Axial Current Generation by $\mathcal{P}$-Odd Domains in QCD Matter

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The dynamics of topological domains which break parity ($\mathcal{P}$) and charge-parity ($\mathcal{CP}$) symmetry of QCD are studied. We derive in a general setting that those local domains will generate an axial current and quantify the strength of the induced axial current. Our findings are verified in a top-down holographic model. The relation between the real time dynamics of those local domains and the chiral magnetic field is also elucidated. We finally argue that such an induced axial current would be phenomenologically important in a heavy-ion collisions experiment.

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Introduction.—One remarkable and intriguing feature of non-Abelian gauge theories such as the gluonic sector of quantum chromodynamics (QCD) is the existence of topologically nontrivial configurations of gauge fields. These configurations are associated with tunneling between different states which are characterized by a topological winding number:

$$Q_W = \int d^4xq, \qquad q = \frac{g^2\epsilon^{\mu\nu\rho\sigma}}{32\pi^2}\mathrm{tr}(G_{\mu\nu}G_{\rho\sigma}), \qquad (1)$$

with $G_{\mu\nu}$ the color field strength. While the amplitudes of the transition between those topological states are exponentially suppressed at zero temperature, such exponential suppression might disappear at high temperature or high density [1]. In particular, for hot QCD matter created in the high energy heavy-ion collisions, there could be metastable domains occupied by such a topological gauge field configuration that violates parity($\mathcal{P}$) and charge-parity ($\mathcal{CP}$) locally. We will refer to those topological domains as the "θ domain" in this Letter (see also Refs. [2] and references therein for more discussion on the nature of the θ domain).

Because of its deep connection to the fundamental aspects of QCD, namely, the nature of P and CP violation, with far-reaching impacts on other branches of physics, in particular, cosmology, the search for possible manifestation of those θ domains in heavy-ion collisions has attracted much interest recently [3,4] (see also [5] for interesting effects of P and CP violation in a related system). A θ domain will generate chiral charge imbalance through the axial anomaly relation

$$\partial_\mu J^\mu_A = -2q. \qquad (2)$$

Furthermore, the intriguing interplay between a $U(1)$ triangle anomaly (in the electromagnetic sector) and chiral charge imbalance would lead to novel $\mathcal{P}$ and $\mathcal{CP}$ odd effects which provide promising mechanisms for the experimental detection of θ domains. For example, a vector current and, consequently, the vector charge separation will be induced in the presence of a magnetic field and chiral charge imbalance. Such an effect is referred to as the chiral magnetic effect (CME) [6] (see Ref. [7] for a recent review). In terms of chiral charge imbalance parametrized by the axial chemical potential μ_A, the CME current is given by $\boldsymbol{j}_V = (N_c e\boldsymbol{B}\mu_A)/(2\pi^2)$.

To decipher the nature of the θ domain through vector charge separation effects such as CME, it is essential to understand not only the distribution of such chiral charge imbalance, but their dynamical evolution as well. Previously, most studies were based on introducing chiral asymmetry by hand, after which the equilibrium response to a magnetic field (or vorticity) is investigated (see Ref. [8] for the case in which the chirality is generated dynamically due to a particular color flux tube configuration). In reality, such as in a heavy-ion collisions experiment, however, the chiral imbalance is dynamically generated through the presence of the θ domain. In this Letter, we study the axial current induced by inhomogeneity of the θ domain, which can be conveniently described by introducing a space-time dependent θ angle $\theta(t,x)$ [cf. Refs. [3,9]]. One may interpret $\theta(t,\boldsymbol{x})$ as an effective axion field creating a θ domain. We show that the presence of $\theta(t,\boldsymbol{x})$ will not only generate chiral charge imbalance, it will also lead to an axial current (cf. Fig. 1):

$$\boldsymbol{j}_A = \kappa_{\mathrm{CS}}\boldsymbol{\nabla}\theta(t,\boldsymbol{x}). \qquad (3)$$

Such an axial current, to the best of our knowledge, has not been considered in the literature so far.

As will be shown later, our results are valid as far as the variation of $\theta(t,\boldsymbol{x})$ in space is on the scale larger than $1/T$ (or mean free path of the system), and the variation of

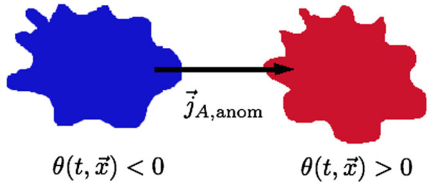

FIG. 1 (color online). A schematic view of the axial current due to the gradient of the effective axion field $\theta(t,\boldsymbol{x})$ [cf. Eq. (3)]. Shaded areas illustrate θ domains (bubbles) with positive θ (red) and negative θ (blue). The axial current flows from θ domains with a smaller value of θ to those with a larger value.

$\theta(t,\boldsymbol{x})$ in time is on the scale longer than the relaxation time of the system, but shorter than the lifetime of the θ domain. It is therefore independent of the microscopic details of the system. While we are considering a system which is in the deconfined phase of QCD, the resulting current bears a close resemblance to that in the superfluid. One may interpret the gradient $\boldsymbol{\nabla}\theta(t,\boldsymbol{x})$ in Eq. (3) as the "velocity" of the θ domain—similar to the case of the superfluid—that the gradient of the phase of the condensate is related to the superfluid velocity. Moreover, we will show that the changing rate $\partial_t\theta(t,\boldsymbol{x})$ is related to the axial chemical potential appearing in the chiral magnetic current, again similar to the "Josephson-type equation" in the superfluid. The relation between μ_A and $\partial_t\theta(t,\boldsymbol{x})$ is suggested in Ref. [6]. We will show how such a connection is realized in a nontrivial way.

The axial current in the presence of $\theta(t,\boldsymbol{x})$.—In this section, we will derive Eq. (3) and the constitutive relation of j_A^μ in the presence of $\theta(t,\boldsymbol{x})$. The expectation value of q induced by θ in Fourier space, is given by $q(\omega,\boldsymbol{k}) = -G_R^{qq}(\omega,\boldsymbol{k})\theta(\omega,\boldsymbol{k})$, where $G^R_{qq}(\omega,k) \equiv -i\int d^4x e^{-i\boldsymbol{k}\cdot\boldsymbol{x}+i\omega t}\langle[q(t,\boldsymbol{x}),q(0,0)]\rangle\Theta(t)$ is the retarded correlator of the density of the topological charge density q. For $\omega,\boldsymbol{k}\ll T$ (or inverse of the mean free path), one may expand $G_R^{qq}(\omega,k)$ up to $O(\omega^2,k^2)$:

$$G_R^{qq}(\omega,k) = -\chi_{\text{Top}} + \frac{1}{2}\left[-i\frac{\Gamma_{\text{CS}}}{T}\omega - \kappa_{\text{CS}}k^2 + \tau_{\text{CS}}\omega^2\right]. \quad (4)$$

Here the first term is the topological susceptibility. It is highly suppressed in the deconfined phase, as indicated by both lattice measurement and holographic calculation [10,11]. We will ignore χ_{Top} from below. Γ_{CS} in the second term is the Chern-Simons diffusion rate and κ_{CS} and τ_{CS} are new transport coefficients. Combining Eq. (4) and the anomaly relation (2), we have in real space

$$\partial_\mu j_A^\mu = -2q(t,\vec{x}) = \left(\frac{\Gamma_{\text{CS}}}{T}\partial_t + \kappa_{\text{CS}}\boldsymbol{\nabla}_x^2 - \tau_{\text{CS}}\partial_t^2\right)\theta(t,\boldsymbol{x}). \quad (5)$$

To proceed, we divide j_A^μ into two parts: $j_A^\mu = j^\mu_{A,\text{anom}} + j^\mu_{A,\text{norm}}$. Here, we require $j^\mu_{A,\text{anom}}$ to satisfy the anomaly equation, i.e., $\partial_\mu j^\mu_{A,\text{anom}} = -2q$. Consequently, the remaining part $j^\mu_{A,\text{norm}}$ is conserved: $\partial_\mu j^\mu_{A,\text{norm}} = 0$. In general, the above division is not unique. However, if we further require that $j^\mu_{A,\text{anom}}$ to be local in θ, i.e., $n_{A,\text{anom}}, \boldsymbol{j}_{A,\text{anom}}$ must be expressed in terms of $\theta(t,\boldsymbol{x})$ and its gradients, j^μ_{anom} can then be determined uniquely from Eq. (5) as follows. We start our analysis with $\boldsymbol{j}_{A,\text{anom}}$. By taking the static limit of Eq. (5) and noting $\boldsymbol{j}_{A,\text{anom}}$ transforms as a vector under $SO(3)$ spatial rotation, one finds that $\boldsymbol{j}_{A,\text{anom}}$ have to be expressed in a gradient of θ with the magnitude fixed by Eq. (5):

$$\boldsymbol{j}_{A,\text{anom}} = \kappa_{\text{CS}}\boldsymbol{\nabla}\theta + \mathcal{O}(\partial^2), \quad (6)$$

as was advertised earlier. Similarly, taking the homogeneous limit of Eq. (5) gives the zeroth component of $j^\mu_{A,\text{anom}}$:

$$j^t_{A,\text{anom}} = \frac{\Gamma_{\text{CS}}}{T}\theta - \tau_{\text{CS}}\partial_t\theta + \mathcal{O}(\partial^2). \quad (7)$$

It is worth pointing out that κ_{CS} appearing in Eq. (4) is accessible by the lattice. To see that, we note in the static limit

$$G^R_{qq}(\omega=0,\boldsymbol{k}) = -\chi_{\text{Top}} - \frac{1}{2}\kappa_{\text{CS}}k^2, \qquad k = |\boldsymbol{k}|. \quad (8)$$

It is related to the Euclidean correlator G^{E}_{qq} by $G^R_{qq}(\omega=0,k) = -G^{\text{E}}_{qq}(\omega=0,k)$, which promises the possibility of measuring κ_{CS} on the lattice through the following Kubo formula:

$$\kappa_{\text{CS}} = \lim_{k\to 0}\frac{d^2}{dk^2}G^{\text{E}}_{qq}(\omega=0,k). \quad (9)$$

At zero temperature, κ_{CS} would coincide with the so-called "zero-momentum slope" of the topological correlation function and is of phenomenological relevance in connection with the spin content of the proton (see Ref. [12] and references therein). However, the importance of κ_{CS} in the deconfined phase of QCD, to the best of our knowledge, has not yet been appreciated. While χ_{Top} is highly suppressed in the deconfined phase, there is no reason for the suppression of κ_{CS}. Equation (6) gives an explicit example where κ_{CS} is phenomenologically relevant.

Chiral charge imbalance, axial chemical potential μ_A, and the real time dynamics of θ.—We are now ready to quantify the chiral charge imbalance due to the presence of $\theta(t,x)$. We concentrate on the first term on the right-hand side of Eq. (7) and define the axial density generated by $\theta(t,\boldsymbol{x})$ as

$$n_{A,\text{anom}}(t,\boldsymbol{x}) \equiv j^t_{A,\text{anom}}(t,\boldsymbol{x}) = \frac{\Gamma_{\text{CS}}}{T}\theta(t,\boldsymbol{x}) + \mathcal{O}(\partial). \quad (10)$$

Equation (10) implies that a local θ domain(bubble) will induce a local axial charge density. Further insight can be obtained by looking at the axial chemical potential μ_A corresponding to $n_{A,\mathrm{anom}}$ in Eq. (10). Using the linearized equation of state $n_A = \chi\mu_A$, where χ is the susceptibility, we have

$$\mu_A = \left(\frac{\Gamma_{\mathrm{CS}}}{\chi T}\right)\theta = \frac{\theta}{2\tau_{\mathrm{sph}}}, \tag{11}$$

where we have introduced the sphaleron damping rate τ_{sph}, which can be related to the Chern-Simons diffusion rate Γ_{CS} by the standard fluctuation-dissipation analysis [13] (see also Ref. [14]): $\tau_{\mathrm{sph}} \equiv (2\chi T)/\Gamma_{\mathrm{CS}}$. Equation (11) relating μ_A and θ is *new* in the literature. It can be connected to the argument of Ref. [6] in which μ_A is identified with $\partial_t\theta$. Equation (11) implies that due to dynamical effects, one should replace ∂_t in the identification $\mu \sim \partial_t\theta$ with $1/\tau_{\mathrm{sph}}$, the characteristic time scale of sphaleron damping. The above analysis suggests that relation Eqs. (10), (11) have already captured the real time dynamics of the effective axion field $\theta(t,\boldsymbol{x})$, namely, the sphaleron damping.

Finally, let us briefly comment on the conserved part of the axial current $j^{\mu}_{A,\mathrm{norm}}$. Because of diffusion, we expect from Eq. (10) that

$$\boldsymbol{j}_{A,\mathrm{norm}} = -D\boldsymbol{\nabla} n_{A,\mathrm{anom}} = -D\frac{\Gamma_{\mathrm{CS}}}{T}\boldsymbol{\nabla}\theta. \tag{12}$$

The conservation of the normal part determines the time component as $j^t_{A,\mathrm{norm}} = -\int dt \boldsymbol{\nabla}\boldsymbol{j}_{A,\mathrm{norm}}$. It depends on the history of the normal part current, thus it is nonlocal in θ. It is also higher order compared to $j^t_{A,\mathrm{anom}}$. For positive κ_{CS}, axial current induced by the θ domain (3) is opposite to the diffusive current (12). We now argue that κ_{CS} is always positive by noting that a nonzero θ will shift the action of the system by $S_\theta = \int d^4xq\theta$. Using the expression for q in Eq. (8), one finds that in the static limit, $S_\theta = -(k_{\mathrm{CS}}/2)\int d^4x(\boldsymbol{\nabla}\theta)^2$. Therefore, κ_{CS} might be interpreted as the coefficient of the kinetic term of the "axion field" θ and must be positive [15].

The holographic model.—The discussion above does not rely on the microscopic details of the theory. We would like to confirm our findings in a top-down holographic model, namely, the Sakai-Sugimoto model [17,18], which at low energy is dual to the four-dimensional $SU(N_c)$ Yang-Mills theory with massless quarks in large N_c and strong coupling. The deconfined phase of the field theory is dual to the $D4$ black-brane metric, which is a warped product of a $5d$ black hole and $S^1 \times S^4$ [19,20]. For the present work, we will consider field fluctuations with trivial dependence on $S^1 \times S^4$, thus, we only need the $5d$ black hole part of the metric:

$$ds^2 = \left(\frac{u}{R}\right)^{\frac{3}{2}}(-f(u)dt^2 + d\vec{x}^2) + \left(\frac{R}{u}\right)^{\frac{3}{2}}\frac{du^2}{f(u)}, \tag{13}$$

where $f(u) = 1-(u_H/u)^3$ and u is the holographic coordinate with $u=\infty$ the boundary and $u=u_H$ the horizon. u_H are related to the temperature of the system by $4\pi T = 3\sqrt{u_H/R^3}$. The flavor degrees of freedom are introduced by a pair of $D8/\bar{D}8$ probe branes, separated along the S^1 direction [17]. The probe branes do not backreact on the geometry.

We will compute axial density n_A and axial current j_A along one particular spatial direction, say the "x" direction in the presence of a source, $\theta(t,x)$. To this end, we consider excitation of the axial gauge field A_M of the $D8/\bar{D}8$-branes, with its field strength $F_{MN} = \partial_M A_N - \partial_N A_M$ and the Ramond-Ramond C_1 form. The index M runs over t,x,u and the rest of the components can be consistently set to zero. The source $\theta(t,x)$ is related to $C_1^{(4)}$, the component of C_1 along S_1 by $2\pi R_4 C_1^{(4)} = \theta$, where R_4 is the radius of S_1. Following the holographic correspondence, the axial current j^{μ}_A is dual to the axial gauge field, A_M and the topological charge density q is dual to $C_1^{(4)}$. In the presence of A_M, we consider instead components of Ramond-Ramond C_7 form (cf. Ref. [17]) B_M. The field strength of B_M, $G_{MN} = \partial_M B_N - \partial_N B_M$, is related to combination of $A_M, C_1^{(4)}$ by: $(\mathcal{N}_G)/(uK)\epsilon^{\mathrm{LMN}}(2\pi R_4\partial_L C_1^{(4)} + 2A_L) = G^{MN}$ by Hodge duality between the C_7 form and C_1 form. Here, $K = 4\pi/3$ and $\mathcal{N}_G = (729\pi K^3 u_H^2)/(4\lambda^3 T^4 R_4^2)$ with λ the 't Hooft coupling.

After integrating over $S^1 \times S^4$ and noting fields depend only on t,x,u, we obtain the effective action, which contains the kinetic terms of F_{MN}, G_{MN}, and Wess-Zumino coupling between F_{MN} and B_M [14]:

$$S = \int d^4x du \frac{1}{4}\left(-\mathcal{N}_F u^{5/2}F^{MN}F_{MN} - \frac{\mathcal{N}_G}{u}G^{MN}G_{MN} - 4K\epsilon^{\mathrm{LMN}}B_L F_{MN}\right), \tag{14}$$

In action (14), $\mathcal{N}_F = (8N_c\lambda^2T^3R_4)/(81u_H^3)$; the indices in Eq. (14) are raised by the $5d$ black hole part of the full metric. The equations of motion following from Eq. (14) are given by

$$\partial_M(G^{MN}/u) = K/(\mathcal{N}_G)\epsilon^{\mathrm{NPQ}}F_{PQ},$$
$$\partial_M(u^{5/2}F^{MN}) = K/(\mathcal{N}_F)\epsilon^{\mathrm{NPQ}}G_{PQ}. \tag{15}$$

According to holographic correspondence, the one point functions n_A, j_A are given by the functional derivative of the gravity on-shell action with respect to the boundary values of A_t, A_x. Using Eq. (15), we can then express n_A, j_A in terms of G_{tx}, F_{tx} [21]:

$$n_A = \frac{2Ki\omega G_{tx} - ik(\mathcal{N}_F u^{5/2} f \partial_u F_{tx})}{\omega^2 - k^2 f}\bigg|_{u\to\infty},$$
$$j_A = \frac{2KikfG_{tx} - i\omega(\mathcal{N}_F u^{5/2} f \partial_u F_{tx})}{\omega^2 - k^2 f}\bigg|_{u\to\infty}. \quad (16)$$

We now need to solve the bulk equation of motion for G_{tx} and F_{tx} [see Eq. (18) and Eq. (19) below] with appropriate boundary condition. We impose the infalling wave condition at the black hole horizon. On the boundary, G_{tx} has the following asymptotic expansion

$$G_{tx} = \frac{K}{2\mathcal{N}_G}(\omega^2 - k^2)\theta(\omega,k)u^2 + \cdots - \frac{q(\omega,k)}{K} + \cdots. \quad (17)$$

The u^2 term is proportional to θ and the constant term gives q. One could verify that Eqs. (16) and (17) indeed reproduce the anomaly equation, $\partial_t n_A + \partial_x j_A = 2KG_{tx}(u\to\infty) = -2q$. We only keep the constant term in near boundary expansion of G_{tx} in the limit. The divergent terms should be removed by holographic renormalization procedure: e.g., the $\omega^2 - k^2$ factor in the leading u^2 term, which is completely determined by the near boundary behavior of the bulk equation of motion, indicates that it is a contact term that can be subtracted by a boundary counterterm. In the case of nonconformal backgrounds, such as the Witten-Sakai-Sugimoto bulk space-time, the holographic renormalization procedure is carefully described in Ref. [22]. On the other hand, F_{tx} is not sourced on the boundary, thus we set $F_{tx}(u\to\infty) = 0$. Note that $K/\mathcal{N}_F \sim O(1/N_c)$, $K/\mathcal{N}_G \sim O(1)$, The back-reaction of F_{tx} to G_{tx} is $1/N_c$ suppressed. Keeping the leading contribution in N_c, we find the following equations of motion for G_{tx} and F_{tx} from action (14):

$$\left[\partial_u\left(\frac{f}{u(\omega^2 - k^2 f)}\partial_u\right) - \frac{R^3}{u^4 f}\right]G_{tx} = 0, \quad (18)$$

$$\left[\partial_u\left(\frac{u^{5/2} f}{\omega^2 - k^2 f}\partial_u\right) - \frac{R^3}{u^{1/2} f}\right]F_{tx} = 2K\frac{k}{\omega}\partial_u\left(\frac{f}{\omega^2 - k^2 f}\right)G_{tx}. \quad (19)$$

Results of the holographic calculation.—We are interested in the solutions to Eq. (18) and Eq. (19) in the hydrodynamic regime, i.e., $\omega, k \ll 1/T$. They can be found analytically, order by order in $(\omega/T, k/T)$, following the standard procedure in the literature (cf. Refs. [23,24]). The full expressions and details of the calculations are straightforward but lengthy and will be reported in a forthcoming paper [14]. In order to compute n_A, j_A, we only need their near-boundary expansions:

$$G_{tx} = \frac{K}{2\mathcal{N}_G}(\omega^2 - k^2)\theta u^2 + \frac{K u_H^2 \theta}{\mathcal{N}_G}\left[-i\omega\left(\frac{u_H}{R^3}\right)^{1/2} + \frac{1}{2}(\omega^2 - k^2) - c_0\omega^2\right], \quad (20)$$

$$F_{tx} = -\frac{4K^2 u_H^2 k\theta}{3\mathcal{N}_G\mathcal{N}_F u^{3/2}}\left[-i\left(\frac{u_H}{R^3}\right)^{1/2} + \frac{2(\omega^2 - k^2)}{3\omega} - c_0\omega\right], \quad (21)$$

where $c_0 = (\sqrt{3}\pi + 3\ln 3)/18$. From Eq. (20), we immediately read q by using Eq. (17). Further comparison with Eq. (5) gives $\Gamma_{\rm CS}$, $\kappa_{\rm CS}$ in the Sakai-Sugimoto model [25]:

$$\Gamma_{\rm CS} = \frac{2u_H^2 K^3 T^2}{\mathcal{N}_G} = \frac{8\lambda^3 T^6}{729\pi M_{\rm KK}^2}, \qquad \kappa_{\rm CS} = \frac{3\Gamma_{\rm CS}}{8\pi T^2}, \quad (22)$$

where $M_{\rm KK} = 1/R_4$ is the mas gap of the theory. Now plugging Eq. (20) and Eq. (21) into Eqs. (16), we recover the time component of the axial current in Eq. (10) and the spatial component as a sum of Eq. (6) and (12):

$$n_A = \frac{\Gamma_{\rm CS}}{T}\theta, \qquad j_A = -ik\left(D\frac{\Gamma_{\rm CS}}{T} - \kappa_{\rm CS}\right)\theta, \quad (23)$$

where the diffusion constant $D = 1/(2\pi T)$ in the Sakai-Sugimoto model [27].

Phenomenological implication in heavy-ion collisions.—In this Letter, we found a new mechanism for generating axial current (3) due to the inhomogeneity of effective θ domains. We now estimate its magnitude in a hot quark-gluon plasma (QGP) and examine its phenomenological importance in heavy-ion collisions. We start by relating θ to μ_A using Eq. (11). In terms of L_θ, the characteristic size of a θ domain, Eq. (3), can be then estimated as

$$j_{A,\theta} \sim (\mu_A \kappa_{\rm CS})\left(\frac{\tau_{\rm sph}}{L_\theta}\right) \sim (\mu_A T^2)\left(\frac{\tau_{\rm sph}}{L_\theta}\right), \quad (24)$$

where in the last step we have taken our holographic results (22), which implies $\kappa_{\rm CS} \sim T^2$ as a crude estimate of $\kappa_{\rm CS}$ in QCD plasma.

We now compare Eq. (24) to axial current from other sources. For QGP in the presence of magnetic field, axial current can be generated by chiral charge separation effects (CCSE) [28]. Similar to CME, the CCSE current is given by $\boldsymbol{j}_{A,\rm CCSE} = (N_c \mu_V e\boldsymbol{B})/(2\pi^2)$. In top energy collisions at relativistic heavy ion collider, eB at the early stage is of a few m_π^2 and, consequently, $N_c e^2 B/2\pi^2$ is at most the same order as T^2. Moreover, in those collisions, most of μ_V (or μ_B) is generated from fluctuations and is expected to be of the same order as μ_A. We therefore conclude that axial current is at least comparable to CCSE current if $\tau_{\rm sph}/L_\theta \sim \mathcal{O}(1)$, but could be larger if $L_\theta < \tau_{\rm sph}$. A similar argument also applies to the comparison to the chiral electric separation effect [29].

The axial current (3) studied in this work is induced by topological fluctuation. In plasma with chiral charge, axial charge can also be generated by thermal fluctuation, which is nontopological. Axial current can also exist as diffusion of such a charge. Assuming the corresponding μ_A is the same order as the one from topological fluctuation, we can estimate the current as

$$\boldsymbol{j}_A = -D\boldsymbol{\nabla} n_A \sim D\chi\frac{\mu_A}{L} \sim T\frac{\mu_A}{L}, \tag{25}$$

where L is the mean free path of fermions and we have taken $D \sim 1/T$ and $\chi \sim T^2$. Comparing with Eq. (24), we conclude if the θ domain parameter $\tau_{\rm sph}/L_\theta$ is larger than T/L, the current (3) would dominate over the axial current generated by thermal diffusion.

To sum up, if the condition $\tau_{\rm sph}/L_\theta \gtrsim 1$, $\tau_{\rm sph}/L_\theta \gtrsim T/L$ is achieved in heavy-ion collisions, the new current (3) proposed in this Letter would become phenomenologically important.

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