

**Erratum: Radiative Improvement of the Lattice Nonrelativistic QCD Action
Using the Background Field Method and Application to the Hyperfine
Splitting of Quarkonium States
[Phys. Rev. Lett. 107, 112002 (2011)]**

T. C. Hammant, A. G. Hart, G. M. von Hippel, R. R. Horgan, and C. J. Monahan
(Received 19 June 2015; published 14 July 2015)

DOI: 10.1103/PhysRevLett.115.039901

PACS numbers: 12.38.Gc, 12.38.Bx, 99.10.Cd

Our Letter contained two errors, one typographical, the other numerical: First, the sign of the coefficient d_2 was reproduced incorrectly in Eq. (14); secondly, the finite part of the result in Eq. (13) for the quark-quark spin-dependent scattering matrix element in relativistic QCD was wrong by a factor of 3, which carried over into Eq. (14), and into the numbers given in Tables I and II.

Consequently, Eq. (13) should read

$$A_{f1}^R = \frac{8}{9}, \quad (1)$$

while Eq. (14) should read

$$\begin{aligned} d_1 &= -3d_2 - \frac{2}{9}(2 - 2 \log 2), \\ d_2 &= -\frac{8}{27} + \frac{1}{3}A_{f1}^{NR} - \frac{1}{3} \log Ma. \end{aligned} \quad (2)$$

The numbers in Tables I and II should be corrected as per the tables shown here (note that only the numbers related to d_1 and d_2 are changed, while the remainder of the table is reproduced for ease of reference).

Correspondingly, the sentence starting “The result is to reduce ...” in the conclusions should read “The result is to reduce the lattice spacing dependence to within errors and to give an estimate for this hyperfine splitting of 68(3)(5)(6) MeV to be compared with the experimental value of 69.3(2.8) MeV.” The general conclusions of our Letter remain unchanged.

TABLE I. Renormalization parameters of the $\sigma \cdot \mathbf{B}$ and the four-fermion terms defined, respectively, in Eqs. (9) and (11) of our Letter.

Ma	1.95	2.8	4.0
$\delta\tilde{Z}_\sigma + \delta\tilde{Z}_2$	-5.164(7)	-4.913(6)	-4.739(6)
$\delta\tilde{Z}_m$	1.512(1)	1.022(3)	0.723(2)
$\delta Z_\sigma^{\text{tad}}$	4.387	4.077	3.841
δZ_m^{tad}	-1.092	-0.787	-0.641
$c_4^{(1)}$	0.728(7)	0.799(7)	0.842(6)
d_1	0.638(7)	-0.109(14)	-1.138(25)
d_2	-0.258(2)	-0.009(5)	0.334(8)

TABLE II. Corrections to the bottomonium hyperfine splitting results of [1] arising from the radiative improvement of the action. In the last column the errors are statistical, $O(\alpha^2)$, and relativistic corrections.

Ma	$\alpha_V(q^*)$	Correction %		hfs (MeV)	hfs (MeV)
		4-fermion	$\sigma \cdot B$	Ref. [1]	Corrected
1.95	0.216	-10.3(1)	+31.4(3)	56(2)	68(3)(5)(6)
2.8	0.249	+1.3(2)	+39.8(3)	50(2)	71(3)(6)(5)
4.0	0.293	+23.2(4)	+49.3(3)	41(2)	71(3)(7)(4)

We thank the authors of Ref. [2] for drawing our attention to these mistakes.

- [1] A. Gray, I. Allison, C. T. H. Davies, E. Gulez, G. P. Lepage, J. Shigemitsu, and M. Wingate, *Phys. Rev. D* **72**, 094507 (2005).
- [2] M. Baker, A. Penin, D. Seidel, and N. Zerf, [arXiv:1504.05979](https://arxiv.org/abs/1504.05979).