Ultrahigh-Energy Debris from the Collisional Penrose Process

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Soon after the discovery of the Kerr metric, Penrose realized that superradiance can be exploited to extract energy from black holes. The original idea (involving the breakup of a single particle) yields only modest energy gains. A variant of the Penrose process consists of particle collisions in the ergoregion. The collisional Penrose process has been explored recently in the context of dark matter searches, with the conclusion that the ratio η between the energy of postcollision particles detected at infinity and the energy of the colliding particles should be modest ($\eta \lesssim 1.5$). Schnittman [Phys. Rev. Lett. 113, 261102 (2014)] has shown that these studies underestimated the maximum efficiency by about 1 order of magnitude (i.e., $\eta \lesssim 15$). In this work we show that particle collisions in the vicinity of rapidly rotating black holes can produce high-energy ejecta and result in high efficiencies under much more generic conditions. The astrophysical likelihood of these events deserves further scrutiny, but our study hints at the tantalizing possibility that the collisional Penrose process may power gamma rays and ultrahigh-energy cosmic rays.

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Introduction.—It is tempting to say that modern relativistic astrophysics was born in 1963, when Kerr discovered the famous solution of Einstein's equations describing rotating black holes [1] and Schmidt identified the first quasar [2]. The connection between these theoretical and observational breakthroughs became clear much later, but astronomers now agree that black holes are the engines behind many high-energy events in the Universe. Black holes are the most compact objects in nature, and therefore particles in their vicinities can attain relativistic velocities, producing observable phenomena such as jets and gamma-ray bursts. The mechanisms converting the gravitational energy stored by black holes into high-energy fluxes of matter and radiation are a major area of investigation in relativistic astrophysics, and particle collisions near black holes could have important implications for dark matter searches (see, e.g., Refs. [3,4]).

In 1969, Penrose discovered a remarkable way to extract energy from Kerr black holes [5]. In the so-called "ergoregion" surrounding rotating black holes particles can have negative energies ($\epsilon_i < 0$) as measured by an observer at infinity. Therefore, an object of energy ϵ_0 can fragment into two pieces, one of which escapes to infinity with energy ϵ_3, while the other is absorbed at the horizon with energy $\epsilon_4 < 0$ (the reason for the odd choice of subscripts will be clear soon). This results in $\epsilon_3 = \epsilon_0 - \epsilon_4 > \epsilon_0$, a net energy gain at the expense of the rotational energy of the hole. Unfortunately, this process yields a modest efficiency $\epsilon_3/\epsilon_0 \leq (1+\sqrt{2})/2 \simeq 1.207$, where the maximum is achieved for disintegration into two massless particles [6–9].

The original Penrose process relies on the somewhat unrealistic disintegration of a single particle. An astrophysically more promising variant of the idea is the collisional Penrose process [8]: two bodies with energy (ϵ_1, ϵ_2)—which could be elementary particles—collide resulting in two bodies with energy (ϵ_3, ϵ_4). This process has recently attracted interest in connection with dark matter searches as a result of work by Banados, Silk, and West [10], who found that particle collisions near rapidly rotating black holes could yield (in principle) unbound center-of-mass energies, perhaps producing observable exotic ejecta (see also Refs. [9,11,12] for caveats, and [13] for a review). A crucial issue is that the highest center-of-mass energies are achieved close to the horizon, and therefore the collision products experience large gravitational redshifts, resulting in modest efficiencies

$$\eta \equiv \frac{\epsilon_3}{\epsilon_1 + \epsilon_2} \tag{1}$$

for the escaping particle (that we will assume, without loss of generality, to be particle 3); $\eta \lesssim 1.5$, where the precise upper bound depends on the nature of the colliding particles [9,14,15].

Surprisingly, Schnittman [16] recently reported an order-of-magnitude increase in the efficiencies ($\eta \lesssim 15$) by allowing for a small kinematic change in the collision, illustrated schematically in Fig. 1. One of the colliding particles (say, particle 1) falls in the effective potential for radial motion around an extremal black hole, defined in Eq. (2) below and depicted by a dash-dotted (blue online)

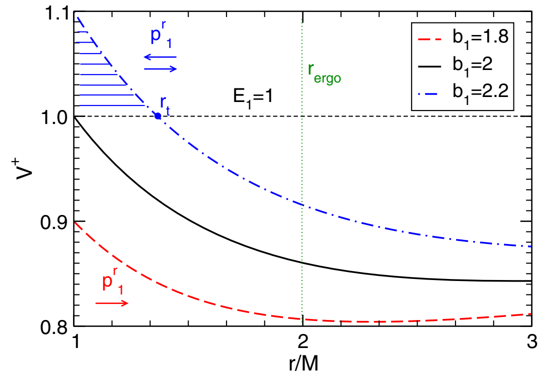

FIG. 1 (color online). Effective radial potential V^+ [cf. Eq. (2)] for an extremal black hole and a particle with energy $E_1 = 1$ and angular momentum $L_1 = b_1 E_1$. The horizon is located at $r = 1$, and the ergoregion corresponds to $r < r_{\rm ergo} = 2$. The dash-dotted (blue online) curve shows the potential felt by particle 1 for the process discussed in Ref. [16]; particle 1 cannot access the dashed (blue online) region with $r < r_t$. The dashed (red online) curve corresponds to the process considered in this Letter. The solid (black online) curve depicts the potential for the critical value of b_1 separating these two processes.

curve in Fig. 1. Particle 1 rebounds at the turning point $r = r_t$, so it has outgoing radial momentum ($p_1^r > 0$) when it collides (at some radius r_0 such that $r_t < r_0 < r_{\rm ergo}$) with incoming particle 2. This outgoing momentum favors ejection of high-energy particle 3 after the collision.

In this Letter we confirm the results of Ref. [16], and we reach the striking conclusion that arbitrarily large efficiencies can be achieved when we do not require both of the colliding particles to fall into the black hole from infinity. In our "super-Penrose" processes outgoing particle 1 has angular momentum below the critical value for escape (corresponding to the solid black line in Fig. 1) when it collides with ingoing particle 2. The radial motion of particle 1 is determined by the dashed (red online) effective potential: particle 1 is confined to the vicinity of the black hole, and it must have been created in the ergoregion via previous scattering events (see Ref. [17] for a similar proposal). The likelihood of such multiple scatterings is a delicate matter to be resolved by detailed cross-section calculations for specific processes, but we demonstrate that there is nothing preventing the creation of such particles in the ergosphere: our super-Penrose collisions are at least kinematically allowed. From now on we will use geometrical units ($G = c = 1$).

Setup.—Our setup is similar to that of Ref. [16]. We consider two bodies (or particles) 1 and 2 colliding with four-momenta p_1^μ and p_2^μ at the Boyer-Lindquist coordinate position $r = r_0$ in the equatorial plane ($\theta = \pi/2$) of a Kerr black hole. The final state consists of two bodies 3 and 4 with four-momenta p_3^μ and p_4^μ, also moving in the equatorial plane. In Boyer-Lindquist coordinates, the geodesic equations for equatorial particles with rest mass m, energy $\epsilon \equiv -g_{t\mu} p^\mu$, and angular momentum $\ell \equiv g_{\varphi\mu} p^\mu$ (as seen by an observer at infinity) are given by

$$\dot{r}^2 = \frac{r^3 + a^2(r+2M)}{r^3}(E - V^+)(E - V^-),$$
$$V^\pm = \frac{2aLM \pm \sqrt{r\Delta[L^2 r + (r^3 + a^2(r+2M))\delta]}}{r^3 + a^2(r+2M)},$$
$$\dot{\vartheta} = 0,$$
$$\dot{\varphi} = \frac{1}{\Delta}\left[\left(1 - \frac{2M}{r}\right)L + \frac{2aM}{r}E\right], \tag{2}$$

where the dot denotes differentiation with respect to the geodesic affine parameter λ and $\Delta = r^2 - 2Mr + a^2$. Here, $E \equiv \epsilon/m$ and $L \equiv \ell/m$ denote the energy and angular momentum per unit mass for massive particles, while for massless particles this distinction does not apply ($E = \epsilon$ and $L = \ell$). The constant $\delta = 1(0)$ for massive (massless) particles, respectively. For massless particles the four-momentum is $p^\mu = \dot{x}^\mu$; for massive particles we can choose $\lambda = \tau/m$ (τ being proper time), so that $p_\mu p^\mu = -m^2$. To enforce causality and guarantee that the locally measured energy is always positive, physical geodesics must satisfy $\dot{t} > 0$.

Local conservation of four-momentum implies

$$p_1^\mu + p_2^\mu = p_3^\mu + p_4^\mu. \tag{3}$$

Using Eqs. (2), this condition can be written as a set of three equations:

$$\epsilon_1 + \epsilon_2 = \epsilon_3 + \epsilon_4, \tag{4}$$

$$\ell_1 + \ell_2 = \ell_3 + \ell_4, \tag{5}$$

$$p_1^r + p_2^r = p_3^r + p_4^r. \tag{6}$$

The initial state is completely specified by fixing $E_{1,2}$ and $L_{1,2}$ (and $m_{1,2}$ for massive particles). For massive particles, we can specify the final state by providing a relation between m_3 and m_4: for example, for two identical collision products we can set $m_4 = m_3$. Finally, we provide the Lorentz boost E_4 and specific angular momentum L_4 and we are left with three unknowns (m_3, E_3, L_3) that can be determined by solving Eqs. (4)–(6). To be observationally interesting, the solutions of the system above must satisfy the requirement that particle 3 can escape and reach an observer at infinity (i.e., there cannot be any turning points r_3^t for particle 3 with $r_3^t > r_0$). To find the maximum efficiency $\eta_{\rm max}$, we simply repeat the previous procedure for a range of values of L_4. We find that the efficiency only depends on the impact parameters of the initial particles $b_{1,2} \equiv L_{1,2}/E_{1,2}$ and on the ratio of their energies $R \equiv \epsilon_1/\epsilon_2$. Therefore, all of our results will be shown as

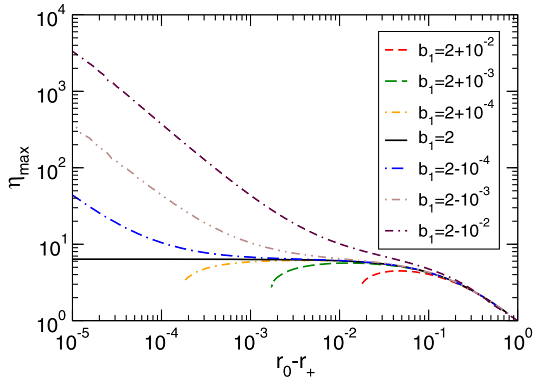

FIG. 2 (color online). Maximum efficiency $\eta_{\max}$ for the collision of equal-energy particles ($R = 1$) as a function of the radius at which the reaction occurs. We consider $p_1^r > 0$, $p_2^r < 0$, $b_2 = -2(1+\sqrt{2})$, and an extremal black hole ($a = 1$). The curves for $b_1 > 2$ terminate at the turning point of particle 1.

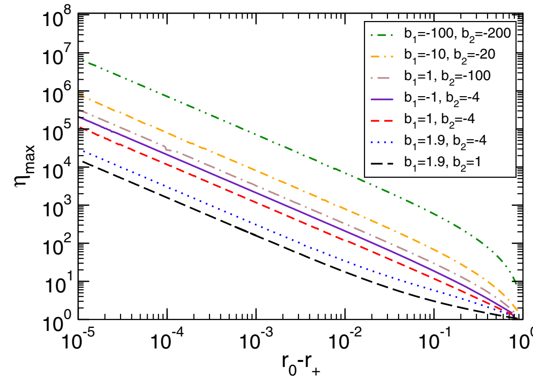

FIG. 3 (color online). Maximum efficiency $\eta_{\max}$ for the same process considered in Fig. 2 as a function of the radius at which the reaction occurs, for $a = 1$. When $b_2 < -2(1+\sqrt{2})$, particle 2 must also be produced close to the black hole.

functions of these quantities. For simplicity, we will also set the black-hole mass $M = 1$.

Superamplification collisions.—We focus on collisions for which particle 1 is outgoing (i.e., $p_1^r > 0$) while particle 2 is ingoing. Our results are summarized in Figs. 2–5. We start by using the same initial conditions as in Ref. [16]: an extremal black hole and $E_1 = E_2$. For $b_1 = 2$ we confirm that in this case the peak efficiency is $\eta_{\max} \sim 6.4$ (a value sensibly larger than the predictions of previous studies [13–15]) for collisions that occur close enough to the horizon. In Fig. 2 we show the maximum efficiency, varying b_1 close to the critical value $b_1 = 2$ and fixing the impact parameter of particle 2 to the value $b_2 = -2(1+\sqrt{2})$. We find that the maximum efficiency quickly decreases with increasing b_1, because particle 1 is now allowed to move in a smaller region inside the ergosphere (see Fig. 1). For subcritical particles ($b_1 < 2$) the scenario is completely different, and the efficiency becomes arbitrarily large as we approach the horizon. In our searches we found that these super-Penrose collisions occur for $b_2 < b_1 < 2$.

Figure 3 shows that the efficiencies of super-Penrose collisions can be orders of magnitude larger than those found in Ref. [16] in a large region of the parameter space of initial conditions. For very large $(|b_1|, |b_2|)$ and collisions close to $r = r_+$, the peak efficiency scales as $\eta_{\max} \sim 0.5\sqrt{(2-b_1)(2-b_2)}/(r_0 - r_+)$. Efficiencies $\eta \gtrsim 10^3$ are easily achieved for maximally spinning black holes.

Figure 4 shows the dependence of the superamplification efficiency on the black-hole spin a. For $a < 1$ the maximum gain quickly drops (bottom panel), and particle 3 can only escape for collisions occurring at $r_0 > r_c$, where r_c (Fig. 4, top panel) is the Boyer-Lindquist radius of the photon sphere in the equatorial plane. The maximum efficiency is attained when the collision occurs precisely at the photon sphere, where we find the scaling $\eta_{\max} \sim (1-a^2)^{-1/2}$. Nonetheless, very high efficiencies are still allowed for astrophysically relevant black holes, as long as particles with large negative impact parameters b_1 and b_2 are created inside the ergosphere and collide. For collisions where particle 2 falls from spatial infinity and black holes spinning at the Thorne limit $a \sim 0.998$ [18] we still find efficiencies as large as $\eta \sim 10^2$.

Finally, in Fig. 5 we consider the Compton-like scattering of a massless particle with $p_1^r > 0$ and a massive particle with $p_2^r < 0$ and we show the dependence of the maximum efficiency on the energy ratio $R \equiv \epsilon_1/\epsilon_2$ of the initial particles. For reactions occurring near the edge of the ergoregion the efficiency is largest when $R = 1$, while near the horizon ($r/r_+ - 1 \ll 1$) the maximum efficiency is achieved for $R \approx 10$. By comparing the solid (blue online) and dash-dash-dotted (green online) curves, we see that the maximum efficiency at the horizon occurs for $1 < R \lesssim 50$, and for $R \gtrsim 50$ the efficiency decreases again. In conclusion,

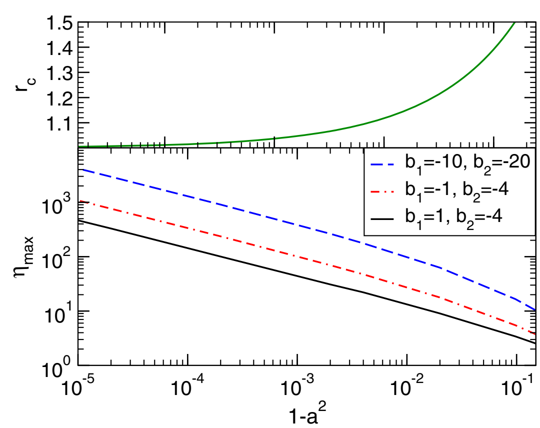

FIG. 4 (color online). Maximum efficiency as a function of the black hole spin a for the same process as in Fig. 2. The maximum efficiency is attained at the photon sphere r_c, below which infalling particle 3 can not escape. For $a \simeq 1$, the maximum efficiency scales as $1/\sqrt{1-a^2}$.

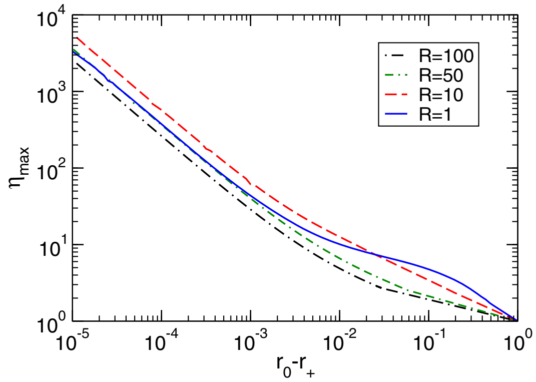

FIG. 5 (color online). Maximum gain for a Compton-like scattering between a massless particle with $p_1^r > 0$, $b_1 = 1.99$ and a massive particle with $p_2^r < 0$, $b_2 = -2(1 + \sqrt{2})$. We plot the maximum gain for different values of $R \equiv \epsilon_1/\epsilon_2$.

reactions such as the inverse Compton scattering between photons and massive particles could (at least in principle) produce highly energetic photons by multiple scattering processes.

The origin of the colliding particles.—Super-Penrose amplification cannot result from the collision of two particles coming from spatial infinity: as shown schematically in Fig. 1, particle 1 (with $p_1^r > 0$ and $b_1 < 2$ for $a = 1$) must be created inside the ergosphere by previous scattering events. Furthermore, for $b_{1,2} < -2(1 + \sqrt{2})$ there are turning points of the motion outside the ergoregion, and particles would be deflected back before reaching the ergoregion (cf. Fig. 1 of Ref. [16]).

The colliding particles giving rise to superamplification are physically relevant initial states, because they can be created (for example) by the collision of particles coming from infinity. In fact, there is a wide range of realistic initial conditions that can result in super-Penrose initial conditions. For example, a particle with $E_1 = 1, m_1 = 1$, and $L_1 = 1.9$ (dotted blue line in Fig. 3) in the extremal Kerr background can be generated by two particles with rest masses $(1, m_*)$ falling from rest at infinity with angular momenta $(L_1 = -4, L_2 = 2)$ and colliding at $r = 1.01$, as long as $m_* > 6$. The threshold mass ratio m_* depends on the angular momentum of the colliding particles and on the spin of the black hole: it is proportional to $|L_1|$, and it scales like $(r - 1)^{-2}$ for extremal black holes. Note in particular that we can have super-Penrose collisions for arbitrarily large negative angular momenta of particle 2. Because of frame dragging, the cross section for counterrotating incoming particles is much larger. Counterrotation may be a common feature in astrophysics, e.g., because of disk fragmentation, as in the "chaotic accretion" scenario [19,20]. Finally, we could (very conservatively) define a combined efficiency η' so that $\epsilon_{1,2}$ in Eq. (1) refers to the "parent" particles falling from infinity. With this redefinition, the maximum efficiency typically seems to lower to the levels predicted in Ref. [16] (see below).

In summary, the initial conditions giving rise to superamplification are kinematically allowed as the result of collisions of particles falling into the hole from large distances.

Multiple scattering.—A consequence of the argument presented above is that multiple scattering events can increase the energetic gain achievable with (and the astrophysical relevance of) the "traditional" collisional Penrose process [8–10] and of Schnittman's variant of the process [16]. This is because the energy of particles that cannot escape to infinity may be substantially larger than the energy of those that can. Even if "trapped" and unable to escape themselves, these particles may collide with other particles and give rise to high-energy collision products that may escape and be detectable.

Multiple energy-extracting collisions may lead to very large efficiencies [16]. Each Penrose scattering decreases the black-hole spin, but efficiencies can still be moderately high, even away from $a = 1$. These events may well be rare, but it is tempting to propose that they could play a role in the production of observable gamma rays or ultrahigh-energy cosmic rays. A detailed assessment of these possibilities is beyond the scope of this Letter.

Discussion.—Previous studies [9,14–16] (which we reproduced) focused on a region of the parameter space that excludes by construction the amplification mechanism studied in our work. The astrophysical likelihood of the super-Penrose amplification process proposed here is obviously a critical issue that requires further work. For very large efficiencies, the energy of the escaping particle can be as large as the black hole's, and our geodesic approximation clearly breaks down. In this regime back-reaction effects must be included. Furthermore, numerical simulations and detailed calculations of production rates (along the lines of Refs. [9,21,22]) are needed to make conclusive statements about the astrophysical relevance of these results.

We made the simplifying assumption that the reaction occurs in the equatorial plane. Very few off-equatorial calculations of the Penrose process have been performed in the past, mainly due to computational difficulties. The results reported here, together with recent studies of the off-equatorial collisional Penrose process [23,24], suggest that surprises may be in store, and a generalization of our calculations to the off-equatorial case is urgently needed.

Another important extension of our work would be the inclusion of external magnetic fields, a common feature of astrophysical black holes. The Penrose process for charged particles in the presence of electromagnetic fields is known to be more efficient than the original process [25], with efficiencies as large as $\eta \sim 10$ [26]. The possibility that electromagnetic fields could trigger and enhance super-Penrose collisions makes this an important line of research for the future.

We hope that this Letter will stimulate further work in these directions and improve our understanding of some of the most energetic events in the Universe.

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Note added.—Recently, we learned about work by Zaslavskii [27] that confirms our main findings.

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