Electron-Phonon Interactions, Metal-Insulator Transitions, and Holographic Massive Gravity

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(Received 20 November 2014; revised manuscript received 6 May 2015; published 23 June 2015)

Massive gravity is holographically dual to “realistic” materials with momentum relaxation. The dual graviton potential encodes the phonon dynamics, and it allows for a much broader diversity than considered so far. We construct a simple family of isotropic and homogeneous materials that exhibit an interaction-driven metal-insulator transition. The transition relates to the formation of polarons—phonon-electron quasibound states that dominate the conductivities, shifting the spectral weight above a mass gap. We characterize the polaron gap, width, and dispersion.

DOI: 10.1103/PhysRevLett.114.251602 PACS numbers: 11.25.Tq, 04.50.Kd, 71.30.+h, 71.38.−k

Introduction.—Strongly correlated materials are interesting because they give rise to rich collective behavior and emergent phenomena. Classic examples are high-temperature superconductors and their parent materials, often strange metals and Mott-like insulators [1,2]. The purpose of this Letter is to use the gauge-gravity duality (GGD) to shed some light onto the collective behavior behind the interaction-driven metal-insulator transition (MIT).

The central observation that justifies the use of the GGD beyond the few duality pairs that have been identified so far is that the key features of the AdS/CFT duality [3] seem to hold rather generally. Conformal field theories (CFTs) that admit a gravity dual are believed to be a subclass of the CFTs with a large N limit and a large ‘t Hooft-like coupling such that the spectrum of light operators in the CFT reduces to just the stress tensor $T_{\mu\nu}$ and a few more. The (quantum) dynamics in these CFTs is expected to simplify, enormously, into a classical gravitational theory in asymptotically anti–de Sitter (AdS) space in one more dimension, that plays the role of the renormalization scale. Here, we will use this notion of the GGD, working directly in the gravity dual and extracting its CFT interpretation.

An interesting recent development is that, in order to model realistic materials, holography needs to incorporate momentum relaxation, and an appealing way to do so is through the gravity dual being massive [4].

Momentum nonconservation requires breaking translation invariance, which is part of a gauged symmetry (diffeomorphism invariance) in the gravity dual. The simplest way to break it is via a graviton mass term (see [5–8] for previous attempts). Importantly, the physical content of this breaking is the introduction of new degrees of freedom. In the present context, these are identified as the phonons.

Holographic massive gravity (HMG).—There are two ways to define a massive gravity theory. One is a diffeomorphism noninvariant language where we add a potential term for the metric. To fix ideas, let us concentrate on a choice similar to the one suggested in [4]

$$m^2 V(P_{\mu\nu} g^{\mu\nu}),$$

where $P_{\mu\nu}$ is a fixed external matrix that is assumed to project only on the spatial coordinates, x^i, probed by the CFT [9]. Thus, $P_{ij} = \delta_{ij}$ and zero, otherwise, which leads to a Lorentz noninvariant mass term.

This theory propagates extra degrees of freedom. It is very convenient to make them explicit resorting to a diffeomorphism invariant presentation. This was systematized in [10,11] for general Lorentz noninvariant mass terms. The minimal covariantization in the present case requires a set of scalar fields Φ^I transforming under an internal Euclidean group of translations and rotations in field space. The massive gravity action is recovered as the truncation to two-derivative operators

$$S_{\mathrm{HMG}} = M_P^2 \int d^4x \sqrt{-g}\left[\frac{R}{2} + \frac{3}{\ell^2} - m^2 V(X)\right], \tag{1}$$

with $X \equiv \frac{1}{2} g^{\mu\nu}\partial_\mu \Phi^I \partial_\nu \Phi^I$ and ℓ the AdS radius. The theory admits solutions where the scalars take on linear vacuum expectation values $\bar{\Phi}^I = \alpha \delta_i^I x^i$ with δ_i^I the Kronecker delta. On these solutions, the previous projector $P_{\mu\nu}$ is identified as $\propto \partial_\mu \bar{\Phi}^I \partial_\nu \bar{\Phi}^I$. This procedure was also described in [12,13] for the potential assumed in [4]. Note that the Lagrangian for the scalars Φ^I is formally identical to (a particular case of) the effective Lagrangian describing phonons—the Goldstone bosons of spontaneously broken translation invariance [14]—in solids [15]. Below, we discuss when the Φ sector includes a phonon mode.

Our first point is that Ref. [4], and most of the literature on HMG to date, unnecessarily restricts to a very narrow family of potentials. They assume, formally, the same potential for the metric as in the de Rham-Gabadadze-Tolley massive gravity [16]. That choice has a number of advantages when the external metric $P_{\mu\nu}$ is Minkowskian, but in the Lorentz noninvariant case that is relevant for condensed matter, the set of consistent choices is much broader [10,11].

Let us discuss the constraints on the form of $V(X)$. First, we require absence of ghosts and other pathological instabilities. Around configurations with rotational symmetries, such as the black branes (BBs) (7), it is convenient to separate the analysis of linear perturbation in scalar, vector, and tensor modes. The full analysis of the scalar sector is a bit lengthy because of the profusion of scalar modes (in the metric, vector, and Goldstone bosons). Instead, we will now perform the analysis in the "decoupling limit" where we turn on only Goldstone fluctuations, which should be valid for small m. We perturb the Goldstones, $\Phi^I = \bar{\Phi}^I + \phi^I$, and expand the Lagrangian to second order. Dropping tadpole terms and the distinction between I and i indices, one gets

$$V'(\bar{X})\partial_\mu\phi^i\partial^\mu\phi^i + \bar{X}V''(\bar{X})(\partial_i\phi^i)^2. \qquad (2)$$

The absence of ghosts, then, leads to monotonic potentials

$$V'(\bar{X}) > 0. \qquad (3)$$

The local (sound) speed of longitudinal phonons is

$$c_S^2 = 1 + \frac{\bar{X}V''(\bar{X})}{V'(\bar{X})} > 0. \qquad (4)$$

While this is not yet the speed of any physical excitation, it is safest to require it to be positive everywhere to guarantee the absence of gradient instabilities [17].

No further constraints arise from the vector and tensor sectors nor at the nonlinear level in the scalars, which is not surprising since a substantial advantage of the Lorentz noninvariant mass terms is that they can be free from the Boulware-Deser ghost [10,11]. We obtain further constraints by requiring that the theory admits asymptotically AdS solutions with $X \to 0$, implying that $V(0) = 0$. In addition, requiring that the Goldstone sector is weakly coupled for $X = 0$ requires that $V'(0)$ is not too small.

All in all, an economic and safe way to satisfy all constraints is to assume that $V(X)$ is monotonic and linear close to $X = 0$. To fix ideas, we shall consider potentials of the form $V_N(X) = X + \beta X^N/N$ with $\beta > 0$ and $N > 1$.

In any asymptotically AdS solution [like the planar black branes in Eq. (7)], $\bar{X} = \alpha^2 u^2$ and close to the AdS boundary ($u = 0$) the Goldstone gradient $\bar{X}$ vanishes. This implies that the mass term for metric modes is also vanishing. So, this is a weaker form of massive gravity than is usually discussed in cosmology—the Compton wavelength is, at most, of the order of the curvature radius of the spacetime. In the CFT interpretation, the stress tensor $T_{\mu\nu}$ does not develop an anomalous dimension [18]. Still, as shown in [4], this is enough to lead to momentum relaxation in the CFT. The good news is that the Goldstone sector required for that is very healthy—again, because it is enough that the mass term is "active" only in the interior of AdS. This is why the form of the potential (the kinetic function of the Goldstone bosons) is almost unconstrained. For the same reason, one can be confident that a proper analysis of the scalar sector will not change the conditions (3), (4) much.

Impurities and phonon dynamics.—Let us now translate this into CFT language. Demanding that the CFT includes momentum relaxation, we ended up with a rather general form of massive gravity. This theory contains a universal sector consisting of the Goldstone degrees of freedom. In CFT language, these are a multiplet of operators $\mathcal{O}^I$ with internal shift symmetries and which are related with phonons and impurities [4]. This can be understood at once by realizing that the scalar profiles realize both an explicit and a spontaneous breaking of x^i—translations. The amount of translation breaking is controlled by the graviton mass term $\bar{m}^2(u) \equiv m^2\alpha^2 V'(\alpha^2 u^2)$, which is scale dependent. Thus, the strength of explicit breaking is given by $\ell\bar{m}(0)$. It suffices that $\ell\bar{m}(0) \ll 1$ to realize a mostly spontaneous breaking. In that case, the presence of light (transverse and longitudinal) phonon modes is guaranteed [19]. This can be explicitly checked by finding the quasinormal modes in the Φ^I sector, see [20].

In the $\ell\bar{m}(0) \ll 1$ limit, then, $\mathcal{O}^I$ includes the phonons [21]. Their dispersion relations encode the material elastic moduli, see [22]. On the other hand, the amount of explicit breaking $\ell\bar{m}(0)$ can be thought to effectively encode a homogeneous distribution of impurities or disorder.

Now, let us look at the quadratic action (2) more closely. Turning to the canonically normalized field $\sqrt{V'(\bar{X})}\phi^i$, one sees that both transverse and longitudinal phonons develop a "scale-dependent" mass ($\bar{X} \sim u^2$)

$$M_\Phi^2(u) = (\partial\bar{a})^2 + \Box\bar{a} = \frac{f(u\partial_u\bar{a})^2}{\ell^2} + \frac{u^2\partial_u(f\partial_u\bar{a})}{\ell^2}, \qquad (5)$$

where $\bar{a}(u) = \frac{1}{2}\log V'(\bar{X})$, and, in the second equation, we specialized to the BB metric (7). We note a few interesting properties: (i) $M_\Phi \neq 0$ only for nonlinear V; (ii) close to the UV boundary, $M_\Phi^2 \sim [V''(0)/V'(0)]u^2$ so δO^I admits a unitary standard quantization with zero anomalous dimension; and (iii) on the horizon ($f = 0$)

$$M_\Phi^2 \sim -|f'(u)| \left.\frac{V''(\bar{X})}{V'(\bar{X})}\right|_{\text{horizon}} < 0.$$

It follows that a large enough V''/V' leads to a mild form of instability near the horizon. This triggers a shift to nonzero mass of the spectral weight in the response functions, with the associated formation of a gap and, eventually, a sharp resonance [23]. Physically, the phonon mode develops an energy gap. In the presence of charge carriers, such phonon resonant modes trap the carriers, forming proper polarons, and can effectively suppress the dc conductivity. (See [1,2,24,25] for reviews on the polaron concept [26] and its applications.) In the rest of this Letter, we elaborate on the interpretation of such quasiparticle states as strong-coupling versions of polarons and on their role in a certain class of metal-insulator transitions. The numerical computation of the optical conductivity confirms this picture, see Fig. 1. Indeed, for sizeable V''/V', we find a spectral weight transfer into a "midinfrared" (polaron) peak.

The "instability" $M_\Phi^2 < 0$ close to the horizon is reminiscent of holographic superconductivity [29], but with an important difference. At $T = 0$, on the horizon $M_\Phi^2 = 0$, so the Breitenlohner-Friedman bound is satisfied. Therefore, there is no condensation or vacuum instability. Still, having $M_\Phi^2 < 0$ at finite T is enough to impact on low energy modes, in our case favoring the formation of the energy gap ω_0 for the phonons or polarons.

Importantly, polaron formation is perfectly compatible with absence of gradient instability because V''/V' needs to be positive. Notice that, in this case, (4) is superluminal. However, this does not imply that there are superluminal excitations. The polaron quasiparticles below are localized close to the horizon. Their physical speed, then, receives an additional redshift factor that can render them subluminal. It seems quite generic, though, that whenever polarons form, the longitudinal phonons or polarons should be faster than transverse ones.

Holographic metal-insulator transition.—In order to model a transition between itinerant and polaron-localized excitations, we need to add one ingredient, the charge carriers. So, we assume that the CFT also contains a conserved current operator J_μ. This is implemented in the gravity dual by adding to the model a Maxwell field

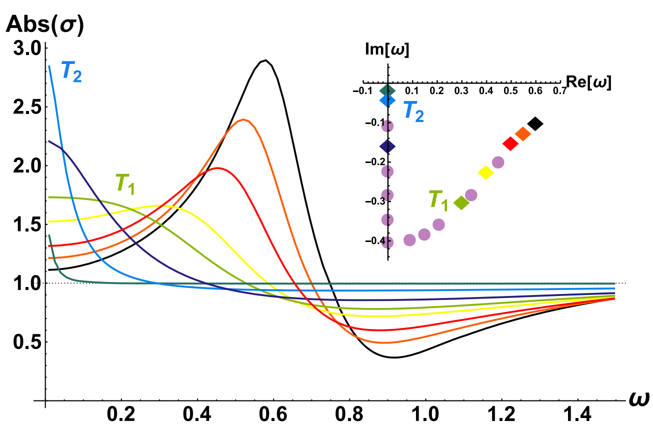


FIG. 1 (color online). Development of polaron formation as temperature is decreased. $T = 1, 0.46(T_2), 0.35, 0.3(T_1), 0.27, 0.22, 0.17, 0.04$. Inset: motion of the lightest quasi-normal-mode (QNM) for the same temperatures (corresponding colors) and some intermediate additional values (purple dots). At large T, the QNM separates from the real axis with decreasing T, until it collides with the next QNM (near T_1) and forms a pair of conjugated poles with positive and negative real parts—the polaron particle-antiparticle poles. (Similar QNM collisions have been observed, e.g., in [27,28]. In our case, it is crucial that, after collision, the QNM approaches the real axis.)

$$S = S_{\rm HMG} - \frac{M_P^2}{4}\int d^4x\sqrt{-g}F_{\mu\nu}F^{\mu\nu}, \tag{6}$$

with $F_{\mu\nu} = \partial_{[\mu}A_{\nu]}$. As usual, the subleading mode of A_μ towards the AdS boundary is the dual J_μ. To introduce a finite density of charge carriers ρ, we turn on a chemical potential μ. The dual of the CFT ground state at finite μ, temperature T, and impurity strength α is a planar BB with $A_t(u \to 0) = \mu$, $\Phi^I = \alpha\delta_i^I x^i$, and

$$ds^2 = \frac{\ell^2}{u^2}\left[\frac{du^2}{f(u)} - f(u)dt^2 + dx^2 + dy^2\right]. \tag{7}$$

Here, u is the holographic coordinate dual to the renormalization scale $\mu \sim 1/u$.

The Maxwell equations set $A_t = \mu - \rho u$ with ρ the charge-carrier density. The Einstein equations reduce to

$$uf'(u) = -3 + 3f + \rho^2u^4/(2\ell^2) + (m\ell)^2V(\alpha^2u^2/\ell^2). \tag{8}$$

The solution for general V is

$$f(u) = u^3\int_u^{u_H} dv\left[\frac{3}{v^4} - \frac{\rho^2}{2\ell^2} - \frac{(m\ell)^2}{v^4}V\left(\frac{\alpha^2v^2}{\ell^2}\right)\right], \tag{9}$$

where u_H stands for the location of the BB horizon. Regularity of the gauge field on the horizon requires $\rho = (\mu/u_H)$, and that the temperature is (from now on, $\ell = 1$)

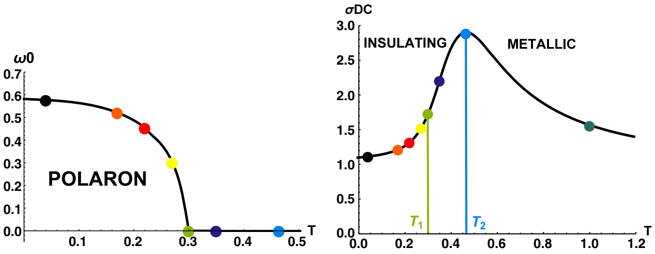
$$T = -\frac{f'(u_H)}{4\pi} = \frac{6 - \mu^2u_H^2 - 2m^2V(\alpha^2u_H^2)}{8\pi u_H}. \tag{10}$$

The energy density is given by $f'''(0)/6$ which can be expressed in terms of μ, α, and u_H. In practice, though, it is more convenient to trade the energy density in favor of u_H, which relates to the entropy density ($s \propto u_H^{-2}$).

Optical conductivities.—Now, let us discuss the behavior of small excitations around the BBs above. We perturb the previous solutions by setting $A_\mu = \bar{A}_\mu + a_\mu$, $g_{\mu\nu} = \bar{g}_{\mu\nu} + h_{\mu\nu}$, and $\Phi^I = \bar{\Phi}^I + \phi^I$, with bars denoting the background solutions, and linearize. In the remainder, we concentrate on transverse vector modes (e.g. $\partial^i a_i = 0$ [30]). Vector perturbations are encoded in $a_i, \phi_i, h_{ti}, h_{ui}$, and $h_{ij} \equiv (1/u^2)\partial_{(i}b_{j)}$. Aside from a_i, we use the gauge-invariant combinations

$$T_i \equiv u^2h_{ti} - \frac{\partial_t\phi_i}{\alpha}, \quad U_i \equiv f(u)\left[h_{ui} - \frac{\partial_u\phi_i}{\alpha u^2}\right], \quad B_i \equiv b_i - \frac{\phi_i}{\alpha}.$$

The linearized equations can be written as [31]



(a) (b)

FIG. 2 (color online). (a) The polaron energy-gap ω_0 as a function of T, extracted numerically as the peak position in $\mathrm{Abs}[\sigma(\omega)]$. ω_0 detaches from 0 at T_1. (b) dc conductivity $\sigma_{\rm dc}(T)$ as a function of T. The continuous line is the analytic result (11) and the dots are the numerical computation. One observes two critical temperatures: the maximum in $\sigma_{\rm dc}(T)$ marking the separation between metallic-insulating behavior (T_2), and the polaron gap formation (T_1).

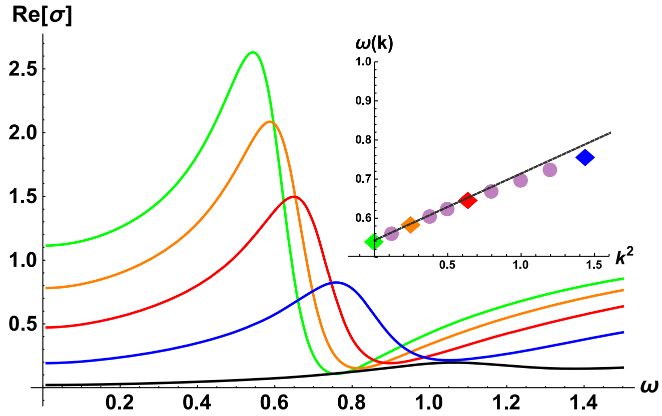


FIG. 3 (color online). Motion of the polaron peak with wave number. $T = 0.04$, $k = 0, 0.5, 0.8, 1.2, 2$. Inset: the extracted dispersion relation.

$$\partial_u(f\partial_u a_i) + \left[\frac{\omega^2}{f} - k^2 - 2u^2\rho^2\right]a_i = \frac{i\rho u^2(2\bar{m}^2 + k^2)}{\omega}U_i - \frac{if\rho k^2}{\omega}\partial_u B_i,$$

$$\frac{1}{u^2}\partial_u\left[\frac{fu^2}{\bar{m}^2}\partial_u(\bar{m}^2 U_i)\right] + \left[\frac{\omega^2}{f} - k^2 - 2\bar{m}^2\right]U_i = -2i\rho\omega a_i + \frac{f'k^2}{u^2}B_i,$$

$$k\left\{u^2\partial_u\left(\frac{f}{u^2}\partial_u B_i\right) + \left[\frac{\omega^2}{f} - k^2 - 2\bar{m}^2\right]B_i = -2\frac{\bar{m}'}{\bar{m}}U_i\right\},$$

where we introduced $\bar{m}^2(u) = \alpha^2 m^2 V'(\alpha^2 u^2)$.

The optical conductivities can be extracted from the numerical integration of these equations. One imposes infalling boundary conditions (dual to the retarded Green function prescription). For the electric conductivity, one imposes that $U_i = 0$ at $u = 0$ and reads off the optical conductivity as $\sigma(\omega) = (\partial_u a_j / i\omega a_j)|_{u\to 0}$ (no summation over repeated indices).

The results are plotted in the figures for $V(X) = X + X^5$, $(m\ell)^2 = 0.05$, $\rho = 1$, and $\alpha = \sqrt{2}$ [20]. Polaron formation is seen in Fig. 1 as T decreases [32]. Figure 2 shows the polaron gap (order parameter), $\omega_0(T)$, and $\sigma_{\rm dc}(T)$. Figure 3 shows the polaron dispersion relation, fitting well to $\omega = \omega_0 + k^2/(2m_*)$ at low k.

In the homogeneous limit $k \to 0$ the gauge-invariant variable B_i decouples, and it is consistent to set $B_i = 0$. Following [33], one arrives at a formula for $\sigma_{\rm dc}$ [33,34]

$$\sigma_{\rm dc} = 1 + \frac{\rho^2 u_H^2}{\alpha^2 m^2 V'(\alpha^2 u_H^2)}. \tag{11}$$

The plot $\sigma_{\rm dc}(T)$ in Fig. 2(b) clearly displays a change from metallic ($d\sigma/dT < 0$) to insulating ($d\sigma/dT > 0$) behavior [35]. Let us emphasize that we take ρ constant in order to isolate the dynamics that drives the transition [36]. From (11), one can extract analytic expressions for the critical temperature, T_2, where $d\sigma/dT = 0$. The condition becomes $\bar{X}V''(\bar{X}) = V'(\bar{X})$, in agreement with the analysis above in terms of $M_\Phi^2 < 0$ near the horizon. For the benchmark model $V_N(X)$ above, one finds $\alpha u(T_2) = [(N-2)\beta]^{-[1/2(N-1)]}$, showing that one needs a high enough exponent, $N > 2$. Models with $N < 2$ tend to give incoherent metals with no MIT.

These results indicate that one can distinguish two different critical temperatures (numerically, not far from each other [37]): T_2 marks the maximum in $\sigma_{\rm dc}$ and T_1 marks the polaron gap formation [defined here as when the peak position, ω_0, in $\mathrm{Abs}\{\sigma(\omega)\}$ separates from 0 [38]].

We must note the quite intriguing intermediate range $T_1 < T < T_2$, showing insulating behavior yet without polarons. A possible interpretation is that polarons form first incoherently for $T_1 < T < T_2$ and coherently for $T < T_1$. See [39–41] for similar ideas.

Discussion.—In summary, we have seen that a simple nonlinear extension of holographic massive gravity captures interesting features of correlated materials such as polaron localization and a (phonon-)interaction-driven MIT. The spectrum of excitations around simple black brane solutions includes clear polaronlike quasilocalized states close to the MIT transition. Unlike previous holographic models [7,42–44], our MIT takes place in homogeneous and isotropic materials, and it does not rely strongly on the running of the electromagnetic coupling (as in [42,45]). See, also, [27,46,47] for the connection between holographic models and quasiparticle descriptions. Several other aspects of the transition are going to be presented elsewhere [22], but one can advance potential correlations: when polarons form, the elastic and piezoelectric response (both linear and nonlinear) is large.

Let us emphasize that polarons are thought to play an important role in cuprate superconductors and colossal magnetoresistance in manganites, see, e.g., [2,24,25,39, 48–50]. Additionally, there is a considerable body of experimental evidence that polarons do occur in a variety of materials [24,25,48–54]. It would be interesting to understand how much can be learned from the new holographic handle on polaron physics. From our perspective, there seems to be potentially many other massive gravity phases that could be similarly relevant for this and, perhaps, other condensed matter applications. We hope to return to these questions soon.

We thank A. Pomarol, N. Magnoli, and S. Sibiryakov for useful discussions. We acknowledge support from MINECO (Ministerio de Economía y Competitividad) under Grant No. FPA2011-25948, DURSI (Departament d'Universitats, Recerca i Societat de la Informació, Barcelona, Spain) under Grant No. 2014SGR1450, and Centro de Excelencia Severo Ochoa program, Grant No. SEV-2012-0234. O. P. is supported by a Ramon y Cajal Fellowship (MICINN-RyC Program).

[1] F. Gebhard, *The Mott metal-insulator transition: Models and methods*, in Springer Tracts in Modern Physics Vol. 137 (Springer-Verlag, Berlin and Heidelberg, 1997).
[2] D. N. Basov, R. D. Averitt, D. van der Marel, M. Dressel, and K. Haule, Electrodynamics of correlated electron materials, Rev. Mod. Phys. **83**, 471 (2011).
[3] J. M. Maldacena, The Large-N limit of superconformal field theories and supergravity, Int. J. Theor. Phys. **38**, 1113 (1999) [Adv. Theor. Math. Phys. **2**, 231 (1998)].
[4] D. Vegh, "Holography without translational symmetry," arXiv:1301.0537.
[5] S. A. Hartnoll and D. M. Hofman, Locally Critical Resistivities from Umklapp Scattering, Phys. Rev. Lett. **108**, 241601 (2012).
[6] S. A. Hartnoll and C. P. Herzog, Ohm's law at strong coupling: S duality and the cyclotron resonance, Phys. Rev. D **76**, 106012 (2007).
[7] A. Donos and S. A. Hartnoll, Interaction-driven localization in holography, Nat. Phys. **9**, 649 (2013).
[8] G. T. Horowitz, J. E. Santos, and D. Tong, Optical conductivity with holographic lattices, J. High Energy Phys. 07 (2012) 168.
[9] For planar materials ($2+1$ CFTs) the gravity dual coordinates are $x^\mu = \{u, t, x^i\}$, with u dual to the RG scale.
[10] V. A. Rubakov, "Lorentz-violating graviton masses: Getting around ghosts, low strong coupling scale and VDVZ discontinuity," arXiv:hep-th/0407104.
[11] S. L. Dubovsky, Phases of massive gravity, J. High Energy Phys. 10 (2004) 076.
[12] T. Andrade and B. Withers, A simple holographic model of momentum relaxation, J. High Energy Phys. 05 (2014) 101.
[13] M. Taylor and W. Woodhead, Inhomogeneity simplified, Eur. Phys. J. C **74**, 3176 (2014).
[14] H. Leutwyler, Nonrelativistic effective Lagrangians, Phys. Rev. D **49**, 3033 (1994); Phonons as goldstone bosons, Helv. Phys. Acta **70**, 275 (1997).
[15] A. Nicolis, R. Penco, and R. A. Rosen, Relativistic fluids, superfluids, solids and supersolids from a coset construction, Phys. Rev. D **89**, 045002 (2014).
[16] C. de Rham, G. Gabadadze, and A. J. Tolley, Resummation of Massive Gravity, Phys. Rev. Lett. **106**, 231101 (2011).
[17] The form of $V(X)$ that sets $c_S^2 = 0$ is $V(X) \propto \log X$.
[18] Generating an anomalous dimension requires $V \sim \log X$, which upsets longitudinal phonons and AdS asymptotics.
[19] Since phonons relate to the breaking of space translations, the CFT observables associated with them are encoded in the energy momentum tensor, in particular in its spatial transverse part $T_{ij}^{(V)}$. In fluids, the $T^{(V)}T^{(V)}$ correlator exhibits a diffusive (overdamped) nonpropagating pole. In solids, it exhibits a propagating (underdamped) excitation—the transverse acoustic phonons. Since T_{ij} overlaps with $\partial_{(i}\mathcal{O}_{j)}$, when Φ^I (that is, $\mathcal{O}^I$) contains propagating transverse modes, so does T_{ij}, and one can identify them as phonons. The massive propagating phonon pole in our case is reminiscent of commensurately pinned crystals. We thank an anonymous referee for these and other valuable comments.
[20] The QNM motion in the same model with $\rho = 0$ is similar to the inset of Fig 1. In that case, δA_i decouples and the QNM is a phonon, which is light because $\ell \bar{m}(0) < 1$.
[21] Because, in the UV, we have a fixed point, not a real lattice, these phonon operators mimic the behavior of real phonons at low energies only. This, though, is enough to extract the low-frequency electroelastic response.
[22] M. Baggioli and O. Pujolas (to be published).
[23] This mechanism can be seen to take place within the validity of the effective field theory, see [22].
[24] J. T. Devreese and A. S. Alexandrov, Frlich polaron and bipolaron: Recent developments, Rep. Prog. Phys. **72**, 066501 (2009); J. T. Devreese, "Lectures on Fröhlich Polarons from 3D to 0D—including detailed theoretical derivations," arXiv:1012.4576.
[25] A. S. Mishchenko, N. Nagaosa, Z.-X. Shen, G. De Filippis, V. Cataudella, T. P. Devereaux, C. Bernhard, K. W. Kim, and J. Zaanen, Charge Dynamics of Doped Holes in High T_c Cuprate Superconductors: A Clue from Optical Conductivity, Phys. Rev. Lett. **100**, 166401 (2008).
[26] By "polarons" we mean a slightly broader concept of charge overdensity bound to lattice distortion, which need not require individual particlelike charge carriers.
[27] R. A. Davison and B. Gouteraux, Momentum dissipation and effective theories of coherent and incoherent transport, J. High Energy Phys. 01 (2015) 039.
[28] I. Amado, D. Areán, A. Jimenez-Alba, K. Landsteiner, L. Melgar, and I. Salazar Landea, Holographic type II Goldstone bosons, J. High Energy Phys. 07 (2013) 108.
[29] S. A. Hartnoll, C. P. Herzog, and G. T. Horowitz, Building a Holographic Superconductor, Phys. Rev. Lett. **101**, 031601 (2008).
[30] In the homogeneous limit, the transverse vector variables become just constant vectors under rotations.
[31] T_i is constrained by an algebraic equation, see [22].
[32] At $T = 0$, there is a polaron formation quantum crossover transition driven by increasing $V''(\bar{X})$, i.e., the nonlinear elastic response.

[33] M. Blake and D. Tong, Universal resistivity from holographic massive gravity, Phys. Rev. D **88**, 106004 (2013).
[34] R. A. Davison, Momentum relaxation in holographic massive gravity, Phys. Rev. D **88**, 086003 (2013).
[35] While, certainly, our geometries are not insulators in the sense that $\sigma_{dc} = 0$ at $T = 0$, they allow several deformations to significantly decrease $\sigma_{dc}(T = 0)$, see [22].
[36] Keeping μ constant, a MIT can also be obtained choosing, e.g., $V(X) = X^{\epsilon} + \beta X^N$ with $0 < \epsilon < 1$, $N > 2$.
[37] The proximity between T_1 and T_2 persists for other parameter choices so long as the MIT is present.
[38] The inset of Fig. 1 reveals another important temperature, separating the presence of an over-damped diffusive pole from the propagating underdamped phonon pole. When viewed in function of $m\ell$, this suggests an exotic behavior whereby the material turns from a viscous liquid to a solid under the influence of an external translation breaking potential.
[39] N. Mannella *et al.*, Nodal quasiparticle in pseudogapped colossal magnetoresistive manganites, Phys. Rev. B **76**, 233102 (2007).
[40] J. Merino and R. H. McKenzie, Transport properties of strongly correlated metals: A dynamical mean-field approach, Phys. Rev. B **61**, 7996 (2000); U. Lundin and R. H. McKenzie, Coherence of polaronic transport in layered metals, J. Phys. Condens. Matter **16**, 6695 (2004).
[41] S Fratini and S Ciuchi, Dynamical Mean-Field Theory of Transport of Small Polarons, Phys. Rev. Lett. **91**, 256403 (2003).
[42] A. Donos and J. P. Gauntlett, Holographic Q-lattices, J. High Energy Phys. 04 (2014) 040; Novel metals and insulators from holography, J. High Energy Phys. 06 (2014) 007.
[43] Y. Ling, C. Niu, J. P. Wu, Z. Y. Xian, and H. Zhang, Metal-Insulator Transition by Holographic Charge Density Waves, Phys. Rev. Lett. **113**, 091602 (2014).
[44] A. Donos, B. Gouteraux, and E. Kiritsis, Holographic metals and insulators with helical symmetry, J. High Energy Phys. 09 (2014) 038.
[45] B. Gouteraux, Charge transport in holography with momentum dissipation, J. High Energy Phys. 04 (2014) 181.
[46] A. Amoretti, A. Braggio, N. Maggiore, N. Magnoli, and D. Musso, Coexistence of two vector order parameters: A holographic model for ferromagnetic superconductivity, J. High Energy Phys. 01 (2014) 054; Thermo-electric transport in gauge/gravity models with momentum dissipation, J. High Energy Phys. 09 (2014) 160.
[47] Y. Liu, K. Schalm, Y.-W. Sun, and J. Zaanen, Lattice potentials and fermions in holographic non Fermi-liquids: Hybridizing local quantum criticality, J. High Energy Phys. 10 (2012) 036.
[48] T. T. M. Palstra, A. P. Ramirez, S-W. Cheong, B. R. Zegarski, P. Schiffer, and J. Zaanen, Transport mechanisms in doped $LaMnO_3$: Evidence for polaron formation, Phys. Rev. B **56**, 5104 (1997).
[49] N. Mannella, W. L. Yang, X. J. Zhou, H. Zheng, J. F. Mitchell, J. Zaanen, T. P. Devereaux, N. Nagaosa, Z. Hussain, and Z.-X. Shen, Nodal quasiparticle in pseudogapped colossal magnetoresistive manganites, Nature (London) **438**, 474 (2005).
[50] F. Massee *et al.*, Bilayer manganites reveal polarons in the midst of a metallic breakdown, Nat. Phys. **7**, 978 (2011).
[51] H. M. Rønnow, Ch. Renner, G. Aeppli, T. Kimura, and Y. Tokura, Polarons and confinement of electronic motion to two dimensions in a layered manganite, Nature (London) **440**, 1025 (2006).
[52] S. J. L. Billinge, R. G. DiFrancesco, G. H. Kwei, J. J. Neumeier, and J. D. Thompson, Direct Observation of Lattice Polaron Formation in the Local Structure of $La_{1-x}Ca_xMnO_3$, Phys. Rev. Lett. **77**, 715 (1996).
[53] M. Koschorreck, D. Pertot, E. Vogt, B. Fröhlich, M. Feld, and M. Köhl, Attractive and repulsive Fermi polarons in two dimensions, Nature (London) **485**, 619 (2012).
[54] C. Kohstall, M. Zaccanti, M. Jag, A. Trenkwalder, P. Massignan, G. M. Bruun, F. Schreck, and R. Grimm, Metastability and coherence of repulsive polarons in a strongly interacting Fermi mixture, Nature (London) **485**, 615 (2012).