Model for Thermal Relic Dark Matter of Strongly Interacting Massive Particles

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(Received 8 January 2015; published 10 July 2015)

A recent proposal is that dark matter could be a thermal relic of $3 \to 2$ scatterings in a strongly coupled hidden sector. We present explicit classes of strongly coupled gauge theories that admit this behavior. These are QCD-like theories of dynamical chiral symmetry breaking, where the pions play the role of dark matter. The number-changing $3 \to 2$ process, which sets the dark matter relic abundance, arises from the Wess-Zumino-Witten term. The theories give an explicit relationship between the $3 \to 2$ annihilation rate and the $2 \to 2$ self-scattering rate, which alters predictions for structure formation. This is a simple calculable realization of the strongly interacting massive-particle mechanism.

DOI: 10.1103/PhysRevLett.115.021301 PACS numbers: 95.35.+d

Introduction.—The majority of the matter content of our Universe is in the form of dark matter (DM). An appealing explanation for its measured abundance is that it is a thermal relic of the early Universe. The most well-studied thermal scenario is that of a weakly interacting massive particle, whose relic abundance is set by $2 \to 2$ annihilations, typically into standard model (SM) particles. This mechanism predicts a dark matter mass of order the weak scale for coupling of order the weak coupling.

Reference [1] proposed a new paradigm for achieving thermal relic dark matter. The requisite features of the mechanism are the following. (i) The dark matter relic abundance is set thermally by the freeze-out of a $3 \to 2$ process that reduces the number of dark matter particles within the dark sector. (ii) At the time of freeze-out, dark matter is in thermal equilibrium with the SM. This setup, termed the strongly interacting massive-particle (SIMP) mechanism, robustly predicts light dark matter with a mass in the MeV to GeV range, with strong self-interactions. Annihilations into SM particles are subdominant during freeze-out, but DM scattering off the SM bath is fast enough to maintain kinetic equilibrium between the dark and visible sectors. The strongly interacting hidden sector is expected to contribute to DM self-scattering cross sections that are relevant for structure formation.

In what follows, we find explicit strongly coupled realizations for the hidden sector that admit the $3 \to 2$ process of the SIMP mechanism. Explicit viable realizations for the mediation mechanism between the dark and visible sectors exist, and will be presented in detail in a forthcoming publication [2].

The SIMPlest realization.—In Ref. [1], a weakly coupled toy model that incorporates the SIMP mechanism and leads to stable dark matter was presented. Here, we present three classes of strongly coupled gauge theories that realize the SIMP mechanism. The basic idea is as follows. We use the well-known 5-point interaction term present in theories of chiral symmetry breaking, first discovered by Wess and Zumino [3] and later studied by Witten [4,5], as the source of the $3 \to 2$ interactions. This term in massless QCD describes the low energy limit of two kaons annihilating into three pions. In a given theory, the existence of the Wess-Zumino-Witten (WZW) term is dictated by a topological condition on the symmetry-breaking pattern; for coset spaces with nontrivial fifth homotopy groups, the WZW term is nonvanishing. This 5-point interaction then generates the $3 \to 2$ freeze-out process. In what follows, we demonstrate this explicitly.

We first consider and review an $\mathrm{Sp}(N_c)$ gauge theory with $2N_f$ Weyl fermions in the fundamental N_c-dimensional representation (with the number of colors N_c even). In the massless limit the UV description takes a simple form

$$\mathcal{L}_{\mathrm{SIMP}} = -\frac{1}{4} F^a_{\mu\nu} F^{\mu\nu a} + \bar{q}_i i \not{D} q_i, \qquad i = 1, \ldots, 2N_f, \quad (1)$$

which admits a global $\mathrm{SU}(2N_f)$ symmetry among the Weyl fermions q_i. It is believed that this model, for moderately small N_f in the asymptotically free range, leads to chiral symmetry breaking with the order parameter

$$\langle q_i q_j \rangle = \mu^3 J_{ij}, \quad (2)$$

where μ is of mass dimension 1 and $J = i\sigma_2 \otimes \mathbb{1}_{N_f}$ is a $2N_f \times 2N_f$ antisymmetric matrix that preserves an $\mathrm{Sp}(2N_f)$ subgroup of the $\mathrm{SU}(2N_f)$ flavor symmetry [5–8]. For $N_f \geq 2$, the topological condition is met,

$$\pi_5(\mathrm{SU}(2N_f)/\mathrm{Sp}(2N_f)) = \mathbb{Z}, \qquad N_f \geq 2, \tag{3}$$

and the WZW term is nonvanishing. The coset space $\mathrm{SU}(2N_f)/\mathrm{Sp}(2N_f)$ is a symmetric space and is parametrized by $N_\pi = 2N_f^2 - N_f - 1$ pion fields π^a, corresponding to the broken generators T^a, with $a = 1, \ldots, N_\pi$. The pions furnish a rank-2 antisymmetric tensor representation of the unbroken $\mathrm{Sp}(2N_f)$, and are stable. Assuming the pions are the lightest states in the theory, dark matter is comprised of these N_π pions.

A simple parametrization is found by performing a transformation on the vacuum and promoting the transformation parameters to fields

$$\langle qq \rangle = \mu^3 J \to \mu^3 V J V^T \equiv \mu^3 \Sigma, \tag{4}$$

where $V = \exp(i\pi/f_\pi)$ and f_π is the decay constant. Since the broken generators obey $\pi J - J\pi^T = 0$ with $\pi = \pi^a T^a$ and $\mathrm{Tr}(T^a T^b) = 2\delta^{ab}$, we have

$$\Sigma = \exp(2i\pi/f_\pi)J. \tag{5}$$

A minimal realization of the $3 \to 2$ mechanism is an $\mathrm{Sp}(2) \simeq \mathrm{SU}(2)$ gauge theory with $N_f = 2$ flavors. Dark matter is comprised of five pions that transform as a 5-plet under the preserved Sp(4) flavor symmetry. The coset space of $\mathrm{SU}(4)/\mathrm{Sp}(4) = \mathrm{SO}(6)/\mathrm{SO}(5)$ is then topologically an S^5. (See, e.g., Refs. [9–20] for lattice work on low-lying spectra in the minimal Sp(2) gauge theory with quarks in the fundamental representation, and Refs. [21–24] for dark-matter examples.)

The relevant pion Lagrangian receives contributions from several terms. The canonically normalized kinetic term yields kinetic and 4-point interactions for the pions

$$\begin{aligned}\mathcal{L}_{\mathrm{kin}} &= \frac{f_\pi^2}{16}\mathrm{Tr}\partial_\mu\Sigma\partial^\mu\Sigma^\dagger = \frac{1}{4}\mathrm{Tr}\partial_\mu\pi\partial^\mu\pi \\ &\quad - \frac{1}{6f_\pi^2}\mathrm{Tr}(\pi^2\partial^\mu\pi\partial_\mu\pi - \pi\partial^\mu\pi\pi\partial_\mu\pi) + \mathcal{O}(\pi^6/f_\pi^4),\end{aligned} \tag{6}$$

where in our normalization $\mathrm{Tr}(\pi^2) = 2\pi^a\pi^a$. The Wess-Zumino-Witten term [3,4] yields 5-point pion interactions. It can be written as an integral on the boundary of a five-dimensional disk, identified with our four-dimensional spacetime

$$\mathcal{S}_{\mathrm{WZW}} = \frac{-iN_c}{240\pi^2}\int \mathrm{Tr}(\Sigma^\dagger d\Sigma)^5. \tag{7}$$

To leading order in pion fields

$$\mathcal{L}_{\mathrm{WZW}} = \frac{2N_c}{15\pi^2 f_\pi^5}\epsilon^{\mu\nu\rho\sigma}\mathrm{Tr}[\pi\partial_\mu\pi\partial_\nu\pi\partial_\rho\pi\partial_\sigma\pi], \tag{8}$$

which is responsible for the required $3 \to 2$ annihilation process. Finally, an $\mathrm{Sp}(2N_f)$-preserving mass term can be written for the quarks:

$$\mathcal{L}_{\mathrm{mass}} = -\frac{1}{2}M^{ij}q_i q_j + \mathrm{c.c.}, \qquad M^{ij} = m_Q J^{ij}. \tag{9}$$

The pions are then pseudo-Goldstone bosons of the broken symmetry and acquire a mass, as well as contact interactions:

$$\begin{aligned}\Delta\mathcal{L}_{\mathrm{eff}} &= -\frac{1}{2}m_Q\mu^3\mathrm{Tr}J\Sigma + \mathrm{c.c.} \\ &= -\frac{m_\pi^2}{4}\mathrm{Tr}\pi^2 + \frac{m_\pi^2}{12f_\pi^2}\mathrm{Tr}\pi^4 + \mathcal{O}(\pi^6/f_\pi^4),\end{aligned} \tag{10}$$

where

$$m_\pi^2 = 8\frac{m_Q\mu^3}{f_\pi^2}. \tag{11}$$

Combining all the above we arrive at the relevant pion Lagrangian

$$\begin{aligned}\mathcal{L}_\pi &= \mathcal{L}_{\mathrm{kin}} + \Delta\mathcal{L}_{\mathrm{eff}} + \mathcal{L}_{\mathrm{WZW}} \\ &= \frac{1}{4}\mathrm{Tr}\partial_\mu\pi\partial^\mu\pi - \frac{m_\pi^2}{4}\mathrm{Tr}\pi^2 + \frac{m_\pi^2}{12f_\pi^2}\mathrm{Tr}\pi^4 \\ &\quad - \frac{1}{6f_\pi^2}\mathrm{Tr}(\pi^2\partial^\mu\pi\partial_\mu\pi - \pi\partial^\mu\pi\pi\partial_\mu\pi) \\ &\quad + \frac{2N_c}{15\pi^2 f_\pi^5}\epsilon^{\mu\nu\rho\sigma}\mathrm{Tr}[\pi\partial_\mu\pi\partial_\nu\pi\partial_\rho\pi\partial_\sigma\pi] + \mathcal{O}(\pi^6).\end{aligned} \tag{12}$$

There can also be $\mathcal{O}(\pi^4)$ terms with four derivatives and higher. These contribute to four-pion self-scattering with a naive-dimensional-analysis [25] suppression of at least $\mathcal{O}(m_\pi^2/\Lambda^2)$, where $\Lambda = 2\pi f_\pi$, compared to those we keep. The $\mathcal{O}(\pi^5)$ terms with four derivatives that we use are the leading 5-point pion interactions of the theory.

The same principle presented above to construct strongly coupled models, which admit $3 \to 2$ interactions and realize the SIMP mechanism, is generalizable to other gauge and flavor symmetries. For instance, one can consider a generalized QCD-like theory with an $\mathrm{SU}(N_c)$ gauge group and N_f Dirac fermions in the fundamental representation. The global flavor symmetry of the theory is $\mathrm{SU}(N_f) \times \mathrm{SU}(N_f)$, which upon chiral symmetry breaking preserves an $\mathrm{SU}(N_f)$ subgroup. Similarly, an $\mathrm{O}(N_c)$ gauge group with N_f fermions in the vector representation exhibits an $\mathrm{SU}(N_f)$ flavor symmetry, which breaks to $\mathrm{SO}(N_f)$ once chiral symmetry breaking occurs. The topological condition on the coset space in each of these cases,

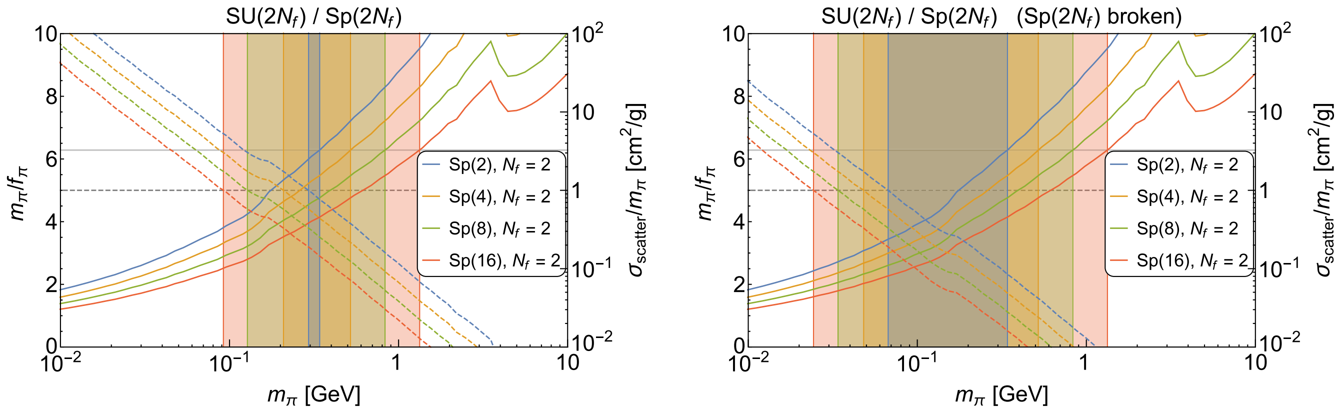

FIG. 1 (color online). Solid curves: the solution to the Boltzmann equation of the $3 \to 2$ system, yielding the measured dark matter relic abundance for the pions, m_π/f_π, as a function of the pion mass (left axis). Dashed curves: the self-scattering cross section along the solution to the Boltzmann equation, $\sigma_{\mathrm{scatter}}/m_\pi$, as a function of the pion mass (right axis). All curves are for selected values of N_c and N_f, for an $\mathrm{Sp}(N_c)$ gauge group with a conserved (left panel) or broken (right panel) $\mathrm{Sp}(2N_f)$ flavor symmetry. The solid horizontal line depicts the perturbative limit of $m_\pi/f_\pi \lesssim 2\pi$, providing a rough upper limit on the pion mass; the dashed horizontal line depicts the bullet-cluster and halo shape constraints on the self-scattering cross section (16), placing a rough lower limit on the pion mass. Each shaded region depicts the resulting approximate range for m_π for the corresponding symmetry structure.

$$\pi_5(\mathrm{SU}(N_f)) = \mathbb{Z}, \qquad N_f \geq 3,$$
$$\pi_5(\mathrm{SU}(N_f)/\mathrm{SO}(N_f)) = \mathbb{Z}, \qquad N_f \geq 3, \tag{13}$$

admits the WZW term (8), and $3 \to 2$ pion interactions are present. The relevant pion Lagrangian terms in each of these cases are readily obtained by replacing J, the $\mathrm{Sp}(2N_f)$ invariant, in Eqs. (2), (4), (5), (9), and (10), by $\mathbb{1}$, which is the $\mathrm{SU}(N_f)$ and $\mathrm{SO}(N_f)$ invariant. Additionally, for the $\mathrm{SU}(N_c)$ case, the quark bilinear in Eqs. (2) and (4) is understood as $q\bar{q}$, with appropriate modification to the transformation in Eq. (4). Reading off the pion interactions accordingly, the Lagrangian of interest remains Eq. (12), with a change of sign in the relation (11).

In what follows, we explicitly study the self-interactions of the pions in the three classes of gauge theories presented, namely $\mathrm{Sp}(N_c)$, $\mathrm{SU}(N_c)$, and $\mathrm{O}(N_c)$. These theories yield qualitatively similar results, with some small quantitative differences described below. An exception arises due to the fact that, in contrast to the $\mathrm{Sp}(N_c)$ class of models, baryons can exist in the $\mathrm{SU}(N_c)$ and $\mathrm{O}(N_c)$ theories. The omission of baryons and other resonances is justified as long as these states are much heavier than the pions.

Results.—The WZW term in Eq. (8) induces the number-changing process that is responsible for the freeze-out of the pions. The Boltzmann equation governing the pion system is given by [1]

$$\dot{n}_\pi + 3Hn_\pi = -(n_\pi^3 - n_\pi^2 n_\pi^{\mathrm{eq}})\langle\sigma v^2\rangle_{3\to2}, \tag{14}$$

where H is the Hubble constant, n_π is the total pion number density, n_π^{eq} is their equilibrium number density, and $\langle\sigma v^2\rangle_{3\to2}$ is the thermally averaged $3 \to 2$ cross section averaged over initial and final state pions, computed from the WZW term in Eq. (8). As discussed in Ref. [1], within the SIMP mechanism, $2 \to 2$ annihilations into SM particles may be neglected above. Interactions maintaining thermal equilibrium with the thermal bath ensure that the dark matter has the same temperature as the photons.

In the left panel of Fig. 1 we plot the results of solving the Boltzmann equation (14), yielding the measured dark matter relic abundance, for an $\mathrm{Sp}(N_c)$ gauge group for several values of N_c and N_f (solid curves). The results for the other gauge groups are very similar [26]. In our convention, an approximate perturbative limit of $m_\pi/f_\pi \lesssim 2\pi$ (depicted by the horizontal solid line) can be set, placing a rough upper limit on the mass of the pions. Note that in the SM $m_K/f_\pi \simeq 2.7$. The upper limit relaxes as N_c increases and N_f decreases. The reason is that the thermally averaged cross section at freeze-out arising from Eq. (8) is

$$\langle\sigma v^2\rangle_{3\to2} = \frac{5\sqrt{5}}{2\pi^5 x_f^2}\frac{N_c^2 m_\pi^5}{f_\pi^{10}}\frac{t^2}{N_\pi^3}, \tag{15}$$

where $x_f = m_\pi/T_f \simeq 20$ with T_f the temperature at freeze-out. The factor of t^2/N_π^3 is combinatorial and decreases with large N_f as $1/N_f$ [26]. Increasing N_c or decreasing N_f enables the perturbative limit on m_π/f_π to be reached for higher values of m_π. For example, for the simplest case of $N_c = N_f = 2$, a dark matter mass of $m_\pi \lesssim 300$ MeV can be reached; for, e.g., $N_c = 8$ and $N_f = 2$, chiral perturbation theory is expected to break down at $m_\pi \sim 800$ MeV.

There are also 4-pion interactions induced by the kinetic and mass terms for the fermions, described via Eqs. (6) and (10). These contribute to the self-scattering cross section of dark matter $\sigma_{\mathrm{scatter}}$, which is constrained by bullet-cluster [27–29] and halo shape [30,31] constraints to obey

$$\frac{\sigma_{\mathrm{scatter}}}{m_{\mathrm{DM}}} \lesssim 1\ \mathrm{cm}^2/\mathrm{g} \tag{16}$$

with m_{DM} the dark matter mass. The self-scattering cross section, obtained along the solution to the Boltzmann equation, is plotted in the dashed curves of the left panel of Fig. 1 for various values of N_c and N_f for an $\mathrm{Sp}(N_c)$ gauge group. The results are similar for the other gauge groups [26]. The experimental constraint is depicted by the horizontal dashed line, and provides a rough lower bound on the mass of the pions. This lower bound decreases as N_c increases and N_f decreases. The reason is that the self-interaction scattering cross section scales as

$$\sigma_{\mathrm{scatter}} = \frac{m_\pi^2}{32\pi f_\pi^4}\frac{a^2}{N_\pi^2} \tag{17}$$

with a^2/N_π^2 nearly constant as N_f varies [26]. For a given mass m_π, as N_c increases or N_f decreases, a larger value of f_π solves the Boltzmann equation, which helps suppress the self-scattering cross section below its constrained value. For example, for the minimal case of $N_c = N_f = 2$, structure formation dictates pion masses above $m_\pi \gtrsim 300$ MeV; for $N_c = 8$ and $N_f = 2$ the lower bound is $m_\pi \gtrsim 150$ MeV.

Combined with the upper bound from chiral perturbation theory, a rough range for the mass of the pions is obtained. For example, as depicted in the left panel of Fig. 1, the minimal case of two flavors in an $\mathrm{Sp}(2) \simeq \mathrm{SU}(2)$ gauge group points to a pion dark matter mass of order ~300 MeV; for $N_c = 8$ with $N_f = 2$, the range is widened to ~150–800 MeV. Similar results are obtained for the $\mathrm{SU}(N_c)$ and $\mathrm{O}(N_c)$ gauge groups as well [26]. We note, however, that the minimal case of $N_f = 3$ in SU(3) and O(3) gauge groups exhibits a tension between the rough upper bound on m_π from perturbativity and the rough lower bound on m_π stemming from self-scattering constraints.

A comment is in order regarding higher-derivative corrections. Throughout we have used the 4-point interaction terms stemming from the mass and kinetic terms (6) and (10). As is evident, the theory is pushed to the strongly interacting regime where m_π is not far from the effective cutoff $\Lambda = 2\pi f_\pi$; here, higher-derivative terms may induce $\mathcal{O}(1)$ effects, shifting the lower bound on the pion mass accordingly. The self-scattering cross section of Eq. (17) is thus a proxy, which suffices for the purpose of obtaining a characteristic pion mass range.

Modifications to the presented canonical realization of the SIMP mechanism are possible. For instance, it is possible to write a mass term for the confining fermions that explicitly breaks the flavor symmetry of $\mathrm{Sp}(2N_f)$, $\mathrm{SU}(N_f)$, or $\mathrm{SO}(N_f)$ in the class of $\mathrm{Sp}(N_c)$, $\mathrm{SU}(N_c)$, or $\mathrm{O}(N_c)$ gauge theories. If one pion is lighter than the others, this pion will be the dark matter. Since the WZW term (8) induces $3 \to 2$ interactions between five different flavors of pions, the decay of the other pions to the lightest one must occur after freeze-out, and their masses must be close. Considering the 4-pion interactions, there are no self-interaction terms between pions of the same flavor originating from the kinetic term. In contrast, the fermion mass term of Eq. (11) does induce same-flavor self-scattering for the lightest pion. The resulting self-scattering cross section for the dark matter state $\sigma'_{\mathrm{scatter}}$ is suppressed numerically between a factor of a few to an order of magnitude, depending on the gauge group, compared to the degenerate-pion case [26]. The rough lower bound on the mass of the dark matter is then reduced compared to the degenerate-pion scenario, expanding the allowed dark matter mass window towards lower masses.

The results for an $\mathrm{Sp}(N_c)$ gauge group with a broken flavor symmetry are depicted in the right panel of Fig. 1 for various values of N_c and N_f. Similar results are obtained for the $\mathrm{SU}(N_c)$ and $\mathrm{O}(N_c)$ gauge groups [26]. For instance, in the simplest case of an $\mathrm{Sp}(2) \simeq \mathrm{SU}(2)$ gauge group with two flavors, explicit breaking of the Sp(4) flavor symmetry relaxes the self-scattering cross section constraint by an order of magnitude, such that pion masses in the range ~70–300 MeV are allowed. Similarly, with a broken flavor symmetry, the QCD-like case of an SU(3) gauge group with three flavors is now viable and points to pion masses of order $m_\pi \sim 150$–350 MeV.

Discussion.—The two basic features of the SIMP setup—strong $3 \to 2$ interactions within the dark sector and thermal equilibrium between the dark and visible sectors—dictate observable signals for this mechanism.

The strong interactions in the dark sector give an unavoidable contribution to a $2 \to 2$ self-scattering cross section amongst the pions, which is constrained *à la* Eq. (16). The failure of N-body simulation to reproduce the small scale structure of Galactic halos has led to the "core versus cusp" and "too big to fail" puzzles (see, e.g., Refs. [32–34] for discussion and references). These motivate self-interacting dark matter with a scattering cross section of [30,31,35,36]

$$\left(\frac{\sigma_{\mathrm{scatter}}}{m_{\mathrm{DM}}}\right)_{\mathrm{obs}} = (0.1\text{–}10)\ \mathrm{cm}^2/\mathrm{g}. \tag{18}$$

As is evident from the dashed curves in Fig. 1, the simplest realization of the dark sector presented in this Letter automatically yields a contribution of the right size to this cross section. (For other strongly interacting dark matter models that can accommodate small scale structure puzzles, see, e.g., Refs. [37–48].) Such behavior was anticipated in Ref. [1], though in the absence of explicit realizations of the strongly coupled sector only a qualitative statement could be made. The explicit realization of the dark sector presented in this Letter now proves this statement quantitatively. Altered predictions for structure formation are a signal of the SIMP mechanism.

In addition, the required thermal equilibrium between the visible and dark sectors dictates non-negligible interactions between the two. As a result, observable signals are predicted in direct and indirect detection, colliders, and cosmology. In contrast to structure formation discussed above, here the precise signatures depend on the mediation mechanism between the visible and dark sectors, and will be explored in detail in future work [2].

We find it intriguing that the resulting mass scales indicated by the SIMP mechanism are surprisingly close to the QCD scale. This suggests a possible joint dynamical origin for both the hidden and visible strong scales (see. e.g., Ref. [49]). Complete SIMP models of this kind will be presented in an upcoming publication [2].

The work of Y. H. is supported by the U.S. National Science Foundation under Grant No. PHY-1002399. Y. H. received support as an Awardee of the Weizmann Institute of Science—National Postdoctoral Award Program for Advancing Women in Science. E. K. is supported by the NSF under Grant No. PHY-1316222. H. M. was supported by the U.S. DOE under Contract No. DE-AC02-05CH11231, and by the NSF under Grants No. PHY-1002399 and No. PHY-1316783. H. M. was also supported by the JSPS Grant-in-Aid for Scientific Research (C) (Grant No. 26400241), Scientific Research on Innovative Areas (Grant No. 26105507), and by WPI, MEXT, Japan. T. V. is supported by the US-Israel Binational Science Foundation, by the EU-FP7 Marie Curie, CIG Fellowship, and by the I-CORE Program of the Planning Budgeting Committee and the Israel Science Foundation (Grant No. 1937/12). J. G. W. is supported in part by the NSF under Grant No. PHY-0756174. This work was supported in part by the NSF under Grant No. PHYS-1066293 and the hospitality of the Aspen Center for Physics.

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[1] Y. Hochberg, E. Kuflik, T. Volansky, and J. G. Wacker, Phys. Rev. Lett. **113**, 171301 (2014).
[2] Y. Hochberg, E. Kuflik, H. Murayama, T. Volansky, and J. G. Wacker (to be published).
[3] J. Wess and B. Zumino, Phys. Lett. **37B**, 95 (1971).
[4] E. Witten, Nucl. Phys. **B223**, 422 (1983).
[5] E. Witten, Nucl. Phys. **B223**, 433 (1983).
[6] M. E. Peskin, Nucl. Phys. **B175**, 197 (1980).
[7] J. Preskill, Nucl. Phys. **B177**, 21 (1981).
[8] D. A. Kosower, Phys. Lett. **144B**, 215 (1984).
[9] S. Hands, J. B. Kogut, M. P. Lombardo, and S. E. Morrison, Nucl. Phys. **B558**, 327 (1999).
[10] J. B. Kogut, M. A. Stephanov, and D. Toublan, Phys. Lett. B **464**, 183 (1999).
[11] S. Hands, I. Montvay, S. Morrison, M. Oevers, L. Scorzato, and J. Skullerud, Eur. Phys. J. C **17**, 285 (2000).
[12] R. Aloisio, V. Azcoiti, G. Di Carlo, A. Galante, and A. F. Grillo, Phys. Lett. B **493**, 189 (2000).
[13] J. B. Kogut, D. K. Sinclair, S. J. Hands, and S. E. Morrison, Phys. Rev. D **64**, 094505 (2001).
[14] S. Hands, I. Montvay, M. Oevers, L. Scorzato, and J. Skullerud, Nucl. Phys. B, Proc. Suppl. **94**, 461 (2001).
[15] R. Aloisio, V. Azcoiti, G. Di Carlo, A. Galante, and A. F. Grillo, Nucl. Phys. **B606**, 322 (2001).
[16] J. B. Kogut, D. Toublan, and D. K. Sinclair, Phys. Lett. B **514**, 77 (2001).
[17] J. B. Kogut, D. Toublan, and D. K. Sinclair, Nucl. Phys. **B642**, 181 (2002).
[18] J. B. Kogut, D. Toublan, and D. K. Sinclair, Phys. Rev. D **68**, 054507 (2003).
[19] Y. Nishida, K. Fukushima, and T. Hatsuda, Phys. Rep. **398**, 281 (2004).
[20] M. P. Lombardo, M. L. Paciello, S. Petrarca, and B. Taglienti, Eur. Phys. J. C **58**, 69 (2008).
[21] T. A. Ryttov and F. Sannino, Phys. Rev. D **78**, 115010 (2008).
[22] R. Lewis, C. Pica, and F. Sannino, Phys. Rev. D **85**, 014504 (2012).
[23] W. Detmold, M. McCullough, and A. Pochinsky, Phys. Rev. D **90**, 115013 (2014).
[24] W. Detmold, M. McCullough, and A. Pochinsky, Phys. Rev. D **90**, 114506 (2014).
[25] H. Georgi, Phys. Lett. B **298**, 187 (1993).
[26] See Supplemental Material at http://link.aps.org/supplemental/10.1103/PhysRevLett.115.021301 for further details.
[27] D. Clowe, A. Gonzalez, and M. Markevitch, Astrophys. J. **604**, 596 (2004).
[28] M. Markevitch, A. H. Gonzalez, D. Clowe, A. Vikhlinin, W. Forman, C. Jones, S. Murray, and W. Tucker, Astrophys. J. **606**, 819 (2004).
[29] S. W. Randall, M. Markevitch, D. Clowe, A. H. Gonzalez, and M. Bradac, Astrophys. J. **679**, 1173 (2008).
[30] M. Rocha, A. H. G. Peter, J. S. Bullock, M. Kaplinghat, S. Garrison-Kimmel, J. Onorbe, and L. A. Moustakas, Mon. Not. R. Astron. Soc. **430**, 81 (2013).
[31] A. H. Peter, M. Rocha, J. S. Bullock, and M. Kaplinghat, Mon. Not. R. Astron. Soc. **430**, 105 (2013).
[32] D. N. Spergel and P. J. Steinhardt, Phys. Rev. Lett. **84**, 3760 (2000).
[33] W. de Blok, Adv. Astron. **2010**, 789293 (2010).
[34] M. Boylan-Kolchin, J. S. Bullock, and M. Kaplinghat, Mon. Not. R. Astron. Soc. **415**, L40 (2011).
[35] M. Vogelsberger, J. Zavala, and A. Loeb, Mon. Not. R. Astron. Soc. **423**, 3740 (2012).
[36] J. Zavala, M. Vogelsberger, and M. G. Walker, Mon. Not. R. Astron. Soc. Lett. **431**, L20 (2013).
[37] G. D. Kribs, T. S. Roy, J. Terning, and K. M. Zurek, Phys. Rev. D **81**, 095001 (2010).
[38] D. S. M. Alves, S. R. Behbahani, P. Schuster, and J. G. Wacker, Phys. Lett. B **692**, 323 (2010).
[39] J. M. Cline, Z. Liu, G. Moore, and W. Xue, Phys. Rev. D **90**, 015023 (2014).
[40] K. Kumar, A. Menon, and T. M. P. Tait, J. High Energy Phys. 02 (2012) 131.
[41] D. Spier Moreira Alves, S. R. Behbahani, P. Schuster, and J. G. Wacker, J. High Energy Phys. 06 (2010) 113.
[42] M. Lisanti and J. G. Wacker, Phys. Rev. D **82**, 055023 (2010).
[43] F.-Y. Cyr-Racine and K. Sigurdson, Phys. Rev. D **87**, 103515 (2013).
[44] D. E. Kaplan, G. Z. Krnjaic, K. R. Rehermann, and C. M. Wells, J. Cosmol. Astropart. Phys. 05 (2010) 021.
[45] S. Tulin, H.-B. Yu, and K. M. Zurek, Phys. Rev. D **87**, 115007 (2013).
[46] M. Kaplinghat, S. Tulin, and H.-B. Yu, arXiv:1308.0618.
[47] K. K. Boddy, J. L. Feng, M. Kaplinghat, and T. M. P. Tait, Phys. Rev. D **89**, 115017 (2014).
[48] K. K. Boddy, J. L. Feng, M. Kaplinghat, Y. Shadmi, and T. M. P. Tait, Phys. Rev. D **90**, 095016 (2014).
[49] Y. Bai and P. Schwaller, Phys. Rev. D **89**, 063522 (2014).