Fate of Many-Body Localization Under Periodic Driving

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We study many-body localized quantum systems subject to periodic driving. We find that the presence of a mobility edge *anywhere* in the spectrum is enough to lead to delocalization for any driving strength and frequency. By contrast, for a fully localized many-body system, a delocalization transition occurs at a finite driving frequency. We present numerical studies on a system of interacting one-dimensional bosons and the quantum random energy model, as well as simple physical pictures accounting for those results.

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Introduction.—The study of disorder and localization has a long and productive history, beginning with the seminal work of Anderson [1]. More recently, the effects of disorder on interacting systems have been considered under the heading of many-body localization (MBL) [2,3], in part motivated by fundamental questions relating to thermalization in closed quantum systems.

At the same time, significant theoretical effort has been devoted to understanding thermalization in periodically driven systems. There has been work recently on the long-time behavior of both integrable [4–6] and nonintegrable [7–9] systems (with Ref. [9] also studying *locally* driven MBL systems). For clean systems or MBL systems in their delocalized phase, it has been found that driving leads to a state equivalent to a fully mixed state, satisfying a special case of the eigenstate thermalization hypothesis (ETH) (see Refs. [7–13]). Local periodic driving of MBL systems in their localized phase, on the other hand, has been argued not to have any global effects [9].

In this work, we study the effects of *global* periodic driving, and find that there exists a regime where MBL survives. We identify two mechanisms by which periodic driving might destroy MBL, depending on the existence or nonexistence of a mobility edge. The first, rather robust, mechanism is the mixing of undriven eigenstates from everywhere in the spectrum by the driving; if there is a mobility edge, this results in delocalization of all states of the effective Hamiltonian. The second mechanism is more subtle and involves strong mixing of states [8] which cause a delocalization transition at finite frequency. Our findings are summarized in Table I.

In what follows, we begin by studying the case of no mobility edge. We introduce and numerically solve a system described by a local nonintegrable Hamiltonian. After establishing the existence of the aforementioned critical frequency using level statistics, we demonstrate that ETH is (is not) satisfied below (above) this frequency and present a physical picture explaining this phenomenon. We then move to the case where a mobility edge exists. As a case study, we use the quantum random energy model (QREM) which has recently been shown to display a mobility edge. A direct numerical solution confirms that driving delocalizes the entire spectrum, consistent with an intuitive argument we sketch. Finally, we point out open questions.

We shall concentrate throughout on systems described by Hamiltonians of the form

$$H(t) = H_0 + H_D(t), \tag{1}$$

so that their time evolution is described by an effective Hamiltonian $H_{\mathrm{eff}}(\epsilon)$ for each instant ϵ during the period T, defined by

$$\exp[-iH_{\mathrm{eff}}(\epsilon)T] = \mathcal{T}\exp\left(-i\int_{\epsilon}^{\epsilon+T} dtH(t)\right). \tag{2}$$

Without loss of generality, we set $\epsilon = 0$ (see Ref. [4]). The eigenvalues of $H_{\mathrm{eff}}(\epsilon)$, called the quasienergies, are independent of ϵ and effectively play the role of energy eigenvalues.

We now define what we mean by localized and delocalized phases. In a *localized* phase, the (quasi)energy level statistics do not display level repulsion, and the expectation values of operators in the eigenstates of the (effective) Hamiltonian fluctuate wildly from eigenstate to eigenstate. In a *delocalized* phase, the opposite is true: the levels repel each other, and the expectation values of physical, local operators in nearby energy or quasienergy states are similar. Other definitions are possible and, in general, equivalent

TABLE I. Effect of driving frequency in the presence of and in the absence of a mobility edge.

Mobility edge	Low frequency	High frequency
Present	Delocalized	Delocalized
Absent	Delocalized	*Localized*

(see, e.g., Ref. [14]). The connection between the eigenstates of H_{eff} and the applicability of ETH was elucidated in Ref. [8]. The framework developed there turns out to be natural for discussing the case of a system which, in the absence of driving, is in the MBL phase.

No mobility edge: Local model.—Let us introduce a model of interacting hard-core bosons described by a driven, local Hamiltonian [Eq. (1)] with

$$H_0 = H_{\mathrm{hop}} + \sum_{r=1}^{2} V_r \sum_{i=1}^{L-1} n_i n_{i+r} + \sum_{i=1}^{L} U_i n_i, \qquad (3)$$

where $H_{\mathrm{hop}} = [-\frac{1}{2}J\sum_{i=1}^{L-1}(b_i^\dagger b_{i+1} + b_{i+1}^\dagger b_i + \mathrm{H.c.})]$ is a hopping operator, the b are hard-core bosonic operators, U_i an on-site random potential uniformly distributed between $-w$ and $+w$, and $H_D(t)$ a time-periodic hopping term

$$H_D(t) = \delta\tilde{\delta}(t)H_{\mathrm{hop}}, \qquad (4)$$

with δ a dimensionless constant, $\tilde{\delta}(t) = -1(+1)$ in the first (second) half of each period $T = 2\pi/\omega$. Via Jordan-Wigner transformations, this model is related to a fermionic interacting system as well as to a spin-1/2 chain. Throughout this work we will concentrate on the specific case $V_1/J = V_2/J = 1$, although our qualitative conclusions are not sensitive to this.

To locate the transition in the undriven model, we use the standard technique [3] involving finite-size scaling of the level statistics (see inset of Fig. 2 and Supplemental Material [15]). At half filling there thus appears to be a transition at a disorder amplitude w_c^u/J (≈6 for our interaction parameters $V_1/J = V_2/J = 1$) [18].

We now drive this system $\delta \neq 0$. The level statistics of the quasienergies of H_{eff} [Eq. (2)] show level repulsion in the clean limit [7] but are found to cross freely (indicating localization) in the MBL regime if driven locally, as reported in Ref. [9]. Here, we show that globally periodically driving the system in the MBL regime delocalizes the system if the driving frequency is below a (system size independent) critical value. We argue that this is a consequence of the structure of the effective Hamiltonian for a MBL system [19–23].

As established above, the undriven system is in the delocalized phase for disorder amplitude $w < w_c^u$; driving at this disorder is qualitatively similar to driving any nonintegrable system [8], a case that has been studied in Ref. [8]. We have indeed confirmed quasienergy level repulsion for $w < w_c^u$.

To study the MBL regime, $w > w_c^u$, we switch on periodic driving [Eq. (4)] with amplitude $\delta/J = 0.1$ (our results do not change qualitatively for different δ provided the system is large enough that the local level spacing is less than δ). We directly calculate H_{eff} and its level statistics. As our central result we find that for each disorder amplitude there exists a driving frequency $\omega_c(w)$ above which the system remains in the localized phase under driving (see Supplemental Material [15]), while for $\omega < \omega_c(w)$ the system delocalizes. This frequency is plotted in Fig. 1 as a function of disorder amplitude w, while examples of the level statistics results are shown in the inset of Fig. 2. We expect $\omega_c(w)$ to diverge as w approaches w_c^u from above.

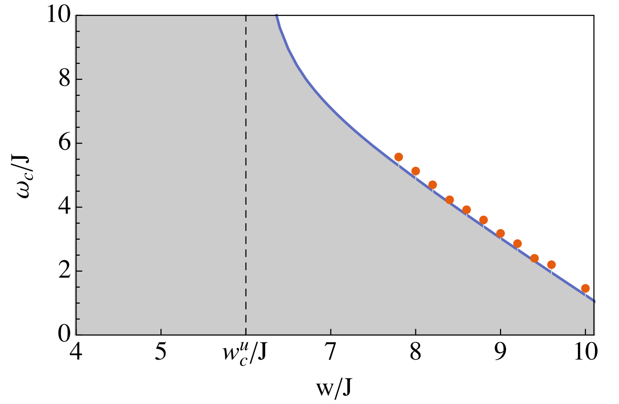

FIG. 1 (color online). Plot of driving frequency ω_c below which the system delocalizes as a function of disorder amplitude w. The shaded areas correspond to delocalization. The red dots are obtained from finite-size studies of the level statistics of the system. The disorder amplitude w_c is the value below which the undriven system is delocalized in the absence of driving. The blue line is a guide to the eye.

Having established a transition via the level statistics, we now show in addition that the phases above (below) ω_c do (do not) satisfy the form of ETH discussed in Ref. [8], further reinforcing our interpretation of ω_c as a "delocalization frequency." We consider a localized undriven system and provide in Fig. 2 direct evidence for the fully mixed nature of the *eigenstates* of H_{eff} for slow—but not

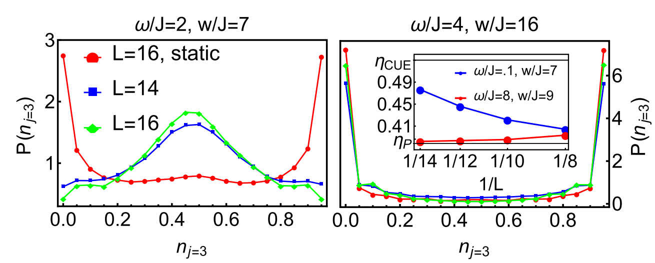

FIG. 2 (color online). Main plot: Probability distribution of the EEVs of the density at site $j = 3$. Left (right): Driving with a low (high) frequency (see Fig. 1) results in the probability distributions which does (does not) develop a central peak upon increasing system size, signaling delocalization [8]. Data are disorder averaged over 10^4 (100) realizations for $L = 14$ ($L = 16$). Inset: Level statistics parameter versus inverse system size in the localized (bottom, red) delocalized (top, blue) phases. The parameter $\eta = \int ds s P(s)$ with $P(s)$ the probability distribution of the level statistics [3,7], taking the value $\eta_{P/CUE}$ in the localized or delocalized regime. Data averaged over 1000 disorder realizations for $L = 8, 10, 12$, 100 realizations for $L = 14$.

for fast—driving. The quantity under consideration is the probability distribution for the eigenstate expectation values (EEVs) [8] of the density operator. Driving faster than the delocalization frequency (right-hand panel) yields little change in the probability distribution. By contrast, driving slowly (left-hand panel), a central peak is seen to develop with increasing system size, corresponding to the EEVs all being equal and given by $n_{j=3} = 0.5$. This is the fully mixed result for our system at half filling, corresponding to delocalization [8].

In passing, let us remark that, since the EEVs of the instantaneous Hamiltonian show the same behavior as in Fig. 2 (data not shown), our localized phase is not unlike the localization in energy space discussed in Ref. [24], even though the underlying physics is quite different.

Physical picture.—We now relate our numerical findings to a physical picture valid for weak driving. In the MBL phase and in the absence of driving, the system is effectively integrable in that there exist extensively many local integrals of motion [19–23]. The system may thus be thought of as a set of local subsystems, of finite spatial extent, therefore of finite energetic bandwidth, as schematically shown in Fig. 3. As a result, if the driving

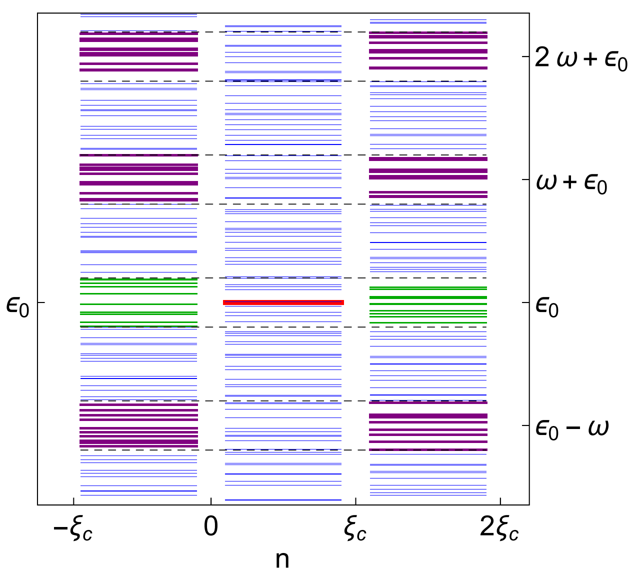

FIG. 3 (color online). Schematic illustration of subsystems and their energy levels in a MBL system. The horizontal axis indexes the conserved quantity (increasing n corresponds to increasing spatial index i); the vertical indexes energy. ξ_c is some localization length, setting the typical spatial size of the subsystems. A periodic coupling of the subsystems with frequency ω couples the red state in the middle block to both the purple and the green levels in the two neighboring blocks, while a time-independent coupling would only couple it to the green ones. The width of the purple and green strips is set by the amplitude of the driving. Critically, the limit of ω greater than the typical subsystem bandwidth is indistinguishable from a time-independent driving. By contrast, the limit of $\omega \to 0$ collapses the local spectra, wiping out the effect of disorder.

frequency is larger than the typical local subsystem bandwidth, the system cannot absorb energy from the driving and does not react. Therefore, driving with a frequency much higher than the typical local bandwidth cannot destroy MBL. In contrast, low-frequency driving may be understood by viewing our driving protocol as a series of quenches: as MBL systems eventually reach a steady state after an instantaneous quench [25,26], periodic driving with the protocol we use can be thought of as a series of nonadiabatic perturbations. It is quite natural then to expect this to cause the system to spread in energy space, delocalizing it.

Let us elaborate this pair of arguments, beginning with high-frequency driving.

High-frequency driving.—The most general form of H_{MBL} consistent with known phenomenology such as vanishing of the conductivity at all energies is

$$H^G_{\mathrm{MBL}} = \sum_n \mathcal{H}^{(\ell)}_n + \sum_{m<n} \mathcal{H}^{(\ell)}_n V^{(\ell)}_{m,n} \mathcal{H}^{(\ell)}_m \ldots, \tag{5}$$

with the $\mathcal{H}^{(\ell)}_n$ Hamiltonians for local subsystems (with local spatial support) and n a spatial index indicating the site about which the subsystem is centered [19–23]. Because of its locality, each $\mathcal{H}^{(\ell)}_n$ has a local spectrum of some typical, *finite* width set by the disorder amplitude and other system details and independent of the other blocks (see Fig. 3, where the spectra for three $\mathcal{H}^{(\ell)}_n$ are sketched schematically).

Driving H^G_{MBL} with a sum of local terms such as in Eq. (1) couples each $\mathcal{H}^{(\ell)}_n$ to its neighbors [27] via terms allowing energy and matter transfer. Consider a single energy level for $n = 0$ (middle block, Fig. 3), indicated by the red line in the middle block. A time-independent coupling between the blocks couples it to the green blocks on each side, while a periodic coupling with frequency ω couples it to both the green and purple blocks by virtue of folding the energy spectrum into the ω-periodic quasienergies. Crucially, for ω larger than the typical width of the blocks, folding the local spectra has no effect [28] and a weak coupling does not delocalize the system, as it acts similarly to a time-independent perturbation [29]. In other words, the system can react to the driving by absorbing energy quanta ω only if there exist levels separated by this energy. In the presence of MBL the typical local bandwidth sets the maximum driving frequency to which the system can react [15].

Low frequency.—In the limit of low-frequency driving disorder is effectively suppressed and the delocalized phase is always reached.

This phenomenon is best understood in the time domain as follows. Consider time evolving with Hamiltonian $H_{1(2)}$ for the first (second) half of the period. This series of nonadiabatic changes to the system generically results in a broadening of the energy distribution, provided that the half-period $T/2$ is longer than the characteristic relaxation time [25,26]. Typically, this eventually leads to a fully mixed state occupying the entire Hilbert space equiprobably.

There are two central ingredients to this argument. The first is that the relaxation time does not diverge with system size so that the half-period $T/2$ *can* be longer. The existence of a dephasing time scale independent of system size [25,26] ensures that this is the case. The second is that repeatedly dephasing in the two different eigenstate bases does lead to energy delocalization. Since $H_{1,2}$ are both MBL Hamiltonians, the eigenstates of one are in general localized in terms of the eigenstates of the other. Nevertheless, repeated cycles of dephasing to alternating bases do indeed eventually lead to a fully mixed state, as is shown in the Supplemental Material [15].

A mobility edge: QREM as a case study.—We now turn to the case in which a mobility edge is present in the undriven spectrum. Our central result is based on the observation [8] that a periodic perturbation acting on a system couples each undriven state to states spread uniformly throughout the spectrum of H_0. As a result, if part of the spectrum corresponds to delocalized eigenstates, then all eigenstates of $H_{\rm eff}$ will necessarily be delocalized. We numerically confirm this by studying the QREM, recently studied in Ref. [31] where it was shown to have a mobility edge. This model is described in Ref. [31]: it is defined for N Ising spins with the Hamiltonian $H = E(\{\sigma_j^z\}) - \Gamma\sum_j \sigma_j^x$, where E is a random operator diagonal in the σ^z basis (that is, it assigns a random energy to each spin configuration) and Γ is a transverse field. Extensivity of the many-body spectrum is satisfied if the random energies are drawn from a distribution $P(E) = (1/\sqrt{\pi N})\exp(-E^2/N)$.

The diagnostic of localization we use is the participation ratio, defined for the state $|\psi\rangle$ as $\phi = \sum_n |\langle n|\psi\rangle|^4$, with n enumerating Fock states. ϕ approaches unity for a state localized on a single Fock state and 2^{-N} for one fully delocalized in the Fock space. The leftmost panel in Fig. 4 shows the average ϕ versus energy (scaled with system size) of the 256 eigenstates of an undriven $N = 8$ system averaged over 1000 disorder realizations, demonstrating the existence of a mobility edge.

Next, we drive the system by modulating $\Gamma(t) = \Gamma_0(1 + \delta\tilde{\delta}(t))$, $\tilde{\delta}(t) = +1$ (-1) for the first (second) half of the period with an amplitude $\delta = 0.2$ and frequency $\omega = 2\pi/T = 0.1$. The participation ratio of the eigenstates of $H_{\rm eff}$ are shown in the lower panel of Fig. 4. As expected, periodic driving causes delocalization of the entire spectrum so long as part of the undriven spectrum at the same Γ_0 is delocalised.

Outlook.—We have shown that many-body systems can remain many-body localized, with Poissonian level statistics, when they are subjected to slow driving. On the other hand, for fast driving or in the presence of a mobility edge, delocalization will occur, with driving inducing level repulsion.

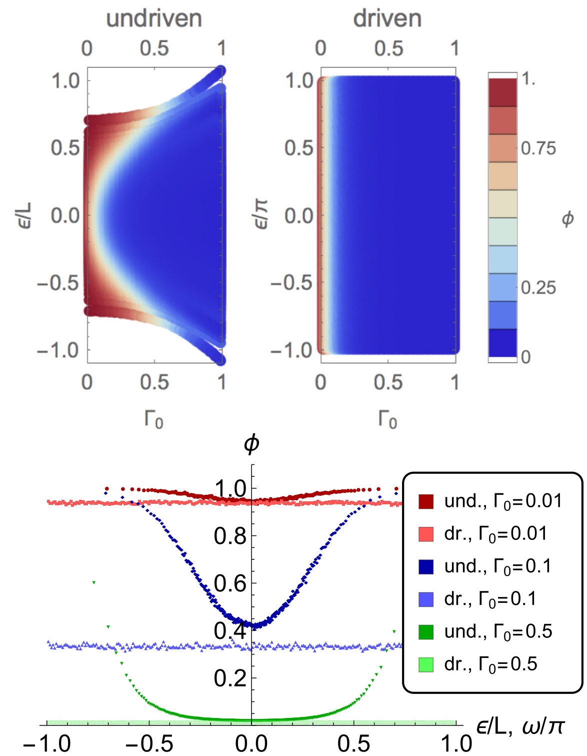

FIG. 4 (color online). Driving the QREM. The top left-hand figure shows the participation ratio ϕ for the eigenstates of the undriven model, showing a mobile region (blue) surrounded by a localized region (red). Driving with frequency $\omega/J = 0.1$ and amplitude $\delta/J = 0.2$ (top right) causes all states at a given Γ_0 to become as delocalized as the least localized state at that Γ_0 in the undriven model. This is also shown in the bottom panel, which shows ϕ for $\Gamma_0 = 0.01, 0.1, 0.5$ (red, blue, and green lines, from top to bottom) in the absence (presence) of driving with darker (lighter) color. The driven points always lie below the undriven points for the corresponding Γ_0. This is due to the strong mixing of all undriven eigenstates by the driving. All data in this figure are for eight spins and averaged over 1000 disorder realizations.

This "classification" of the behavior of MBL systems under driving immediately raises further questions. What are the time scales involved in reaching the long-time state we have discussed, how do they depend on the driving amplitude and frequency, and how do they differ between the localized and the delocalized limit? What is the precise difference between local and global driving as far as both the long-time state and the approach to it are concerned? More broadly, we have concentrated on systems with a bounded local spectrum. What happens if it is unbounded, as in the cases of a continuum system or of a lattice boson system? What if we bring the system in contact with a heat bath?

We believe that the dual out-of-equilibrium situation—driving and MBL—is only beginning to be explored and will prove to be fertile ground for future research.

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Note added.—Recently, two related works [32,33] have appeared. Each of these takes a somewhat different perspective, but they both establish phenomenologies essentially consistent with the one we report.

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