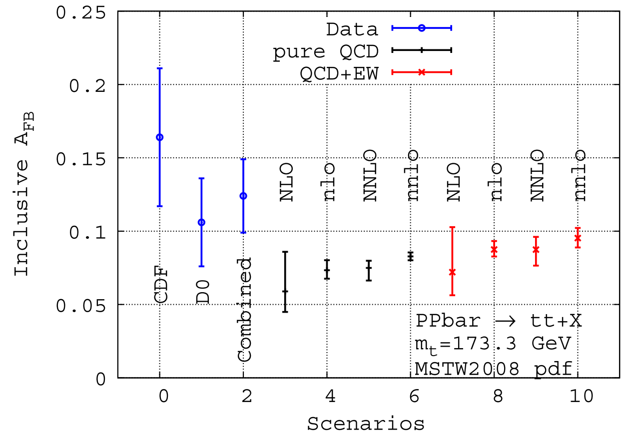

FIG. 1 (color online). The inclusive asymmetry in pure QCD (black) and QCD + EW [28] (red). Capital letters (NLO, NNLO) correspond to the unexpanded definition Eq (2), while small letters (nlo, nnlo) to the definition Eq. (3). The CDF/DØ (naive) average is from Ref. [29]. Error bands are from scale variation only. Our final prediction corresponds to scenario 10.

Results.—Following Ref. [6], the differential asymmetry is defined as

$$A_{\rm FB}=\frac{\sigma^+_{\rm bin}-\sigma^-_{\rm bin}}{\sigma^+_{\rm bin}+\sigma^-_{\rm bin}},\qquad \sigma^\pm_{\rm bin}=\int\theta(\pm\Delta y)\theta_{\rm bin}d\sigma,\qquad (1)$$

with the rapidity difference $\Delta y\equiv y_t-y_{\bar{t}}$. The binning function $\theta_{\rm bin}$ restricts the kinematics of the $t\bar{t}$ pair to the corresponding bins in Figs. 2–4. Setting $\theta_{\rm bin}=1$ in Eq. (1) yields the inclusive asymmetry $A_{\rm FB}$.

The fully differential cross section $d\sigma$ appearing in Eq. (1) for the process $p\bar{p}\to t\bar{t}+X$ is computed through NNLO in the strong coupling α_S. We use the top pole mass $m_t=173.3$ GeV, the MSTW2008 pdf set [45], and kinematics-independent scales with central value

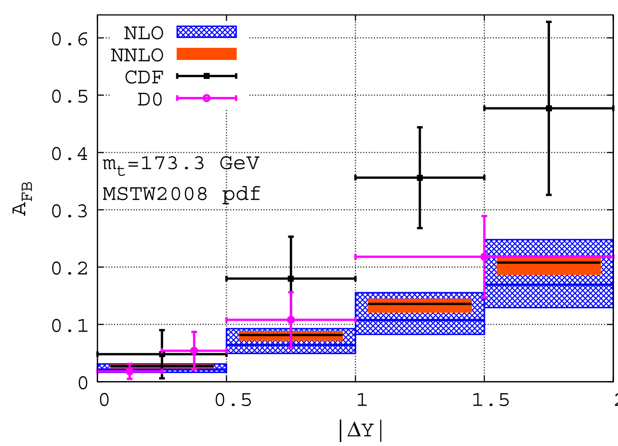

FIG. 2 (color online). The $|\Delta y|$ differential asymmetry in pure QCD at NLO (blue) and NNLO (orange) versus CDF [6] and DØ [15,44] data. Error bands are from scale variation only. For improved readability some bins are plotted slightly narrower. The highest bin contains overflow events.

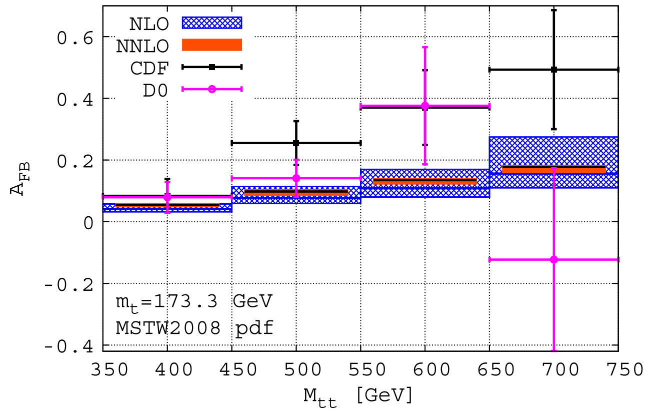

FIG. 3 (color online). As in Fig. 2 but for the $M_{t\bar{t}}$ differential asymmetry. The highest bin contains overflow events and the lowest bin includes all events down to the production threshold $2m_t$.

$\mu_R = \mu_F = m_t$. The theoretical uncertainty is estimated with restricted scale variation $\mu_R \neq \mu_F \in (m_t/2, 2m_t)$ [46] which was validated with the NNLO $t\bar{t}$ cross section [36–39]. The pdf uncertainty is small and is not included.

The differential cross section $d\sigma$ is computed following the setup of Refs. [36–39]: the two-loop virtual corrections are evaluated as in Refs. [47,48], utilizing the analytical form for the poles [49]. The one-loop squared amplitude has been calculated previously [50] and confirmed by us. The real-virtual (RV) corrections are derived by integrating the one-loop amplitude with a counterterm that regulates all its singular limits [51]. The finite part of the one-loop amplitude is computed with a code used in the calculation of $pp \to t\bar{t}j$ at NLO [31]. The double real (RR) corrections are computed as in Refs. [52,53].

Our calculation includes *all* partonic reactions that contribute to inclusive $t\bar{t}$ production in pure QCD without making any approximations. We have checked that our calculation reproduces $\sigma_{\rm tot}$ from Refs. [36–39] for each value of μ_R, μ_F with a precision better than one per mil. We also observe the cancellation of infrared singularities in each bin. At NLO our calculation agrees with the MCFM Monte Carlo generator [25,54]. The predicted NNLO $P_{T,t\bar{t}}$ dependence of $A_{\rm FB}$ for nonvanishing transverse momentum, $P_{T,t\bar{t}} \geq 10$ GeV (see Fig. 4), is consistent with results for the NLO QCD corrections to $pp \to t\bar{t}j$ from Refs. [32,55,56] and agrees perfectly with an independent evaluation using HELAC-NLO [57].

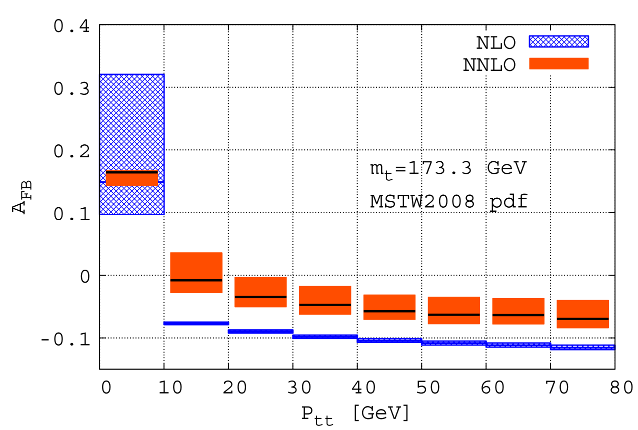

FIG. 4 (color online). As in Fig. 2 but for the $P_{T,t\bar{t}}$ differential asymmetry.

In this Letter we use two definitions for $A_{\rm FB}$ that are formally equivalent through NNLO and allow for EW corrections

$$A_{\rm FB} \equiv \frac{N_{\rm EW} + \alpha_S^3 N_3 + \alpha_S^4 N_4 + \mathcal{O}(\alpha_S^5)}{\alpha_S^2 D_2 + \alpha_S^3 D_3 + \alpha_S^4 D_4 + \mathcal{O}(\alpha_S^5)}, \tag{2}$$

$$= \alpha_S \frac{N_3}{D_2} + \frac{N_{\rm EW}}{\alpha_S^2 D_2} + \alpha_S^2 \left(\frac{N_4}{D_2} - \frac{N_3 D_3}{D_2^2}\right) - \frac{N_{\rm EW} D_3}{\alpha_S D_2^2} + \mathcal{O}(\alpha_S^3). \tag{3}$$

[The term $N_{\rm EW}$ contains some terms that involve powers of α_S. We ignore this α_S dependence in the power counting in Eq. (3).] The first definition, Eq. (2), uses exact results in both the numerator and denominator of Eq. (1), while the second, Eq. (3), is the expansion of the ratio Eq. (2) in powers of α_S. (Such an expansion is not, strictly speaking, fully consistent since the α_S expansion is performed after convolution with pdfs. Nevertheless, following the existing literature, we consider it as an indication of the sensitivity of $A_{\rm FB}$ to missing higher order terms.)

In the present Letter, we present differential asymmetries with the unexpanded definition (2) and without EW corrections (see Figs. 2, 3, 4). The inclusive asymmetry, see Fig. 1, is computed with both definitions, Eq. (2) and Eq. (3), including EW corrections. (EW corrections to D_i are neglected since EW effects to the total cross section are very small $\mathcal{O}(1\%)$, see Refs. [58–62].) The numerator factor $N_{\rm EW}$ is taken from Table II in Ref. [28]. (We have checked that the different pdf and m_t used in Ref. [28] have negligible impact on the QCD numerator N_3 and so we expect the same to hold for $N_{\rm EW}$.) Only for the inclusive asymmetry we determine the scale variation by keeping $\mu_R = \mu_F$ (since the scale dependence of $N_{\rm EW}$ is published [28] only for $\mu_R = \mu_F$). (We have checked that for the pure QCD corrections to the total asymmetry the difference with respect to scale uncertainty derived with $\mu_R \neq \mu_F$ variation is negligible.) We also note that the scale variation of $A_{\rm FB}$ is derived from the consistent scale variation of the ratio; i.e., both the numerator and denominator in Eqs. (2) and (3) are computed for each scale value.

Discussion and conclusions.—In Fig. 1 we observe that the central values of the expanded [Eq. (3)] and unexpanded [Eq. (2)] definitions of inclusive $A_{\rm FB}$ differ significantly at NLO but less so at NNLO. While the unexpanded

TABLE I. Principal contributions to the numerator N_4.

	Factorization	RR	RV	VV
(princ contr)/$(\alpha_S^4 N_4)$	−0.47	5.34	−3.90	0.03

definition, Eq. (2), closely resembles the experimental setup, the consistency of the two definitions *within uncertainties* renders the question about the more appropriate choice largely irrelevant. We also note the small scale error for the expanded A_{FB} definition, Eq. (3), in pure QCD at both NLO and NNLO, which appears too small to be realistic. The inclusion of EW corrections, however, breaks this pattern and brings the scale dependence in line with the unexpanded definition, Eq. (2). Therefore, following the previous literature, we choose as our final prediction $A_{\mathrm{FB}}^{\mathrm{SM}} = 0.095 \pm 0.007$ (scenario 10 in Fig. 1) which is derived with the expanded definition, Eq. (3), and includes EW [28] corrections.

The inclusion of higher order QCD corrections reduces the scale uncertainty of the differential asymmetry. The only exception is the $P_{T,t\bar{t}}$ dependent asymmetry whose scale behavior at NLO QCD is atypical.

The relative contributions of the principal NNLO corrections to the inclusive numerator in Eq. (2) are given in Table I. (Note that this separation is not unambiguous, just as at NLO.) Clearly, the inclusive asymmetry at NNLO is driven by a strong cancellation between RR and RV contributions. The contribution from collinear factorization is sizeable while the pure virtual (VV) correction is quite small. We have also checked that the numerator $\alpha_S^4 N_4$ almost exclusively originates in the $q\bar{q}$ partonic channel. (The contribution due to collinear factorization is not included in this comparison.) Where present, the contribution to $\alpha_S^4 N_4$ due to the qg reaction is 2 orders of magnitude smaller than $q\bar{q}$. The remaining qq'-type partonic reactions are another 2 orders of magnitude smaller. This pattern is in line with the contributions of these partonic reactions to the total cross-section [36–39].

In contrast to the negligible approximate NNLO QCD correction to A_{FB} implied by soft-gluon resummation [20,22], we find that the exact NNLO QCD correction to the inclusive A_{FB} is, in fact, large. (We note that the prediction of Ref. [21] differs from the one of [20,22], presumably due to different subleading terms.) Specifically, in Table II we compare the exact results for A_{FB} and its numerator [defined as the QCD part of the numerator in Eq (2)] through NNLO in QCD, with the NLO + NNLL predictions of Ref. [22]. (The settings in both papers are the same, except for a small difference of 0.2 GeV in the value of m_t which we neglect.) The ratio $A_{\mathrm{FB}}^{(\mathrm{NNLO})}/A_{\mathrm{FB}}^{(\mathrm{NLO})}$ is 1.27 (1.13) for A_{FB} defined through Eqs. (2) and (3). The corresponding ratio for the numerator of the asymmetry is 1.33, which is even larger than that for A_{FB}. Clearly the corrections to both quantities are significantly different from those of approximate NNLO, which yield 0.99 for the A_{FB} and 1.13 for the numerator ratio. [We refrain from directly comparing differential asymmetries because in this work we define them through Eq. (2) while the ones in Ref. [22] are defined through Eq. (3).]

TABLE II. Comparison of the numerator in Eq. (2) and the inclusive asymmetry A_{FB} computed in pure QCD at NLO (with NLO pdf set), NNLO and NLO + NNLL [22]. Only errors from $\mu_F = \mu_R$ scale variation are shown.

	NLO	NNLO	NLO+NNLL
$\alpha_S^3 N_3 + \alpha_S^4 N_4$ [pb]	$0.394^{+0.211}_{-0.127}$	$0.525^{+0.055}_{-0.085}$	$0.448^{+0.080}_{-0.071}$
$\alpha_S^4 N_4$ [pb]	$\cdots$	0.148	$\cdots$
$A_{\mathrm{FB}}[\%]$ (Eq. (3))	$7.34^{+0.68}_{-0.58}$	$8.28^{+0.27}_{-0.26}$	$7.24^{+1.04}_{-0.67}$
$A_{\mathrm{FB}}[\%]$ (Eq. (2))	$5.89^{+2.70}_{-1.40}$	$7.49^{+0.49}_{-0.86}$	$\cdots$

The large difference between A_{FB} predicted in exact and approximate NNLO can be understood from its $P_{T,t\bar{t}}$ dependence. We recall that soft gluon resummation applies to kinematical configurations that resemble those at the Born level; i.e., it should mainly contribute to the small $P_{T,t\bar{t}}$ bins. As Fig. 4 suggests, harder radiation generates a significant portion of the NNLO corrections. Studying the cumulative differential asymmetry $A_{\mathrm{FB}}(P_{T,t\bar{t}} \leq P_{T,t\bar{t}}^{\mathrm{cut}})$ and the corresponding cumulative numerator we observe that in the first bin $P_{T,t\bar{t}}^{\mathrm{cut}} \leq 10$ GeV (where soft gluon resummation should be most relevant) the NLO and NNLO numerators are practically equal; i.e., the 10% shift from NLO to NNLO in the first bin in Fig. 4 is exclusively due to the difference between NLO and NNLO denominators. With the inclusion of the next bins, however, the NLO and NNLO cumulative numerators start to differ quite rapidly. Indeed, about 50% of their difference is generated by the addition of the second bin $P_{T,t\bar{t}}^{\mathrm{cut}} = 20$ GeV.

Analyzing the $P_{T,t\bar{t}}$ dependence of A_{FB}, the CDF Collaboration [6] noted that the discrepancy between data and NLO QCD appears to be independent of $P_{T,t\bar{t}}$. It is easy to see from Fig. 4 that the difference between NNLO and NLO corrections to the $P_{T,t\bar{t}}$ asymmetry for $P_{T,t\bar{t}} \geq 10$ GeV follows precisely this pattern and is, furthermore, consistent with the analysis of Ref. [63].

The pdf uncertainty is generally small and has not been included in our results. For its estimation, we have first computed A_{FB} in NLO QCD with a NNLO pdf set (at 68% C.L.) and then rescaled it with the appropriate K factor based on central scale values. In inclusive quantities such as the inclusive A_{FB} and the numerator in Eq. (2), the pdf uncertainty is smaller than the scale uncertainty by a factor of 3 or more. Similarly, the pdf error in the differential asymmetry is typically much smaller than the one from scale variation, although in some bins it can be as large as half the scale error. Therefore, for most A_{FB}-related applications we can envisage, one can safely neglect pdf errors. However, if a precise error estimate is essential, the pdf errors might need to be revisited.

The Monte Carlo (MC) integration error in all our results is insignificant. Specifically, its relative contribution to the inclusive asymmetry and cross section is at the per mil and sub-per mil levels, respectively. The relative MC error in the differential asymmetry is typically below 1% in each bin, with the exception of the largest $M_{t\bar{t}}$ bin and the 60 GeV $\leq P_{T,t\bar{t}} \leq$ 70 GeV bin where it is about 1.5% (for central scales).

Finally, we would like to emphasise the connection between the top quark $A_{\rm FB}$ and the perturbatively generated strange asymmetry of the proton [64]. For example, the asymmetry-generating diagrams are the same in both cases (compare Fig. 1 from Ref. [64] with Fig. 3(a) of Ref. [2]) up to crossing legs from the initial to the final state and setting m_t to zero. In fact, *in the absence of other predictions*, one might speculate that our results indicate that the currently unknown four-loop corrections to the spacelike splitting functions may bring non-negligible corrections to the perturbatively generated s, c, b, t asymmetries of the proton.

Summary.—We compute the largest missing SM correction to top quark $A_{\rm FB}$ originating in NNLO QCD. Our calculation includes all contributing partonic channels exactly, which makes it the first-ever complete NNLO fully differential calculation in a process with four colored partons. In contrast to previous approximations we observe a significant NNLO correction to $A_{\rm FB}$ which brings the SM prediction for the inclusive asymmetry in agreement with the measurement of the DØ Collaboration and about 1.5σ below the value measured by the CDF Collaboration. The predicted differential asymmetry, even without EW corrections, is in agreement with the corresponding DØ measurements.

We thank Dante Amidei, Tom Ferbel, Amnon Harel, Regina Demina, and Jon Wilson for clarifications about the experimental results. We also thank Stefan Dittmaier for kindly providing us with his code for the evaluation of the one-loop virtual corrections, Markus Schulze for a comparison with the results of Refs. [32,55], Jan Winter for a comparison with Ref. [56], and Manfred Kraus for a comparison with HELAC-NLO. The work of M. C. and P. F. was supported by the German Research Foundation (DFG) via the Sonderforschungsbereich/Transregio SFB/TR-9 "Computational Particle Physics," and the Heisenberg programme. The work of A. M. is supported by the UK Science and Technology Facilities Council [Grants No. ST/L002760/1 and No. ST/K004883/1].

[1] J. H. Kühn and G. Rodrigo, Phys. Rev. Lett. **81**, 49 (1998).
[2] J. H. Kühn and G. Rodrigo, Phys. Rev. D **59**, 054017 (1999).
[3] J. A. Aguilar-Saavedra, W. Bernreuther, and Z. G. Si, Phys. Rev. D **86**, 115020 (2012).
[4] T. A. Aaltonen *et al.* (CDF Collaboration), Phys. Rev. D **83**, 112003 (2011).
[5] T. A Aaltonen*et al.* (CDF Collaboration), Phys. Rev. Lett. **101**, 202001 (2008).
[6] T. Aaltonen *et al.* (CDF Collaboration), Phys. Rev. D **87**, 092002 (2013).
[7] T. A. Aaltonen *et al.* (CDF Collaboration), Phys. Rev. Lett. **111**, 182002 (2013).
[8] T. A. Aaltonen *et al.* (CDF Collaboration), Phys. Rev. D **88**, 072003 (2013).
[9] T. A. Aaltonen *et al.* (CDF Collaboration), Phys. Rev. Lett. **113**, 042001 (2014).
[10] V. M. Abazov *et al.* (D0 Collaboration), Phys. Rev. Lett. **100**, 142002 (2008).
[11] V. M. Abazov *et al.* (D0 Collaboration), Phys. Rev. D **84**, 112005 (2011).
[12] V. M. Abazov *et al.* (D0 Collaboration), Phys. Rev. D **87**, 011103 (2013).
[13] V. M. Abazov *et al.* (D0 Collaboration), Phys. Rev. D **88**, 112002 (2013).
[14] V. M. Abazov *et al.* (D0 Collaboration),Phys. Rev. D **90**, 072001 (2014).
[15] V. M. Abazov *et al.* (D0 Collaboration), Phys. Rev. D **90**, 072011 (2014).
[16] J. F. Kamenik, J. Shu, and J. Zupan, Eur. Phys. J. C **72**, 2102 (2012).
[17] E. L. Berger, Q. H. Cao, C. R. Chen, C. S. Li, and H. Zhang, Phys. Rev. Lett. **106**, 201801 (2011).
[18] O. Antunano, J. H. Kühn, and G. Rodrigo, Phys. Rev. D **77**, 014003 (2008).
[19] A. V. Manohar and M. Trott, Phys. Lett. B **711**, 313 (2012).
[20] L. G. Almeida, G. F. Sterman, and W. Vogelsang, Phys. Rev. D **78**, 014008 (2008).
[21] N. Kidonakis, Phys. Rev. D **84**, 011504 (2011).
[22] V. Ahrens, A. Ferroglia, M. Neubert, B. D. Pecjak, and L. L. Yang, Phys. Rev. D **84**, 074004 (2011).
[23] W. Hollik and D. Pagani, Phys. Rev. D **84**, 093003 (2011).
[24] J. H. Kühn and G. Rodrigo, J. High Energy Phys. 01 (2012) 063.
[25] J. M. Campbell and R. K. Ellis, J. Phys. G **42**, 015005 (2015).
[26] S. J. Brodsky and X. G. Wu, Phys. Rev. D **85**, 114040 (2012).
[27] P. Skands, B. Webber, and J. Winter, J. High Energy Phys. 07 (2012) 151.
[28] W. Bernreuther and Z. G. Si, Phys. Rev. D **86**, 034026 (2012).
[29] J. A. Aguilar-Saavedra, D. Amidei, A. Juste, and M. Perez-Victoria, Rev. Mod. Phys. **87**, 421 (2015)
[30] A. Falkowski, M. L. Mangano, A. Martin, G. Perez, and J. Winter, Phys. Rev. D **87**, 034039 (2013).
[31] S. Dittmaier, P. Uwer, and S. Weinzierl, Phys. Rev. Lett. **98**, 262002 (2007); **59**, 625 (2009).
[32] K. Melnikov and M. Schulze, Nucl. Phys. **B840**, 129 (2010).
[33] S. J. Brodsky and X. G. Wu, Phys. Rev. Lett. **109**, 042002 (2012).
[34] S. J. Brodsky, G. P. Lepage, and P. B. Mackenzie, Phys. Rev. D **28**, 228 (1983).
[35] K. Melnikov and A. Mitov, Phys. Lett. B **620**, 69 (2005).

[36] M. Czakon, P. Fiedler, and A. Mitov, Phys. Rev. Lett. **110**, 252004 (2013).
[37] M. Czakon and A. Mitov, J. High Energy Phys. 01 (2013) 080.
[38] M. Czakon and A. Mitov, J. High Energy Phys. 12 (2012) 054.
[39] P. Bärnreuther, M. Czakon, and A. Mitov, Phys. Rev. Lett. **109**, 132001 (2012).
[40] K. Hagiwara, J. Kanzaki, and Y. Takaesu, Nucl. Phys. B, Proc. Suppl. **234**, 93 (2013).
[41] S. J. Brodsky (private communication).
[42] A. Mitov and G. Sterman, Phys. Rev. D **86**, 114038 (2012).
[43] G. Aad *et al.* (ATLAS Collaboration), Eur. Phys. J. C **72**, 2043 (2012).
[44] Supplemental information from the D0 Collaboration: http://www-d0.fnal.gov/Run2Physics/WWW/results/final/TOP/T14D/
[45] A. D. Martin, W. J. Stirling, R. S. Thorne, and G. Watt, Eur. Phys. J. C **63**, 189 (2009).
[46] M. Cacciari, S. Frixione, M. L. Mangano, P. Nason, and G. Ridolfi, J. High Energy Phys. 09 (2008) 127.
[47] M. Czakon, Phys. Lett. B **664** (2008) 307.
[48] P. Bärnreuther, M. Czakon, and P. Fiedler, J. High Energy Phys. 02 (2014) 078.
[49] A. Ferroglia, M. Neubert, B. D. Pecjak, and L. L. Yang, J. High Energy Phys. 11 (2009) 062.
[50] C. Anastasiou and S. M. Aybat, Phys. Rev. D **78**, 114006 (2008); B. Kniehl, Z. Merebashvili, J. G. Körner, and M. Rogal, Phys. Rev. D **78**, 094013 (2008); J. G. Körner, Z. Merebashvili, and M. Rogal, Phys. Rev. D **71**, 054028 (2005).
[51] Z. Bern, V. Del Duca, W. B. Kilgore, and C. R. Schmidt, Phys. Rev. D **60**, 116001 (1999); S. Catani and M. Grazzini, Nucl. Phys. **B591**, 435 (2000); S. Catani, S. Dittmaier, and Z. Trocsanyi, Phys. Lett. B **500**, 149 (2001); I. Bierenbaum, M. Czakon, and A. Mitov, Nucl. Phys. **B856**, 228 (2012).
[52] M. Czakon, Phys. Lett. B **693**, 259 (2010); Nucl. Phys. **B849**, 250 (2011).
[53] M. Czakon, Nucl. Phys. **B849** (2011) 250.
[54] P. Nason, S. Dawson, and R. K. Ellis, Nucl. Phys. **B303**, 607 (1988); Nucl. Phys. **B327**, 49 (1989); **B335**, 260(E) (1990).
[55] K. Melnikov, A. Scharf, and M. Schulze, Phys. Rev. D **85**, 054002 (2012).
[56] S. Hoeche, J. Huang, G. Luisoni, J. Winter, and M. Schonherr, Phys. Rev. D **88**, 014040 (2013).
[57] G. Bevilacqua, M. Czakon, M. V. Garzelli, A. van Hameren, A. Kardos, C. G. Papadopoulos, R. Pittau, and M. Worek, Comput. Phys. Commun. **184** (2013) 986.
[58] W. Beenakker, A. Denner, W. Hollik, R. Mertig, T. Sack, and D. Wackeroth, Nucl. Phys. **B411**, 343 (1994).
[59] S. Moretti, M. R. Nolten, and D. A. Ross, Phys. Lett. B **639**, 513 (2006); **B660**, 607(E) (2008).
[60] W. Bernreuther, M. Fücker, and Z.-G. Si, Phys. Rev. D **74**, 113005 (2006).
[61] W. Hollik and M. Kollar, Phys. Rev. D **77**, 014008 (2008).
[62] J. H. Kühn, A. Scharf, and P. Uwer, Phys. Rev. D **91**, 014020 (2015).
[63] B. Gripaios, A. Papaefstathiou, and B. Webber, J. High Energy Phys. 11 (2013) 105.
[64] S. Catani, D. de Florian, G. Rodrigo, and W. Vogelsang, Phys. Rev. Lett. **93**, 152003 (2004).