Hybrid Matter-Wave–Microwave Solitons Produced by the Local-Field Effect

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It was recently found that the electric local-field effect (LFE) can lead to a strong coupling of atomic Bose-Einstein condensates (BECs) to off-resonant optical fields. We demonstrate that the magnetic LFE gives rise to a previously unexplored mechanism for coupling a (pseudo-) spinor BEC or fermion gas to microwaves (MWs). We present a theory for the magnetic LFE and find that it gives rise to a short-range attractive interaction between two components of the (pseudo) spinor, and a long-range interaction between them. The latter interaction, resulting from deformation of the magnetic field, is locally repulsive but globally attractive, in sharp contrast with its counterpart for the optical LFE, produced by phase modulation of the electric field. Our analytical results, confirmed by the numerical computations, show that the long-range interaction gives rise to modulational instability of the spatially uniform state, and it creates stable ground states in the form of hybrid matter-wave–microwave solitons (which seem like one-dimensional magnetic monopoles), with a size much smaller than the MW wavelength, even in the presence of arbitrarily strong contact intercomponent repulsion. The setting is somewhat similar to exciton-polaritonic condensates in semiconductor microcavities. The release of matter waves from the soliton may be used for the realization of an atom laser. The analysis also applies to molecular BECs with rotational states coupled by the electric MW field.

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Ultracold atomic gases are used in various areas, including quantum metrology and interferometry [1–3], and the emulation of nonequilibrium quantum dynamics [4] and condensed-matter physics [5–7]. They have also drawn much interest as tunable media for quantum optics. In this vein, comanipulation of quantum light and matter waves has been studied in cavities loaded with atomic Bose-Einstein condensates (BECs) [8,9]. Raman superradiance in ultracold gases trapped in a cavity was used to generate stationary lasing with a bandwidth < 1 MHz, and with the average cavity photon number < 1 [10]. A mirrorless parametric resonance has been demonstrated for atomic BEC loaded into an optical lattice (OL) [11]. Optomechanics-induced large-scale structuring of ultracold atomic gases was reported in Ref. [12]. The resonant interaction of laser fields with BEC was also proposed for generating "photonic bubbles" emulating cosmology settings [13].

An important feature of the interaction of light with ultracold gases is the local-field effect (LFE), i.e., a feedback of the BEC on the light propagation. Strong LFE can be induced in cold-atom experiments, as recently demonstrated with the help of OLs [14–16]. Usually, OLs are sturdy structures, maintaining perfect interference fringes. However, asymmetric matter-wave diffraction on an OL formed by counterpropagating optical fields with unequal intensities [14] could be explained only by taking into regard deformation of the OL by the LFE [15]. Conventional rigid OLs and their deformable counterparts may be categorized as "stiff" and "soft" ones. Polaritonic solitons produced by the hybridization of coupled atomic and optical waves have been predicted in soft OLs [16], and are considered promising for matter-wave interferometry due to their high atom number density [17]. These results demonstrate the potential of the soft OLs in studies of systems combining quantum matter and photons, akin to exciton polaritons in microcavities [18]. Furthermore, BECs built of up to 10^8 atoms are now available [19]. For such massive BECs, the refraction-index change through the perturbation of the atomic density may be significant, even for the laser-frequency detuning from the resonance $\gg 1$ GHz, allowing the LFE to generate hybrid matter-wave–photonic states [16].

The use of spinor gases opens ways for the emulation of spin-orbit coupling [7] and quantum magnetism [20], as well as for the realization of quantum matter-wave [21] and microwave (MW) [22] optics. In this context, the MW magnetic field is used for manipulating spin states. Coupling different hyperfine atomic states by MWs was studied in other contexts, too, including dressed states [23], domain walls [24], and instabilities [25].

However, manifestations of the magnetic LFE (MLFE) in quantum gases have not been studied yet, unlike its electric counterpart. In this Letter, we develop the theory of the MLFE for a MW field coupled to the pseudospinor BEC. The MW wavelength (greater than or equivalent to several millimeters) exceeds the typical size of the BEC by orders of magnitude. In this situation, the BEC was considered before as a thin slice that affects the phase of the MW field. We find that the MLFE causes subwavelength deformations of the MW amplitude profile, too, inducing a long-range interaction between components of the pseudospinor BEC. Unlike the electric LFE [16], where a nonlocal interaction is induced by phase perturbations, the long-range interaction generated by the MLFE is locally repulsive but globally attractive. The same effect leads to local attraction between the components of the BEC, which may compete with collisional repulsion between them. We demonstrate that these interactions create self-trapped ground states (GSs) in the form of hybrid matter-wave–microwave solitons, whose field component seems like that of a magnetic monopole. Actually, the solitons realize a dissipation- and pump-free counterpart of hybridized exciton-polariton complexes in dissipative microcavities, pumped by external laser fields. In opposition to our case, the size of those complexes is much larger than the polaritonic wavelength, while the effective mass of the excitons is usually considered infinite [18]. Note that direct dissipation-free emulation of the exciton-polariton setting is possible, too, in a dual-core optical system [26].

We consider the magnetic coupling of the MW radiation with frequency ω_L to two hyperfine atomic states $|\downarrow\rangle$ and $|\uparrow\rangle$ which compose the BEC pseudo spinor [7,27], with free Hamiltonian $H_0 = \hat{p}^2/(2m) - (\hbar\delta/2)\sigma_3$, where $\hbar\delta$ is the energy difference between the two states, σ_3 is the Pauli matrix, and $\hat{p}$ the atomic momentum. The magnetic interaction is governed by the term

$$H_{\rm int} = -\begin{pmatrix} \mathbf{m}_{\downarrow\downarrow} & \mathbf{m}_{\downarrow\uparrow} \\ \mathbf{m}_{\uparrow\downarrow} & \mathbf{m}_{\uparrow\uparrow} \end{pmatrix} \cdot (\mathbf{B}e^{-i\omega_L t} + \mathbf{B}^* e^{i\omega_L t}). \quad (1)$$

Here, $\mathbf{m}_{\downarrow,\uparrow}$ are matrix elements of the magnetic momentum, and the magnetic induction is $\mathbf{B} = \mu_0\mathbf{H} + \mathbf{M}$ with $\mathbf{M} = \mathbf{m}_{\downarrow\uparrow}\psi_\downarrow^*\psi_\uparrow$, with $\psi_\downarrow$ and $\psi_\uparrow$ building the pseudospinor wave function, $|\Psi\rangle = (\psi_\downarrow \exp(i\omega_L t/2), \psi_\uparrow \exp(-i\omega_L t/2))^T$. In the rotating-wave approximation, $|\psi\rangle = (\psi_\downarrow, \psi_\uparrow)^T$ satisfies the system of coupled Gross-Pitaevskii equations (GPEs):

$$i\hbar\frac{\partial|\psi\rangle}{\partial t} = \Bigg[-\frac{\hbar^2}{2m}\nabla^2 + \frac{\hbar\Delta}{2}\sigma_3 - \mu_0\begin{pmatrix} 0 & \mathbf{m}_{\downarrow\uparrow}\cdot\mathbf{H}^* \\ \mathbf{m}_{\uparrow\downarrow}\cdot\mathbf{H} & 0 \end{pmatrix} - \mathbf{m}_{\uparrow\downarrow}\cdot\mathbf{m}_{\downarrow\uparrow}\begin{pmatrix} |\psi_\uparrow|^2 & 0 \\ 0 & |\psi_\downarrow|^2 \end{pmatrix}\Bigg]|\psi\rangle, \quad (2)$$

with the MW detuning from the atomic transition $\Delta = \omega_L - \delta$. Neglecting the time derivatives of $\mathbf{H}$ and $\mathbf{M}$ for the low-frequency MW, the wave equation for $\mathbf{H}$ reduces to the Helmholtz form, $\nabla^2\mathbf{H} + \omega_L^2/c^2\mathbf{H} = -\varepsilon_0\omega_L^2\mathbf{M}$.

We consider a cigar-shaped condensate with effective cross-section area S, subject, as usual, to tight transverse confinement, and it is irradiated by two linearly polarized counterpropagating microwaves along the cigar's axis x. Eliminating the transverse variation of the fields under these conditions [28], we reduce the coupled GPEs and Helmholtz equation to the normalized form,

$$i\frac{\partial|\phi\rangle}{\partial\tau} = \left[-\frac{1}{2}\frac{\partial^2}{\partial x^2} + \eta\sigma_3 - \begin{pmatrix} \beta|\phi_\uparrow|^2 & \mathcal{H}^* \\ \mathcal{H} & \beta|\phi_\downarrow|^2 \end{pmatrix}\right]|\phi\rangle, \quad (3)$$

$$\partial_x^2\mathcal{H} + \kappa^2\mathcal{H} = -\gamma\phi_\downarrow^*\phi_\uparrow, \quad (4)$$

with $\phi_{\downarrow,\uparrow} \equiv \sqrt{X_0}\psi_{\downarrow,\uparrow}$, $\tau \equiv t/t_0$, $x \equiv X/X_0$, $H \equiv H/h_c$ and $\beta \equiv \mathbf{m}_{\downarrow\uparrow}\cdot\mathbf{m}_{\uparrow\downarrow}t_0/(\hbar S X_0)$, $\eta \equiv t_0\Delta/2$, $\gamma \equiv \mathcal{N}\varepsilon_0\omega_L^2 m_{\downarrow\uparrow}X_0/(h_c S)$, $\kappa \equiv X_0\omega_L/c$ measured in natural units of time and coordinate, $t_0 = \hbar/(\mu_0 m_{\downarrow\uparrow}h_c)$, $X_0 = \sqrt{\hbar t_0/m}$, where h_c is the magnetic-field strength and $\mathcal{N}$ the total atom number. The rescaled wave function is subject to normalization $N_\downarrow + N_\uparrow \equiv \int_{-\infty}^{+\infty}[|\phi_\uparrow(x)|^2 + |\phi_\downarrow(x)|^2]dx = 1$. If collisions between atoms in different spin states are taken into account, which may be controlled by the Feshbach resonance [29], β in Eq. (3) combines contributions from the MLFE and direct interactions. Upon proper rescaling, the same system of Eqs. (3) and (4) applies to a degenerate gas of fermions [30,31] with spin 1/2, in which $\psi_\downarrow$ and $\psi_\uparrow$ represent two spin components, coupled to the magnetic field $\mathcal{H}$, and asymmetry η is imposed by a dc magnetic field.

Equation (4) can be solved using the respective Green's function [32], $\mathcal{H}(x,t) = A\exp(i\kappa x) + C\exp(-i\kappa x) - (\gamma/2\kappa)\int_{-\infty}^{+\infty}\sin(\kappa|x-x'|)\phi_\downarrow^*(x',t)\phi_\uparrow(x',t)dx'$, where constants A and C represent the solution of the corresponding homogeneous equation, i.e., they are amplitudes of two incident counterpropagating microwaves. Since the MW wavelength is far larger than the condensate size ($\kappa \sim 10^{-5}$), one may set $\exp(i\kappa x) \approx 1$ and $\sin(\kappa|x-x'|)/\kappa \approx |x-x'|$ in the domain occupied by the condensate to simplify the solution: $\mathcal{H}(x,t) = \mathcal{H}_0 - (\gamma/2)\int_{-\infty}^{+\infty}|x-x'|\phi_\downarrow^*(x',t)\phi_\uparrow(x',t)dx'$, where $\mathcal{H}_0 \equiv A + C$ is made real by means of a phase shift. The form of the magnetic field at $|x| \to \infty$ resembles that of a one-dimensional *artificial magnetic monopole* [33]: $\mathcal{H}_{\rm asympt}(x) = -\gamma N_\Updownarrow|x|$, $N_\Updownarrow \equiv \int_{-\infty}^{+\infty}\phi_\downarrow^*(x)\phi_\uparrow(x)dx$.

Substituting the solution for $\mathcal{H}$ in Eq. (3), we arrive at the final form of the GPEs:

$$i\frac{\partial\phi_\downarrow}{\partial\tau} = -\frac{1}{2}\frac{\partial^2\phi_\downarrow}{\partial x^2} - \mathcal{H}_0\phi_\uparrow + \eta\phi_\downarrow - \beta|\phi_\uparrow|^2\phi_\downarrow + \frac{\gamma}{2}\phi_\uparrow(x)\int_{-\infty}^{+\infty}|x-x'|\phi_\downarrow(x',t)\phi_\uparrow^*(x',t)dx', \quad (5)$$

$$i\frac{\partial\phi_\uparrow}{\partial\tau} = -\frac{1}{2}\frac{\partial^2\phi_\uparrow}{\partial x^2} - \mathcal{H}_0\phi_\downarrow - \eta\phi_\uparrow - \beta|\phi_\downarrow|^2\phi_\uparrow + \frac{\gamma}{2}\phi_\downarrow(x)\int_{-\infty}^{+\infty}|x-x'|\phi_\downarrow^*(x',t)\phi_\uparrow(x',t)dx'. \quad (6)$$

Thus, the MLFE gives rise to two nonlinear terms: the one $\sim\beta$ accounts for short-range interaction, while the integral term represents the long-range interaction, which is *locally repulsive*, but *globally attractive* because the repulsion kernel, $|x-x'|$, growing at $|x|\to\infty$, suggests a possibility of self-trapping. The mechanism of creating bright solitons by the spatially growing strength of local self-repulsion was proposed in Ref. [34] and then extended for nonlocal dipolar-BEC [35] settings; however, that mechanism was imposed by appropriately engineered spatial modulation of the nonlinearity, while here we consider the self-trapping in free space.

The symmetric version of Eqs. (5) and (6), with $\eta=0$, may be combined into separate equations for $\phi_\pm \equiv \phi_\downarrow \pm \phi_\uparrow$, with trapping (for $+$) and expulsive (for $-$) potentials, respectively. Therefore, this system has only symmetric solutions ($\phi_-=0$). At $|x|\to\infty$, Eqs. (5) and (6) take the linear asymptotic form, $i\partial_\tau\phi_{\downarrow\uparrow} = -(1/2)\partial_{xx}^2\phi_{\downarrow\uparrow} + (\gamma N_\Updownarrow/2)|x|\phi_{\uparrow\downarrow}$; hence, solutions for the asymmetric system ($\eta\neq 0$) have symmetric asymptotic tails, too, $\phi_\downarrow=\phi_\uparrow \sim \exp\left(-(2/3)\sqrt{\gamma N_\Updownarrow}|x|^{3/2}\right)$.

The GS of system (5) and (6) with $\eta=0$ and chemical potential μ is sought for as $\phi_{\downarrow\uparrow}=e^{-i\mu t}\varphi(x)$, where real φ satisfies the equation

$$\tilde{\mu}\varphi = \left[-\frac{1}{2}\frac{d^2}{dx^2} - \beta\varphi^2 + \frac{\gamma}{2}\int_{-\infty}^{+\infty}|x-x'|\varphi^2(x')dx'\right]\varphi, \quad (7)$$

with $\mu\equiv\tilde{\mu}-\mathcal{H}_0$. Thus, $\mathcal{H}_0$ only shifts the chemical potential in the zero-detuning system. For $\beta=0$, it follows from Eq. (7) that $\tilde{\mu}$ and the magnetic field obey scaling relations: $\{\tilde{\mu}(\gamma), \mathcal{H}(x;\gamma)-\mathcal{H}_0\} = (\gamma/\gamma_0)^{2/3}\{\tilde{\mu}(\gamma=\gamma_0), \mathcal{H}(x;\gamma=\gamma_0)-\mathcal{H}_0\}$, where γ_0 is a fixed constant. Thus, for $\eta=0$ and $\beta=0$, all the GSs may be represented by a single one, plotted in Fig. 1(a), which was found by means of the imaginary-time method. This is a hybrid soliton, built of the self-trapped matter wave coupled to the subwavelength deformation of the magnetic field.

In the presence of the local self-repulsion ($\beta<0$), the GS can be found with the help of the Thomas-Fermi approximation (TFA), which neglects the second derivative in Eq. (7):

$$\varphi_{\mathrm{TFA}}^2(x) = \begin{cases} (1/4)\sqrt{\gamma/|\beta|}\cos\xi \text{ at } |\xi|<\pi/2, \\ 0 \text{ at } |\xi|>\pi/2, \end{cases} \quad (8)$$

$$\mathcal{H}_{\mathrm{TFA}}(x)+\mu = \begin{cases} (1/4)\sqrt{\gamma|\beta|}\cos\xi \text{ at } |\xi|<\pi/2, \\ -\sqrt{\gamma|\beta|}(|\xi|-\pi/2)/4 \text{ at } |\xi|>\pi/2, \end{cases} \quad (9)$$

where $\xi\equiv\sqrt{\gamma/|\beta|}x$. An example, displayed in Fig. 1(b), shows very good agreement of the TFA with the numerical solution. Thus, the globally attractive long-range interaction induced by the MLFE creates the self-trapped GS, overcoming the arbitrarily strong self-repulsive contact interaction.

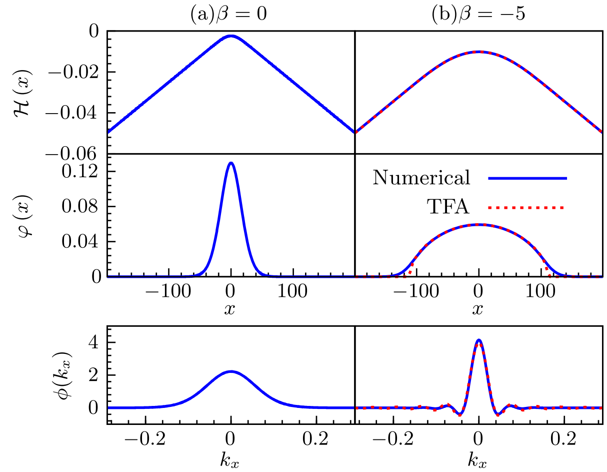

FIG. 1 (color online). (a) The GS wave function, $\varphi(x)$, along with its Fourier transform, $\phi(k_x)$, and magnetic field, $\mathcal{H}(x)$, for $\beta=0$, with $\tilde{\mu}=4.26$. (b) Comparison of the GS, as predicted by the TFA [Eqs. (8), (9), and (10)] (the dashed curves) and found numerically (the solid lines) for $\beta=-5$ with $\mu=0.079$. In both plots, $\eta=0$, $H_0=0$, and $\gamma=10^{-3}$.

The time-of-flight spectrum produced by releasing the condensate can be used in the experiment to detect the solitons predicted here, characterized by their distribution over the longitudinal momentum, $\phi(k_x)\equiv\int_{-\infty}^{+\infty}e^{-ik_x x}\varphi(x)dx$, as shown in Fig. 1 for $\beta=0$ and -5. Specifically, the TFA yields

$$\phi_{\mathrm{TFA}}(k_x) = \frac{\pi(|\beta|/\gamma)^{1/4}}{8\Gamma(5/4+\sqrt{|\beta|/4\gamma}k_x)\Gamma(5/4-\sqrt{|\beta|/4\gamma}k_x)}, \quad (10)$$

where Γ is the gamma function; see the right bottom panel in Fig. 1. Note that expression (10) vanishes at $k_x=\pm 2\sqrt{\gamma/|\beta|}(5/4+n)$, $n=0,1,2,\ldots$. The strong compression of the soliton in the momentum space, evident in Fig. 1(b), may be used for the design of matter-wave lasers [36,37], as the released beam will feature high velocity coherence.

The existence of bright solitons in free space is related to the modulational instability (MI) of flat states [38]. The flat solution to Eq. (7) is $\phi=\Phi_0\exp(-i\mu_0 t)$, with the divergence of μ_0 regularized by temporarily replacing $|x-x'|$ in Eq. (7) by $|x-x'|\exp(-\epsilon|x-x'|)$ with small $\epsilon>0$. The MI analysis for small perturbations with wave number k and MI gain λ proceeds, as usual, by substituting $\phi=\Phi(x,t)\exp[i\chi(x,t)]$, with $\{\Phi;\chi\}=\{\Phi_0;-\mu_0 t\}+\{\Phi_1^{(0)};\chi_1^{(0)}\}\exp(ikx+\lambda t)$, where $\{\Phi_1^{(0)};\chi_1^{(0)}\}$ are perturbation amplitudes. The subsequent linearization and then setting $\epsilon=0$ yields $\lambda^2=-(k^4/4-\beta\Phi_0^2k^2-\gamma\Phi_0^2)$. Because of nonlocality, λ^2 does not vanish at $k^2\to 0$, in

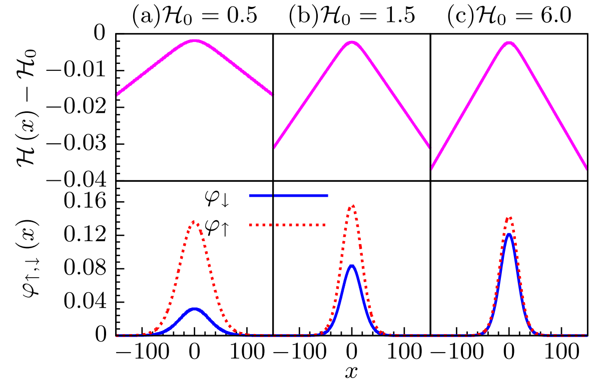

FIG. 2 (color online). Numerically found profiles of the GS wave functions and magnetic field in the system with $\eta = 1$, $\beta = 0$, and $\gamma = 10^{-3}$, for $\mathcal{H}_0 = 0.5$ (a), 1.5 (b), 6.0 (c). The respective chemical potentials are $\mu = -1.12$ (a), -1.80 (b), -6.079 (c). For the strongly asymmetric soliton in (a), the analytical approximation predicts the amplitude ratio of the two components 0.250, while its numerically found counterpart is 0.235.

contrast with local models [38]. The MI is *always present*, as λ^2 remains positive at $k^2 < 2|\Phi_0|(\sqrt{\beta^2\Phi_0^2 + \gamma} + \beta|\Phi_0|)$. Thus, arbitrarily strong local self-repulsion, with $\beta < 0$, does not suppress the MI.

In the system with detuning, i.e., $\eta \neq 0$ in Eqs. (5) and (6), the background magnetic field $\mathcal{H}_0$ is an essential parameter. Figure 2 plots the GS wave functions, $\varphi_{\downarrow\uparrow}(x)$, and the corresponding magnetic field, $\mathcal{H}(x)$, for different values of $\mathcal{H}_0$. The GS exhibits asymmetry between the lower- and higher-energy components at $\mathcal{H}_0 \lesssim \eta$, while large $\mathcal{H}_0$ suppresses the asymmetry, as seen in Fig. 3, which displays the scaled norms of the two components, $N_{\downarrow\uparrow}$, versus $\mathcal{H}_0$.

For $\eta \gg \mathcal{H}_0$, strongly asymmetric GSs can be found using the stationary version of Eqs. (5) and (6) with chemical potential $\mu = -\eta + \Delta\mu$, $|\Delta\mu| \ll \eta$. Then, Eq. (5) eliminates the weak component in favor of the strong one: $\varphi_\downarrow \approx (2\eta)^{-1}\mathcal{H}_0\varphi_\uparrow$, and $N_\downarrow \approx \mathcal{H}_0^2/(4\eta^2)$, which agrees well with the numerical results, as shown by the inset in Fig. 3. The substitution of this into Eq. (6) yields

$$\left(\Delta\mu + \frac{\mathcal{H}_0^2}{2\eta}\right)\varphi_\uparrow = -\frac{1}{2}\frac{d^2\phi_\uparrow}{dx^2} - \frac{\beta\mathcal{H}_0^2}{4\eta^2}\varphi_\uparrow^3 + \frac{\gamma\mathcal{H}_0^2}{8\eta^2}\varphi_\uparrow(x)\int_{-\infty}^{+\infty}|x - x'|\varphi_\uparrow^2(x')dx', \tag{11}$$

which is actually tantamount to Eq. (7). The respective small deformation of the magnetic field is $\mathcal{H}(x) = \mathcal{H}_0 - (\gamma\mathcal{H}_0/4\eta)\int_{-\infty}^{+\infty}\phi_\uparrow^2(x')|x - x'|dx'$.

We have confirmed the stability of all of the GS states by direct simulations of Eqs. (5) and (6) with randomly perturbed initial conditions. For the symmetric system with $\eta = \beta = 0$, the above-mentioned scaling implies that the stability of a single GS guarantees the stability of all GSs, while for the detuned system the stability had to be checked by varying $\mathcal{H}_0$ at a fixed γ and η. Furthermore, the stability for $\eta = \beta = 0$ is predicted by the *anti-Vakhitov-Kolokolov* (anti-VK) criterion, which states that the necessary stability condition for *bright* solitons supported by the *repulsive* nonlinearity is $d\mu/dN > 0$ [39] (the VK criterion proper, which pertains to attractive nonlinearity, is $d\mu/dN < 0$ [38,40]). Although $N = 1$ was fixed above, the criterion can be applied by means of a rescaling which fixes γ and liberates N. As a result, the scaling relation $\tilde{\mu} \sim \gamma^{2/3}$ is replaced by $\tilde{\mu} \sim N^{2/3}$; hence, the criterion holds.

Summarizing, we have explored the MLFE (magnetic local-field effect) in the BEC built of two atomic states coupled by the MW (microwave) field. We have deduced the system of evolution equations for the matter-wave components and the MW magnetic field, which demonstrate that the subwavelength distortion of the magnetic field by perturbations of the local atom density induces short- and long-range interactions between the BEC components. The same equations apply to the spinor wave function of a fermionic gas coupled to the MW magnetic field. The model produces the self-trapped GS in the form of the hybridized BEC-MW subwavelength solitons, which may be considered counterparts of the hybrid exciton-polariton solitons in the dissipation-free system. Basic characteristics of the solitons were obtained analytically. The release of the solitons from the cigar-shaped trap may be used as a source of coherent matter waves for an atom laser. The flat states in the present system are subject to the modulational instability, which is naturally related to the existence of the bright solitons.

It is straightforward to extend the analysis to molecular BECs, with the transition between two rotational states driven by the electric MW field. Such ultracold molecular

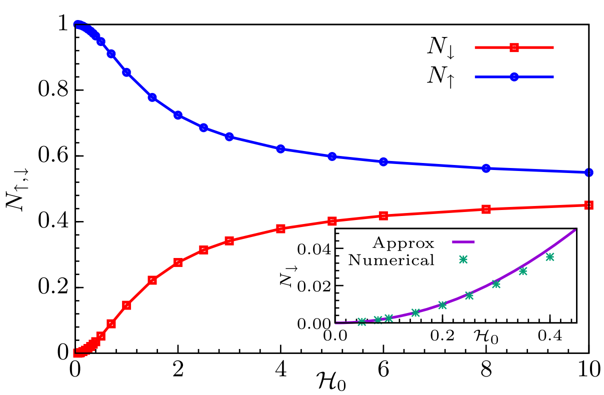

FIG. 3 (color online). The relative share of the total norm of each component in the system with $\gamma = 10^{-3}$, $\beta = 0$, $\eta = 1$ vs the background magnetic field, $\mathcal{H}_0$. The inset shows the dependence for $N_\downarrow$ at small values of $\mathcal{H}_0$ (the stars) *vis-à-vis* the analytical prediction.

gases have a potential for quantum simulations of condensed-matter physics [41]. An interesting extension may be the analysis of the system with a *three-component* bosonic wave function corresponding to spin $F = 1$ [42], in which a single MW field couples components with $m_F = \pm 1$ to the one with $m_F = 0$. Other relevant directions for the extension are a search for excited states in the system, in addition to the GS, and the analysis of the two-dimensional setting. Furthermore, it is relevant to investigate a potential effect of the MLFE on quantum precision measurements.

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