Measuring Quantum Coherence with Entanglement

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Quantum coherence is an essential ingredient in quantum information processing and plays a central role in emergent fields such as nanoscale thermodynamics and quantum biology. However, our understanding and quantitative characterization of coherence as an operational resource are still very limited. Here we show that any degree of coherence with respect to some reference basis can be converted to entanglement via incoherent operations. This finding allows us to define a novel general class of measures of coherence for a quantum system of arbitrary dimension, in terms of the maximum bipartite entanglement that can be generated via incoherent operations applied to the system and an incoherent ancilla. The resulting measures are proven to be valid coherence monotones satisfying all the requirements dictated by the resource theory of quantum coherence. We demonstrate the usefulness of our approach by proving that the fidelity-based geometric measure of coherence is a full convex coherence monotone, and deriving a closed formula for it on arbitrary single-qubit states. Our work provides a clear quantitative and operational connection between coherence and entanglement, two landmark manifestations of quantum theory and both key enablers for quantum technologies.

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Introduction.—Coherence is a fundamental aspect of quantum physics that encapsulates the defining features of the theory [1], from the superposition principle to quantum correlations. It is a key component in various quantum information and estimation protocols and is primarily accountable for the advantage offered by quantum tasks versus classical ones [2,3]. In general, coherence is an important physical resource in low-temperature thermodynamics [4–8], for exciton and electron transport in biomolecular networks [9–14], and for applications in nanoscale physics [15,16]. Experimental detection of coherence in living complexes [17,18] and creation of coherence in hot systems [19] have also been reported.

While the theory of quantum coherence is historically well developed in quantum optics [20–27], a rigorous framework to quantify coherence for general states in information theoretic terms has only been attempted recently [14,26,28–31]. This framework is based on identifying the set of incoherent states and a class of "free" operations, named incoherent quantum channels, that map the set onto itself [14,28]. The resulting resource theory of coherence is in direct analogy with the resource theory of entanglement [32], in which local operations and classical communication are the free operations that map the set of separable states onto itself [33]. Within such a framework for coherence, one can define suitable measures that vanish for any incoherent state, and satisfy specific monotonicity requirements under incoherent channels. Measures that respect these conditions gain the attribute of coherence monotones, in analogy with entanglement monotones [34]. Examples of coherence monotones include the relative entropy and the l_1-norm of coherence [28]. Intuitively, both coherence and entanglement capture quantumness of a physical system, and it is well known that entanglement stems from the superposition principle, which is also the essence of coherence. It is then legitimate to ask how can one resource emerge *quantitatively* from the other [24,26].

In this Letter, we provide a mathematically rigorous approach to resolve the above question, using a common frame to quantify quantumness in terms of coherence and entanglement. In particular, we show that any nonzero amount of coherence in a system S can be converted to (distillable) entanglement between S and an initially incoherent ancilla A, by means of incoherent operations. This allows us to formulate a novel, general method to quantify coherence in terms of entanglement (see Fig. 1). Namely, we prove that, given an arbitrary set of entanglement monotones $\{E\}$, one can define a corresponding class of coherence monotones $\{C_E\}$ that satisfy all the requirements of Ref. [28]. The input coherence C_E of S is defined as the maximum output entanglement E over all incoherent operations on S and A. We explicitly evaluate the maximization in some relevant instances, defining novel coherence monotones such as the fidelity-based geometric measure of coherence. These results provide powerful advances for the operational quantification of coherence.

Characterizing coherence.—For an arbitrary fixed reference basis $\{|i\rangle\}$, the incoherent states are defined as [28]

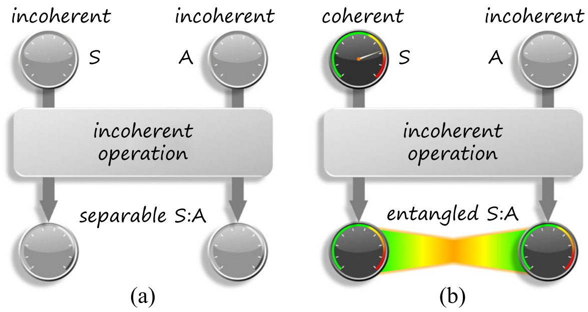

FIG. 1 (color online). (a) Incoherent operations cannot generate entanglement from incoherent input states. (b) Conversely, we show that any nonzero coherence in the input state of a system S can be converted to entanglement via incoherent operations on S and an incoherent ancilla A. Input coherence and output entanglement are quantitatively equivalent: For any entanglement monotone E, the maximum entanglement generated between S and A by incoherent operations defines a faithful coherence monotone C_E on the initial state of S.

$$\sigma = \sum_i p_i |i\rangle\langle i|, \tag{1}$$

where p_i are probabilities. Any state which cannot be written as above is defined *coherent* [28]. Note that, unlike other resources in information theory, coherence is basis dependent. The reference basis with respect to which coherence is measured depends on the physical problem under investigation; it is, e.g., the energy basis for transport phenomena in engineered and biological domains [13], or the eigenbasis of the generator of an unknown phase shift in quantum metrology [2].

A completely positive trace preserving map Λ is said to be an incoherent operation if it can be written as

$$\Lambda[\rho] = \sum_l K_l \rho K_l^\dagger, \tag{2}$$

where the defining operators K_l, called incoherent Kraus operators, map every incoherent state to some other incoherent state, i.e., $K_l \mathcal{I} K_l^\dagger \subseteq \mathcal{I}$, where $\mathcal{I}$ is the set of incoherent states.

Following established notions from entanglement theory [32,35–37], Baumgratz *et al.* proposed the following postulates for a measure of coherence $C(\rho)$ in Ref. [28]: (C1) $C(\rho) \geq 0$, and $C(\sigma) = 0$ if and only if $\sigma \in \mathcal{I}$. (C2) $C(\rho)$ is nonincreasing under incoherent operations, i.e., $C(\rho) \geq C(\Lambda[\rho])$ with $\Lambda[\mathcal{I}] \subseteq \mathcal{I}$. (C3) $C(\rho)$ is nonincreasing on average under selective incoherent operations, i.e., $C(\rho) \geq \sum_l p_l C(\varsigma_l)$, with probabilities $p_l = \mathrm{Tr}[K_l \rho K_l^\dagger]$, states $\varsigma_l = K_l \rho K_l^\dagger / p_l$, and incoherent Kraus operators K_l obeying $K_l \mathcal{I} K_l^\dagger \subseteq \mathcal{I}$. (C4) $C(\rho)$ is a convex function of density matrices, i.e., $C(\sum_i p_i \rho_i) \leq \sum_i p_i C(\rho_i)$. Note that conditions (C3) and (C4) automatically imply condition (C2). The reason we listed all conditions above is that (similar to entanglement measures) there might exist meaningful quantifiers of coherence which satisfy conditions (C1) and (C2), but for which conditions (C3) and (C4) are either violated or cannot be proven. Following the analogous notion from entanglement theory, we call a quantity which satisfies conditions (C1), (C2), and (C3) a coherence monotone.

Examples of functionals that satisfy all the four properties mentioned above include the l_1-norm of coherence [28] $C_{l_1}(\rho) = \sum_{i \neq j} |\rho_{ij}|$ and the relative entropy of coherence [28]

$$C_r(\rho) = \min_{\sigma \in \mathcal{I}} H(\rho \| \sigma), \tag{3}$$

with the quantum relative entropy $H(\rho \| \varsigma) = \mathrm{Tr}[\rho \log_2 \rho] - \mathrm{Tr}[\rho \log_2 \varsigma]$. As was shown in Ref. [28], the relative entropy of coherence can also be written as $C_r(\rho) = H(\rho_d) - H(\rho)$, where ρ_d is the diagonal part of the density matrix ρ in the reference basis $\{|i\rangle\}$ and H is the von Neumann entropy.

Bipartite coherence.—We first extend the framework of coherence to the bipartite scenario (see also Ref. [38]); the following definitions extend straightforwardly to multipartite systems. In particular, for a bipartite system with two subsystems X and Y, and with respect to a fixed reference product basis $\{|i\rangle^X \otimes |j\rangle^Y\}$, we define bipartite incoherent states as follows:

$$\rho^{XY} = \sum_k p_k \sigma_k^X \otimes \tau_k^Y. \tag{4}$$

Here, p_k are probabilities and the states σ_k^X and τ_k^Y are incoherent states on the subsystem X and Y, respectively, i.e., $\sigma_k^X = \sum_i p'_{ik} |i\rangle\langle i|^X$ and $\tau_k^Y = \sum_j p''_{jk} |j\rangle\langle j|^Y$ for probabilities p'_{ik} and p''_{jk}. Note that the states in Eq. (4) are always separable.

We next define bipartite incoherent operations as in Eq. (2), with incoherent Kraus operators K_l, such that $K_l \mathcal{I} K_l^\dagger \subseteq \mathcal{I}$, where $\mathcal{I}$ is now the set of bipartite incoherent states defined in Eq. (4). An example of bipartite incoherent operation is the two-qubit CNOT gate U_{CNOT}. It is not possible to create coherence from an incoherent two-qubit state by using the CNOT gate, since it takes any pure incoherent state $|i\rangle \otimes |j\rangle$ to another pure incoherent state, $U_{\mathrm{CNOT}}(|i\rangle \otimes |j\rangle) = |i\rangle \otimes |\mathrm{mod}(i+j, 2)\rangle$. The CNOT gate can be used, however, to create entanglement; e.g., it is well known that the state $U_{\mathrm{CNOT}}(|\psi\rangle \otimes |0\rangle)$ is entangled for any coherent state $|\psi\rangle$ [3].

Converting coherence to entanglement.—Referring to Fig. 1, we say that the coherence in the initial state ρ^S of a (finite-dimensional) system S can be converted to entanglement via incoherent operations if, by attaching an ancilla A initialized in a reference incoherent state $|0\rangle\langle 0|^A$, the final system-ancilla state $\Lambda^{SA}[\rho^S \otimes |0\rangle\langle 0|^A]$ is entangled for some incoherent operation Λ^{SA}. Note that incoherent system states σ^S cannot be used for conversion to entangled states in this way, since for any incoherent state σ^S the state $\Lambda^{SA}[\sigma^S \otimes |0\rangle\langle 0|^A]$ will be of the form given in Eq. (4), and thus separable.

Entanglement can instead be generated by incoherent operations if the initial ρ^S is coherent, as in the two-qubit CNOT example. It is then natural to ask: Can *any* nonzero amount of coherence be converted to entanglement via incoherent operations? To answer this, we first consider distance-based measures of entanglement E_D and coherence C_D [28,35–38]:

$$E_D(\rho) = \min_{\chi\in\mathcal{S}} D(\rho,\chi), \qquad C_D(\rho) = \min_{\sigma\in\mathcal{I}} D(\rho,\sigma). \quad (5)$$

Here, $\mathcal{S}$ is the set of separable states and $\mathcal{I}$ is the set of incoherent states. Moreover, we demand that the distance D be contractive under quantum operations,

$$D(\Lambda[\rho],\Lambda[\varsigma]) \leq D(\rho,\varsigma) \quad (6)$$

for any completely positive trace preserving map Λ. This implies that E_D does not increase under local operations and classical communication [35,36], and C_D does not increase under incoherent operations [28]. Equipped with these tools we are now in position to present the first result of this Letter.

Theorem 1: For any contractive distance D, the amount of (distance-based) entanglement E_D generated from a state ρ^S via an incoherent operation Λ^{SA} is bounded above by its (distance-based) coherence C_D:

$$E_D^{S:A}(\Lambda^{SA}[\rho^S \otimes |0\rangle\langle 0|^A]) \leq C_D(\rho^S). \quad (7)$$

Proof.—Let σ^S be the closest incoherent state to ρ^S, i.e., $C_D(\rho^S) = D(\rho^S,\sigma^S)$. The contractivity of the distance D further implies the equality: $D(\rho^S,\sigma^S) = D(\rho^S \otimes |0\rangle\langle 0|^A, \sigma^S \otimes |0\rangle\langle 0|^A)$. In the final step, note that the application of an incoherent operation Λ^{SA} to the incoherent state $\sigma^S \otimes |0\rangle\langle 0|^A$ brings it to another incoherent—and thus separable—state. Applying Eq. (6) and combining the aforementioned results we arrive at the desired inequality: $C_D(\rho^S) \geq D(\Lambda^{SA}[\rho^S \otimes |0\rangle\langle 0|^A], \Lambda^{SA}[\sigma^S \otimes |0\rangle\langle 0|^A]) \geq E_D^{S:A}(\Lambda^{SA}[\rho^S \otimes |0\rangle\langle 0|^A])$. □

This result provides a strong link between the resource frameworks of entanglement and coherence. An even stronger link exists when choosing specifically D as the relative entropy. The corresponding quantifiers are the relative entropy of entanglement E_r [35], and the relative entropy of coherence C_r [28] introduced in Eq. (3). Importantly, the inequality (7) can be saturated for these measures if the dimension of the ancilla is not smaller than that of the system, $d_A \geq d_S$. In this case there always exists an incoherent operation Λ^{SA} such that

$$E_r^{S:A}(\Lambda^{SA}[\rho^S \otimes |0\rangle\langle 0|^A]) = C_r(\rho^S). \quad (8)$$

To prove this statement, we consider the unitary operation

$$U = \sum_{i=0}^{d_S-1}\sum_{j=0}^{d_S-1} |i\rangle\langle i|^S \otimes |\mathrm{mod}(i+j,d_S)\rangle\langle j|^A + \sum_{i=0}^{d_S-1}\sum_{j=d_S}^{d_A-1} |i\rangle\langle i|^S \otimes |j\rangle\langle j|^A. \quad (9)$$

Note that for two qubits this unitary is equivalent to the CNOT gate with S as the control qubit and A as the target qubit. It can be seen by inspection that this unitary is incoherent (i.e., the state $\Lambda^{SA}[\rho^{SA}] = U\rho^{SA}U^\dagger$ is incoherent for any incoherent state ρ^{SA}), and maps the state $\rho^S \otimes |0\rangle\langle 0|^A$ to the state

$$\Lambda^{SA}[\rho^S \otimes |0\rangle\langle 0|^A] = \sum_{i,j}\rho_{ij}|i\rangle\langle j|^S \otimes |i\rangle\langle j|^A, \quad (10)$$

where ρ_{ij} are the matrix elements of $\rho^S = \sum_{i,j}\rho_{ij}|i\rangle\langle j|^S$. In the next step we use the fact that for any quantum state ς^{SA} the relative entropy of entanglement is bounded below as follows [39]: $E_r^{S:A}(\varsigma^{SA}) \geq H(\varsigma^S) - H(\varsigma^{SA})$. Applied to the state $\Lambda^{SA}[\rho^S \otimes |0\rangle\langle 0|^A]$, this inequality reduces to

$$E_r^{S:A}(\Lambda^{SA}[\rho^S \otimes |0\rangle\langle 0|^A]) \geq H\left(\sum_i \rho_{ii}|i\rangle\langle i|^S\right) - H(\rho^S). \quad (11)$$

Noting that the right-hand side of this inequality is equal to the relative entropy of coherence $C_r(\rho^S)$ [28], we obtain $E_r^{S:A}(\Lambda^{SA}[\rho^S \otimes |0\rangle\langle 0|^A]) \geq C_r(\rho^S)$. The proof of Eq. (8) is complete by combining this result with Theorem 1.

The results presented above also hold for the distillable entanglement E_d. Namely, the relative entropy of coherence C_r also serves as an upper bound for the conversion to distillable entanglement via incoherent operations, and the equality in Eq. (8) still holds if E_r is replaced by E_d, and the incoherent unitary of Eq. (9) is applied. This follows from Theorem 1, together with the fact that distillable entanglement admits the following bounds [40,41]: $H(\varsigma^S) - H(\varsigma^{SA}) \leq E_d^{S:A} \leq E_r^{S:A}$.

This shows that the degree of (relative entropy of) coherence in the initial state of S can be exactly converted to an equal degree of (distillable or relative entropy of) entanglement between S and the incoherent ancilla A by a suitable incoherent operation, that is a generalized CNOT gate. We can now settle the general question posed above.

Theorem 2: A state ρ^S can be converted to an entangled state via incoherent operations if and only if ρ^S is coherent.

Proof.—If ρ^S is incoherent, it cannot be converted to an entangled state via incoherent operations by Theorem 1. Conversely, if ρ^S is coherent, it has nonzero relative entropy of coherence $C_r(\rho^S) > 0$. By Eq. (8), there exists an incoherent operation Λ^{SA} leading to nonzero relative entropy of entanglement $E_r^{S:A}(\Lambda^{SA}[\rho^S \otimes |0\rangle\langle 0|^A]) > 0$, concluding the proof. □

Quantifying coherence with entanglement.—We are ready to present the central result of the Letter. Reversing the perspective, Theorem 1 can also be seen as providing a lower bound on distance-based measures of coherence through conversion to entanglement: precisely, the coherence degree C_D of a state ρ^S is always bounded below by the maximal entanglement degree E_D generated from it by incoherent operations.

Going now beyond the specific setting of distance-based measures, we will show that such a maximization of the output entanglement, for any fully general entanglement monotone, leads to a quantity which yields a valid quantifier of input coherence in its own right. We specifically define the family of entanglement-based coherence measures $\{C_E\}$ as follows:

$$C_E(\rho^S) = \lim_{d_A\to\infty}\left\{\sup_{\Lambda^{SA}} E^{S:A}(\Lambda^{SA}[\rho^S \otimes |0\rangle\langle 0|^A])\right\}. \qquad (12)$$

Here, E is an arbitrary entanglement measure and the supremum is taken over all incoherent operations Λ^{SA} [42].

It is crucial to benchmark the validity of C_E for any E as a proper measure of coherence. Remarkably, we find that C_E satisfies all the aforementioned conditions (C1)–(C3) given an arbitrary entanglement monotone E, with the addition of (C4) if E is convex as well. We, namely, get the following result:

Theorem 3: C_E is a (convex) coherence monotone for any (convex) entanglement monotone E.

Proof.—Using the arguments presented above it is easy to see that C_E is nonnegative, and zero if and only if the state ρ^S is incoherent. Moreover, C_E does not increase under incoherent operations Λ^S performed on the system S. This can be seen directly from the definition of C_E in Eq. (12), noting that an incoherent operation Λ^S on the system S is also incoherent with respect to SA. The proof that C_E further satisfies condition (C3) is presented in the Supplemental Material [43]. There we also show that C_E is convex for any convex entanglement monotone E; i.e., (C4) is fulfilled as well in this case. □

These powerful results complete the parallel between coherence and entanglement, *de facto* establishing their full quantitative equivalence within the respective resource theories. Thanks to Theorem 3, one can now use the comprehensive knowledge acquired in entanglement theory in the last two decades [34,35,37,44] to address the quantification of coherence in a variety of operational settings, and to define and validate physically motivated coherence monotones. For instance, C_E as defined by Eq. (12) amounts to the previously defined relative entropy of coherence [28], if E is the relative entropy of entanglement or the distillable entanglement.

Furthermore, we can now focus on the relevant case of E being the geometric entanglement [45,46] E_g, defined for a bipartite state ρ as $E_g(\rho) = 1 - \max_{\chi\in\mathcal{S}} F(\rho,\chi)$, where the maximum is taken over all separable states $\chi \in \mathcal{S}$, and $F(\rho,\varsigma) = [\mathrm{Tr}(\sqrt{\rho}\varsigma\sqrt{\rho})^{1/2}]^2$ is the Uhlmann fidelity. The geometric entanglement coincides with its expression obtained via convex roof [46,47], $E_g(\rho) = \min \sum_i p_i E_g(|\psi_i\rangle)$, where the minimum is over all decompositions of $\rho = \sum_i p_i |\psi_i\rangle\langle\psi_i|$. In the Supplemental Material [43], we show that the geometric measure of coherence C_g, associated to E_g via Eq. (12), can be evaluated explicitly and amounts to $C_g(\rho) = 1 - \max_{\sigma\in\mathcal{I}} F(\rho,\sigma)$, where the maximum is taken over all incoherent states $\sigma \in \mathcal{I}$. The incoherent operation which attains the maximum in Eq. (12) is again the generalized CNOT defined by Eq. (9). Because of Theorem 3, since the geometric measure of entanglement is a full convex entanglement monotone [37,45], we have just proven that the geometric measure of coherence C_g is a full convex coherence monotone obeying (C1)–(C4). This settles an important question left open in previous literature [28,38]. Remarkably, the geometric measure C_g is also analytically computable for an arbitrary state ρ of one qubit [43], as follows:

$$C_g(\rho) = \frac{1}{2}\left(1 - \sqrt{1 - 4|\rho_{01}|^2}\right), \qquad (13)$$

where ρ_{01} is the off-diagonal element of ρ with respect to the reference basis. Notice that C_g in this case is a simple monotonic function of the l_1-norm of coherence [28], $C_{l_1}(\rho) = 2|\rho_{01}|$.

Some of these results extend to any distance-based entanglement measure $E_{g(F)}$ defined via Eq. (5) with $D_{g(F)}(\rho,\varsigma) = g[F(\rho,\varsigma)]$, where $g(F)$ is a nonincreasing function of the fidelity F. These include the Bures measure of entanglement [35,36], with $g(F) = 2(1-\sqrt{F})$, and the Groverian measure of entanglement [48,49], with $g(F) = \sqrt{1-F}$. For any such entanglement $E_{g(F)}$, the corresponding quantifier of coherence is $C_{g(F)}(\rho) = \min_{\sigma\in\mathcal{I}} D_{g(F)}(\rho,\sigma)$ [43], and Theorem 1 holds with equality for any matching pair $E_{g(F)}$ and $C_{g(F)}$ [50].

Conclusions.—In this Letter we have established a rigorous and general framework for the interconversion between two quantum resources, coherence on one hand, and entanglement on the other hand, via incoherent operations. Our framework can be interpreted in both ways: on one hand, it demonstrates the formal potential of coherence for entanglement generation (although not necessarily useful in practical applications, as cheaper schemes for entanglement creation might be available); on the other hand, it demonstrates the usefulness of entanglement to obtain and validate measures of coherence. Building on this connection, we proposed, in fact, a family of coherence quantifiers in terms of the maximal entanglement that can be generated by incoherent operations (see Fig. 1). The proposed coherence quantifiers satisfy all the necessary criteria to be *bona fide* coherence monotones [28]. In particular, the relative entropy of coherence and the geometric measure of coherence have been (re)defined and interpreted operationally in terms of the maximum converted distillable and geometric entanglement, respectively.

Our framework bears some resemblance with, and may be regarded as the general finite-dimensional counterpart to, the established (qualitative and quantitative) equivalence between input nonclassicality, intended as superposition of optical coherent states, and output entanglement created by passive quantum optical elements such as beam splitters [23,24,26,52]. The results presented here should also be compared to the scheme for activating distillable entanglement via premeasurement interactions [53–55] from quantum discord, a measure of nonclassical correlations going beyond entanglement [56,57]. In the latter approach, which has attracted a large amount of attention recently [56,58–61], measures of discord in a composite system are defined in terms of the minimum entanglement created with an ancillary system via fixed premeasurement interactions defined as in Eq. (9), where the minimization is over local unitaries on the system regulating the control bases before the interaction. By contrast, in this work the reference basis is fixed, and a maximization of the output entanglement over all incoherent operations returns a measure of coherence for the initial system. One might combine the two approaches in order to define a unified framework for interconversion among coherence, discord, and entanglement, whereby discord-type measures could be interpreted as measures of bipartite coherence suitably minimized over local product reference bases (see, e.g., [38,62]). Exploring these connections further will be the subject of another work.

The theory of entanglement has been the cornerstone of major developments in quantum information theory and has triggered the advancement of modern quantum technologies. The construction of a physically meaningful and mathematically rigorous quantitative theory of coherence can improve our perception of genuine quantumness, and guide our understanding of nascent fields such as quantum biology and nanoscale thermodynamics. By uncovering a powerful operational connection between coherence and entanglement, we believe the present work delivers a substantial step in this direction.

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