Quantum Critical Scaling of Dirty Bosons in Two Dimensions

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We determine the dynamical critical exponent z appearing at the Bose glass to superfluid transition in two dimensions by performing large scale numerical studies of *two* microscopically different quantum models within the universality class: The hard-core boson model and the quantum rotor (soft core) model, both subject to strong on-site disorder. By performing many simulations at different system size L and inverse temperature β close to the quantum critical point, the position of the critical point and the critical exponents, z, ν, and η can be determined *independently* of any implicit assumptions of the numerical value of z, in contrast to most prior studies. This is done by a careful scaling analysis close to the critical point with a particular focus on the temperature dependence of the scaling functions. For the hard-core boson model we find $z = 1.88(8)$, $\nu = 0.99(3)$, and $\eta = -0.16(8)$ with a critical field of $h_c = 4.79(3)$, while for the quantum rotor model we find $z = 1.99(5)$, $\nu = 1.00(2)$, and $\eta = -0.3(1)$ with a critical hopping parameter of $t_c = 0.0760(5)$. In both cases do we find a correlation length exponent consistent with $\nu = 1$, saturating the bound $\nu \geq 2/d$ as well as a value of z significantly larger than previous studies, and for the quantum rotor model consistent with $z = d$.

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Most familiar quantum critical points (QCPs) are characterized by Lorentz invariance implying a symmetry between correlations in space and time and consequently between the respective correlation lengths $\xi \sim \xi_\tau$ [1]. In turn, the dynamical critical exponent, defined through $\xi_\tau \sim \xi^z$, is simply $z = 1$, such as in the crossing of the special multicritical point of the Bose-Hubbard model [1]. Anisotropic systems where $z \neq 1$, implying different scaling of ξ and ξ_τ, are comparatively less common [1,2]. This quantum critical scaling is particularly intriguing if disorder is present, in which case *nonintegral* values of z have been proposed [3,4]. One model for which it is generally believed that $z \neq 1$ is the Bose glass to superfluid (BG-SF) transition describing interacting bosons subject to disorder, the so-called dirty-boson problem, modeled by the Hamiltonian

$$H_{bh} = -t\sum_{\mathbf{r},\mathbf{e}}(b_\mathbf{r}^\dagger b_{\mathbf{r}+\mathbf{e}} + \mathrm{H.c.}) - \sum_\mathbf{r}\mu_\mathbf{r}\tilde{n}_\mathbf{r} + \frac{U}{2}\sum_\mathbf{r}\tilde{n}_\mathbf{r}(\tilde{n}_\mathbf{r} - 1). \tag{1}$$

Here $\mathbf{e} = \mathbf{x}, \mathbf{y}$, and $b_\mathbf{r}^\dagger, b_\mathbf{r}$ are the boson creation and annihilation operators at site $\mathbf{r}$ with $\tilde{n}_\mathbf{r}$ the corresponding number operator. The parameters of the model are the hopping constant t, Hubbard repulsion U, and *site-dependent* chemical potential $\mu_\mathbf{r}$, inducing the disorder.

Experimental setups emulating dirty boson physics include optical lattices [5] adsorbed helium in random media [6], Josephson-junction arrays [7], thin-film superconductors [8], and quantum magnets such as doped dichloro-tetrakis-thiourea-nickel(II) (DTN) [9]. For recent reviews, see Refs. [10,11].

The dynamical critical exponent z appearing at the BG-SF transition has proven exceedingly hard to determine. Assuming the validity of the (quantum) Harris criterion for disordered systems $\nu \geq 2/d$ [12], initial theoretical work [13] argued that $z = d$ in *any* dimension. This has intriguing implications since it implies the absence of an upper critical dimension. Although many initial numerical studies [14–19] were consistent with $z = d = 2$, most were biased by implicit assumptions about z, using it to fix the simulation aspect ratio L^z/β. The exponent z was therefore not *truly* independently determined. However, recent theoretical work by Weichman and collaborators [11,20] has challenged the arguments leading to $z = d$ leaving the value of z an open question. A subsequent numerical study [3] of the hard-core version of Eq. (1) found $z = 1.40(2)$, $\nu = 1.10(4)$ while a recent unbiased state-of-the-art study [4] using an effective classical model of Eq. (1) determined a significantly larger value of $z = 1.75(5)$ and $\nu = 1.15(3)$. Both results violate $z = d$. Intriguingly, in three dimensions both numerical [9,19,21] and experimental [22], studies yield evidence for $z = d = 3$, although these numerical estimates cannot be seen as fully unbiased.

At present, the value of z at the dirty-boson QCP along with many of the other exponents most notably ν can therefore best be regarded as ill determined, at least for the fully quantum mechanical model. It is not known to what extent, if any, the relation $z = d$ is violated or if the relation $\nu \geq 2/d$ [12] is satisfied. Here we try to answer some of these questions by performing large-scale simulations on *two* fully quantum mechanical models in two dimensions within the dirty-boson universality class: A hard-core boson model (HCB) modeled as a transverse field XY

model and a soft-core quantum rotor model (QR), both subject to strong on-site disorder. In all cases do we find it necessary to use 10^4–10^5 disorder realizations over a large range of temperatures extending down to $\beta = 1024$ for system sizes $L = 12$–32. In contrast to Ref. [3] these dramatically improved statistics allow for a significantly better determination of the critical point of the HCB model. We also note that the use of fully quantum models presents significant advantages over the effective classical model used in Ref. [4]. Finally, through a fully unbiased analysis, without implicit assumptions on z, we find for the first time strong evidence that $z = d = 2$ and that $\nu \geq 2/d$ is satisfied as an equality.

We now briefly summarize some of the theoretical discussion. The arguments leading to the equality $z = d$ start with hyperscaling [23] which states that the singular part of the free energy inside a correlation volume is a *universal* dimensionless number, $(f_s/\hbar)\xi^d\xi_\tau = A$. With $\xi \sim \delta^{-\nu}$ it follows that $f_s \sim \delta^{\nu(d+z)}$ with a finite-size form [1]:

$$f_s(\delta, L, \beta) \sim \delta^{\nu(d+z)} F(\xi/L, \xi_\tau/\beta). \tag{2}$$

Imposing a phase gradient $\partial\phi$ along one of the *spatial* directions will then give rise to a free energy difference $\Delta f_s/\hbar = \frac{1}{2}\rho(\partial\phi)^2$ where ρ is the stiffness (superfluid density). Since Δf_s must obey a form similar to Eq. (2) and since $\partial\phi$ has dimension of inverse length implying $\partial\phi \sim 1/\xi$, it follows that $\rho \sim \xi^2\delta^{\nu(d+z)} \sim \delta^{\nu(d-2+z)}$, with a finite-size scaling form of

$$\rho = L^{2-d-z} R(\delta L^{1/\nu}, \beta/L^z). \tag{3}$$

If an analogous argument is used with a twist in the *temporal* direction scaling as $\partial_\tau\phi \sim 1/\xi_\tau$, Fisher *et al.* [1] argued that the compressibility scales as $\kappa \sim \delta^{\nu(d-z)}$ which they then used to argue that $z = d$. In contrast, Weichman and collaborators [11,20] argue that in the presence of disorder $\partial_\tau\phi \sim 1/\xi_\tau$ should not apply, invalidating the relation $\kappa \sim \delta^{\nu(d-z)}$, leaving z unconstrained. Interestingly, a different theoretical argument [9] favoring $z = d$ has also been put forward.

Models.—The first model we study, closely related to Eq. (1), is the QR model. It is defined in terms of conjugate phase and number operators $\theta_{\mathbf{r}}, n_{\mathbf{r}}$ satisfying $[\theta_{\mathbf{r}}, n_{\mathbf{r}'}] = \delta_{\mathbf{r},\mathbf{r}'}$ on a $L \times L$ lattice:

$$H_{\mathrm{qr}} = -\sum_{\mathbf{r},\mathbf{r}+\mathbf{e}} t\cos(\theta_{\mathbf{r}} - \theta_{\mathbf{r}+\mathbf{e}}) - \sum_{\mathbf{r}} \mu_{\mathbf{r}} n_{\mathbf{r}} + \frac{U}{2}\sum_{\mathbf{r}} n_{\mathbf{r}}^2, \tag{4}$$

where U is the on-site repulsion, t is the nearest neighbor tunneling amplitude, and $\mu_r \in [-\Delta, \Delta]$ represents the uniformly distributed on-site disorder in the chemical potential. As before, $\mathbf{e} = \mathbf{x}, \mathbf{y}$. We fix $\Delta = \frac{1}{2}$, $U = 1$, and cross the BG-SF transition by varying t. In contrast to Eq. (1) $n_{\mathbf{r}}$ can take negative as well as positive values and can be interpreted as deviations from the average filling n_0. For convenience we study Eq. (4) using a link-current representation [24] for which *directed* worm algorithms are available [25]. We use lattices ranging from $L = 12$ to $L = 32$, with 5×10^4 disorder realizations for $L = 12, \ldots, 28$ and 10^4 disorder realizations for $L = 32$. In all cases we average over 6×10^4 Monte-Carlo steps (MCS) per disorder realization. For the simulations of the QR model a temporal discretization of $\Delta\tau = 0.1$ was used, sufficiently small that remaining discretization errors could be neglected.

The second model we consider is the $U \to \infty$ HCB limit of Eq. (1) equivalent to the $S = 1/2$ XY model on an $L \times L$ lattice in a random transverse field:

$$H_{xy} = -\frac{1}{2}\sum_{\mathbf{r},\mathbf{e}} (S_{\mathbf{r}}^+ S_{\mathbf{r}+\mathbf{e}}^- + S_{\mathbf{r}}^- S_{\mathbf{r}+\mathbf{e}}^+) + \sum_{\mathbf{r}} h_{\mathbf{r}} S_{\mathbf{r}}^z, \tag{5}$$

with $h_{\mathbf{r}} \in [-h, h]$ uniformly. In this case we traverse the transition by tuning the disorder strength h. We use a directed loop version of the stochastic series expansion (SSE) [26] to simulate this model. This technique is free of discretization errors and efficient directed algorithms [26,27] are available. We further use a beta-doubling scheme [28] that allows for rapid equilibration at large β values. In contrast to the QR model, we employ a microcanonical ensemble for the disorder by constraining each disorder realization to have *exactly* $\sum_{\mathbf{r}} h_{\mathbf{r}} = 0$. This facilitates the analysis without affecting the results [29]. We use at least $\sim 10^5$ disorder realizations per data point, a large improvement over [3]. In the following $[\ldots]$ denotes the disorder average and $\langle\ldots\rangle$ the thermal average.

Observables.—Our main focus is the scaling behavior of the superfluid stiffness ρ for which the finite-size scaling form Eq. (3) was derived. For both models we measure ρ as

$$\rho = \frac{[\langle W_x^2 + W_y^2\rangle]}{2\beta}, \tag{6}$$

where W_x and W_y are the winding numbers in the spatial directions. [For the HCB model Eq. (6) is multiplied by π to yield ρ.] From Eq (6), it follows that $\beta\rho = W^2$ has a particularly attractive scaling form when $d = 2$, which we may write

$$W^2 = \frac{\beta}{L^z} W(\delta L^{1/\nu}, L/\beta^{1/z}), \tag{7}$$

where we define $\delta = (t - t_c)$ (QR model) and $\delta = (h - h_c)$ (HCB model). We also make extensive use of the correlation functions, defined as $C(\mathbf{r} - \mathbf{r}', \tau - \tau') = [\langle \exp\{i(\theta_{\mathbf{r}}(\tau) - \theta_{\mathbf{r}'}(\tau'))\}\rangle]$ for the QR model and as $C(\mathbf{r} - \mathbf{r}', \tau - \tau') = [\langle S_{\mathbf{r}}^+(\tau)\, S_{\mathbf{r}'}^-(\tau')\rangle]$ for the HCB model.

Results, QR.—A large number of independent simulations of Eq. (4) were carried out at many different L, β close to the QCP. Since we expect ρ to approach zero in an exponential manner as L is increased at *fixed* β and since ρ is likely exponentially suppressed in the insulating phase it

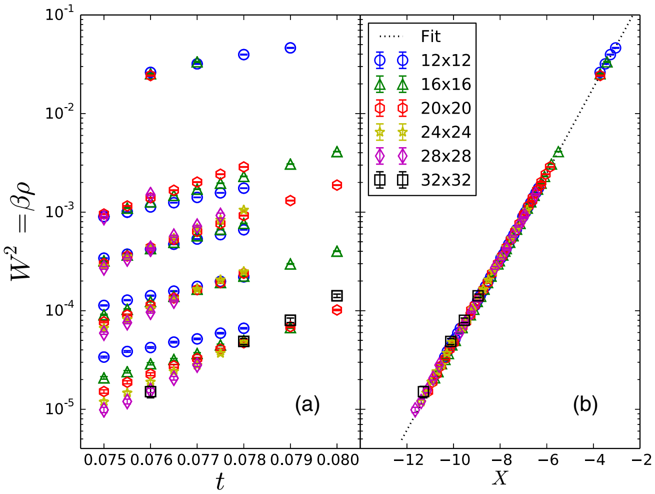

FIG. 1 (color online). Scaling collapse of 142 independent simulations of $W^2 = \beta\rho$ for the QR model. (a) Unscaled data of W^2 versus t. (b) Scaling collapse of the data of panel (a). The data are plotted against the scaling variable $X = \ln(a\beta/L^z) + b(t-t_c)L^{1/\nu} - cL/\beta^{1/z} - d(L/\beta^{1/z})^2$.

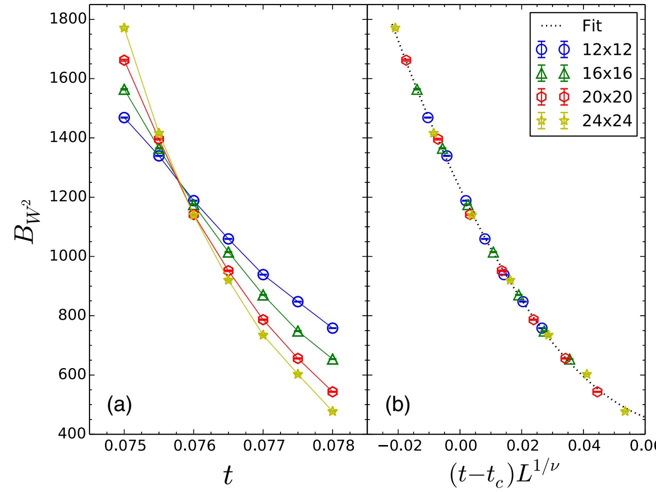

FIG. 2 (color online). The Binder cumulant B_{W^2} for the QR model versus t with $\beta = L^2/4$. (a) Unscaled data showing a crossing close to the critical point $t_c = 0.0760(5)$. (b) Scaling plot versus $(t-t_c)L^{1/\nu}$ obtained by fitting the data in (a) to the form $a + b(t-t_c)L^{1/\nu} + c(t-t_c)^2L^{2/\nu}$ yielding $t_c = 0.758(5)$ and $\nu = 0.98(3)$.

seems reasonable to approximate the function $W(x,y)$ in Eq. (7) as $a\exp[f(x,y)]$ with $x = \delta L^{1/\nu}$, $y = L/\beta^{1/z}$. If the temperature dependence is carefully mapped out [30] one indeed sees that $W(x,y)$ has a clear exponential dependence. As a first step, we then assume $f(x,y) = bx - cy - dy^2$. We can then fit *all* 142 data points to this form determining the coefficients a, b, c, d along with $t_c = 0.0760(5)$ $\nu = 1.00(2)$ and $z = 1.99(5)$. The results are shown in Fig. 1 with a scaling plot using the scaling variable $X = \ln(a\beta/L^z) + b(t-t_c)L^{1/\nu} - cL/\beta^{1/z} - d(L/\beta^{1/z})^2$. A more refined analysis [30] shows that the temperature dependence likely involves a correction term $W^2 = ay^z\exp(bx - cy) + dy^{-w}\exp(bx - c'y)$. The correction term is here proportional to T^w and disappears as T tends to zero. It is straightforward to fit *all* our data to this form which yields identical estimates for t_c, ν, z along with $w = 0.6(2)$. Estimating the AIC (Akaike information criterion) for the two forms heavily favors the latter. We note that our results appear to satisfy $z = d$ and $\nu \geq 2/d$ as equalities in contrast to [4] which finds $z = 1.75(5)$, $\nu = 1.15(3)$.

With a reliable estimate of z we can now *fix* the scaling argument L^z/β. If we then study the Binder cumulant $B_{W^2} = [\langle W^4\rangle]/[\langle W^2\rangle]^2$ we see that at *fixed* L^z/β it should follow a simplified form of Eq. (7), $B_{W^2} = B(\delta L^{1/\nu})$. As shown in Fig. 2, lines for different L will then cross at t_c, thereby confirming our previous estimates.

Our results for the correlation functions for the QR models are shown in Fig. 3 for a $L = 20$ lattice at t_c for a range of temperatures. Asymptotically, one expects [1] $C(\tau) \sim \tau^{-(d-2+z+\eta)/z}$ and $C(\mathbf{r}) \sim r^{-(d-2+z+\eta)}$. Clearly, $C(\mathbf{r})$ drops off much faster than $C(\tau)$ confirming that $z \neq 1$. However, pronounced finite temperature effects are visible in $C(\mathbf{r})$ arising because the limit $\beta \gg L^z$ has not yet been reached which prevents us from reliably determining the power law for $C(\mathbf{r})$. However, from $C(\tau)$ we determine $(z+\eta)/z = y_\tau = 0.85(2)$ and hence $\eta = -0.3(1)$ using our previous estimate $z = 1.99(5)$. This estimate satisfies the rigorous inequality $2-(d+z) < \eta \leq 2-d$ [1] and agrees with the prior result $\eta = -0.3(1)$ [4].

For the QR model we have also verified that the compressibility κ remains finite and independent of L throughout the transition, consistent with $z \leq d$. Furthermore, a direct evaluation of $\partial W^2/\partial t$ directly at t_c for fixed L^z/β, expected from Eq. (7) to scale as $\sim L^{1/\nu}$, yields $\nu = 0.98(4)$ consistent with our previous results.

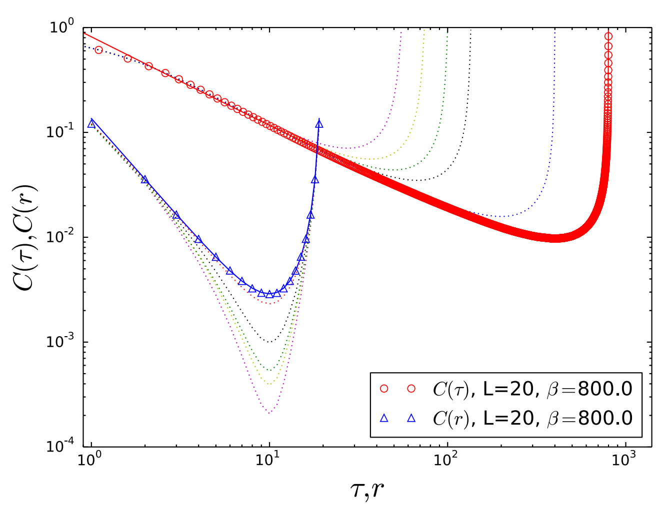

FIG. 3 (color online). The correlation functions $C(\tau)$ and $C(\mathbf{r})$ as a function of τ, $\mathbf{r}$ for a system size $L = 20$. Results are shown for the QR model at the critical point and a range of $\beta = 55, \ldots, 800$. The solid red line is a fit to $\beta = 800$ results for $C(\tau)$ using the form $a[\tau^{-y_\tau} + (\beta-\tau)^{-y_\tau}]$ with pi $(z+\eta)/z = y_\tau = 0.85(2)$.

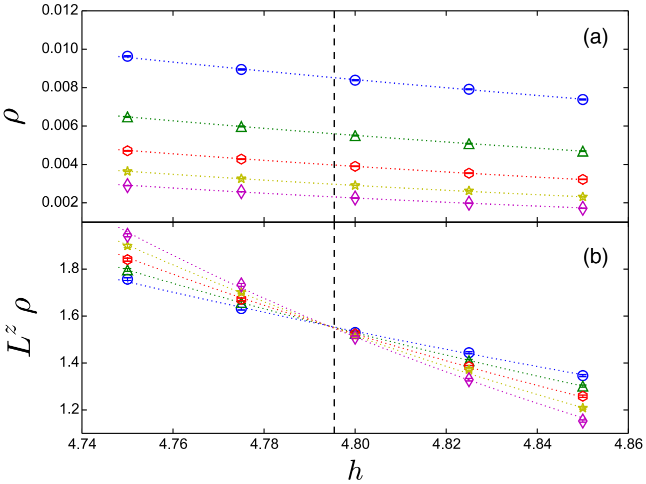

FIG. 4 (color online). Finite-size scaling analysis of the stiffness for the HCB model using data only at $\beta = 512$. (a) ρ vs h. (b) Crossing of the scaling function: $\tilde{R} = L^z \rho$ at the critical point, $h_c = 4.79(3)$ (vertical dashed line). For both panels, the dotted lines are fits to $\tilde{R}(x) = a + bx + cx^2$.

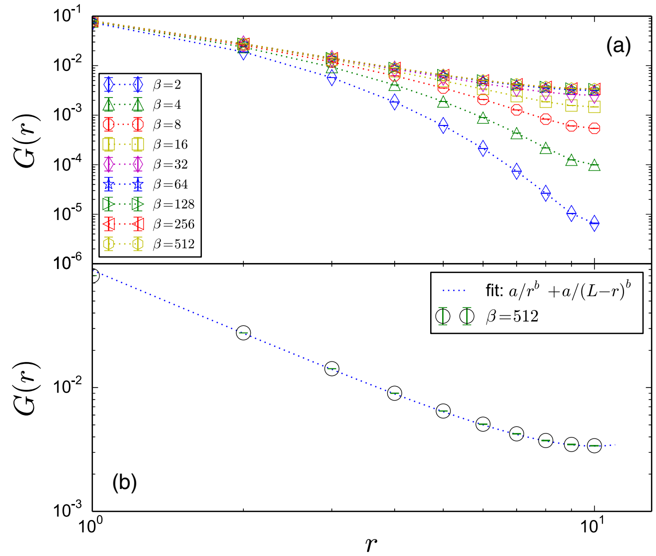

FIG. 5 (color online). The equal-time spatial correlation function, $C(\mathbf{r})$ of a 20×20 lattice for the HCB model at h_c. (a) Convergence of $C(\mathbf{r})$ using the beta-doubling procedure. (b) $C(\mathbf{r})$ (open circles) at $L = 20$, $\beta = 512$ fitted to the form $a(1/r^{y_r} + 1/(L-r)^{y_r})$ (dashed line), yielding $z + \eta = y_r = 1.718(1)$.

Results, HCB.—Because of the hard-core constraint number, fluctuations are dramatically suppressed in the HCB model. Combined with the very effective beta-doubling scheme we can reach much lower temperatures compared to the QR model. Hence, we use a simplified form of Eq. (3),

$$\rho = L^{2-d-z} \tilde{R}(\delta L^{1/\nu}), \tag{8}$$

suppressing the temperature dependence. We have extensively verified that this is permissible for the system sizes used [30] and that our data appear independent of temperature at $\beta = 512$ to within numerical precision. We then fit our data for ρ at $\beta = 512$ to an expansion of $\tilde{R}$ in Eq. (8) to second order $\tilde{R} = a + b\delta L^{1/\nu} + c(\delta L^{1/\nu})^2$, obtaining the estimates $h_c = 4.79(3)$, $z = 1.88(8)$, $\nu = 0.99(3)$ to be contrasted with the values $h_c = 4.42(2)$, $z = 1.40(5)$, and $\nu = 1.10(4)$ determined in Ref. [3] with significantly less statistics. The result of this fit is shown in Fig. 4. In panel *b* of Fig. 4, we show the crossing of the scaling function $\tilde{\rho}$, at $h_c = 4.79(3)$ as carried out for the QR model in Fig. 1 to yield similar results for z, ν, and h_c.

The correlation functions show a pronounced temperature dependence as shown in Fig. 5(a) for $C(\mathbf{r})$. However, as we lower the temperature, $C(\mathbf{r})$ reaches a stable power-law form at $\beta = 512$ for *all* L studied, showing that the regime $\beta \gg L^z$ is reached, confirming that the temperature dependence can be neglected in Eq. (8). To determine the anomalous dimension η we then fit the results in Fig. 5(a) for $L = 20, \beta = 512$ and $h_c = 4.79(3)$ to a power-law form with $z + \eta = y_r = 1.718(1)$ as shown in Fig. 5(b). Using our earlier estimate of z, we obtain $\eta = -0.16(8)$ in reasonable agreement with the QR results. For the HCB model we have also calculated the compressibility κ. It remains roughly constant and independent of L through the transition. We note that our results significantly improve on previous conflicting calculations for the exponents of the HCB model which include $z = 2.0(4)$, $\nu = 0.90(13)$ in [16], $z = 0.5(1)$, $\nu = 2.2(2)$ in [31], and $z = 1.40(2)$, $\nu = 1.10(4)$ in [3].

Conclusion.—Our results for ν for both models studied indicate clearly that $\nu \geq 2/d$ is satisfied as an *equality.* For the dynamical critical exponent z, describing the BG-SF transition, we find a value that is significantly larger than previous estimates. While there is a slight disagreement in the estimate of z for the two models we studied it seems possible that indeed $z = d$. During the final stages of writing this manuscript we became aware of Ref. [32] which for the HCB model reached conclusions similar to ours.

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