Color Confinement and Screening in the θ Vacuum of QCD

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QCD perturbation theory ignores the compact nature of the $SU(3)$ gauge group that gives rise to the periodic θ vacuum of the theory. We propose to modify the gluon propagator to reconcile perturbation theory with the anomalous Ward identities for the topological current in the θ vacuum. As a result, the gluon couples to the Veneziano ghost describing the tunneling transitions between different Chern-Simons sectors of the vacuum; we call the emerging gluon dressed by ghost loops a "glost." We evaluate the glost propagator and find that it has the form $G(p) = (p^2 + \chi_{\mathrm{top}}/p^2)^{-1}$ where χ_{top} is the Yang-Mills topological susceptibility related to the η' mass by the Witten-Veneziano relation; this propagator describes the confinement of gluons at distances $\sim\chi_{\mathrm{top}}^{-1/4} \simeq 1$ fm. The same functional form of the propagator was originally proposed by Gribov as a solution to the gauge copies problem that plagues perturbation theory. The resulting running coupling coincides with the perturbative one at $p^2 \gg \sqrt{\chi_{\mathrm{top}}}$, but in the infrared region either freezes (in pure Yang-Mills theory) or vanishes (in full QCD with light quarks), in accord with experimental evidence. Our scenario makes explicit the connection between confinement and topology of the QCD vacuum; we discuss the implications for spin physics, high energy scattering, and the physics of quark-gluon plasma.

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QCD possesses a compact gauge group that allows for topologically nontrivial gauge field configurations. These configurations realize homotopy maps from the gauge group to the space-time manifold. For example, the homotopy map from the $SU(2)$ subgroup of the gauge group to the Euclidean space-time sphere S^3 describes the instanton solution [1]. However, the compactness of the gauge group is ignored in perturbation theory, and this may be at the origin of problems marring the perturbative approach.

In QCD, one of these problems is the existence of Gribov copies [2]—multiple solutions of the gauge-fixing condition that make the perturbative approach ambiguous. In the Coulomb gauge, the emergence of Gribov copies can be traced back to the existence of energy-degenerate vacua with different Chern-Simons numbers [3]. A natural question arises—is it possible to formulate QCD perturbation theory in a way that is consistent with the topological structure of the theory? In this Letter we argue that the answer to this question is positive. We find that the resulting gluon propagator naturally describes confinement, i.e., nonpropagation of color degrees of freedom, and the running coupling displays the screening of color charge at large distances.

In Minkowski space-time, instanton solutions represent the tunneling events connecting the degenerate vacuum states with different Chern-Simons numbers

$$X(t) = \int d^3x K_0(x,t), \tag{1}$$

where K_0 is the temporal component of topological current

$$K_\mu = \frac{g^2}{16\pi^2}\epsilon_{\mu\nu\rho\sigma}A^{\nu,a}\left(\partial^\rho A^{\sigma,a} + \frac{1}{3}gC^{abc}A_b^\rho A_c^\sigma\right); \tag{2}$$

the first term in K_0 is the density of Abelian "magnetic helicity" while the second term is its non-Abelian generalization.

The chiral anomaly in QCD leads to nonconservation of the axial current

$$\partial_\mu J_A^\mu = 2N_f Q(x) + \sum_f (2im_f)\bar{q}_f\gamma_5 q_f, \tag{3}$$

where m_f are the masses of quarks, N_f is the number of flavors, and

$$Q(x) = \frac{g^2}{32\pi^2}F_{\mu\nu}(x)\tilde{F}^{\mu\nu}(x) \tag{4}$$

is the density of topological charge normalized by $\int d^4xQ(x) = \nu$; for finite action field configurations ν is an integer. The density of topological charge can be represented as a divergence $Q(x) = \partial^\mu K_\mu$ of the gauge-dependent current (2).

Veneziano [4] has demonstrated that the periodic θ-vacuum structure in QCD can be captured by introducing a massless "ghost" in the correlation function of the gauge-dependent topological current (2):

$$K_{\mu\nu}(q) \equiv i\int d^4xe^{iqx}\langle K_\mu(x)K_\nu(0)\rangle \xrightarrow{q^2\ll\mu^2} -\frac{\mu^4}{q^2}g_{\mu\nu}, \tag{5}$$

where $\mu^4 \equiv \chi_{\mathrm{top}}$ is the topological susceptibility of pure Yang-Mills theory. Note that the rhs of Eq. ([4]) has the "wrong" sign; i.e., the ghost does not describe a propagating degree of freedom. This means that the ghost cannot be produced in a physical process; however, the couplings of the ghost (that describe the effect of topological fluctuations) certainly can affect physical amplitudes. A similar "dipole" ghost had been earlier introduced by Kogut and Susskind [5] in the analysis of axial anomaly in the Schwinger model. This procedure has been demonstrated to solve the $U_A(1)$ problem in QCD [6,7].

The physical meaning of Eq. ([4]) becomes apparent if one compares it to the correlation function of the electron's coordinate $x(t)$ in a crystal [8]:

$$i\int dte^{i\omega t}\langle T\{x(t)x(0)\}\rangle \xrightarrow{\omega\to 0} -\frac{1}{\omega^2 m^*} = -\frac{1}{\omega^2}\frac{\partial^2 E(k)}{\partial k^2}\bigg|_{k=0}, \tag{6}$$

where $E = k^2/2m^*$ is the energy of an electron with an effective mass m^* and quasimomentum k in a crystal. The emergence of the pole in Eq. (6) signals the possibility of electron's propagation in the periodic potential of the crystal due to tunneling. Note that the pole emerges not just from a single tunneling event (corresponding to the instanton in QCD), but sums up the effect of many tunnelings throughout the crystalline lattice.

The analogy between Eqs. (5) and (6) can be made even more apparent if we choose the frame with $q^\mu = (\omega, 0)$ and use the analog of coordinate given by Eq. (1) that is invariant with respect to "small" gauge transformations but changes by an integer under "large" gauge transformations (i.e., the transformations that cannot be smoothly deformed to identity). The expression Eq. (5) then takes the form completely analogous to Eq. (6):

$$i\int dte^{i\omega t}\langle T\{X(t)X(0)\}\rangle \xrightarrow{\omega\to 0} -\frac{\mu^4}{\omega^2}V = -\frac{1}{\omega^2}\frac{\partial^2 E(\theta)}{\partial \theta^2}\bigg|_{\theta=0}, \tag{7}$$

where V is the volume of the system, and $E(\theta) = \epsilon(\theta)V$ is the energy of the Yang-Mills vacuum. The energy density of the vacuum $\epsilon(\theta)$ is a periodic function of the θ angle that is analogous to the quasimomentum k in Eq. (6). At small θ, we can expand $\epsilon(\theta)$ and write

$$\epsilon(\theta) = \mu^4\frac{\theta^2}{2}, \tag{8}$$

which exhibits the physical meaning of μ^4 as of the topological susceptibility $\chi_{\rm top} = \mu^4$ of the Yang-Mills theory; note that a term linear in θ is forbidden by P and CP invariances of QCD.

It is well known that topological susceptibility vanishes, order by order, in perturbation theory. Perturbative description thus corresponds to $\mu \to 0$ in Eq. (5), or to the limit of the infinitely heavy electron, $m^* \to \infty$ in Eq. (6). Infinitely heavy electrons do not respond to electromagnetic fields, as the corresponding coupling of electromagnetic current $j = eq/m^*$ to the gauge field $jA \sim 1/m^*$ vanishes in the limit $m^* \to \infty$. In this case the dynamics of photons is not sensitive to the periodic structure of the crystal, and one can build the usual perturbation theory of photons. However, when m^* is finite, and is of the order of the frequency of the external gauge field, photons can be absorbed and reemitted by the electrons, and these processes severely affect the photon propagator. Also, at finite m^*, the static Coulomb field can be screened by the electrons at large distances.

As we will discuss below, the situation in QCD is very similar—as $\mu \to 0$, the periodic structure of the θ vacuum becomes irrelevant. However, in the physical world $\mu \sim \Lambda_{\rm QCD} \sim 200$ MeV, so the ghost (describing the tunneling in the periodic θ vacuum) strongly affects propagation of gluons with frequencies $\omega \sim \mu$. At large distances, the ghost also gives rise to the screening of color charge, leading to the freezing of the effective coupling in the infrared IR limit for pure gauge theory, or to the vanishing of the coupling in the IR in QCD with light quarks. Because $\mu \sim \Lambda_{\rm QCD} \sim 1/R_{\rm conf}$ is on the order of inverse confinement radius $R_{\rm conf}$, these phenomena describe confinement of gluons at distances $R_{\rm conf} \simeq 1$ fm.

The key observation of our Letter is that Eqs. (2) and (5) define an effective ghost-gluon-gluon vertex $\Gamma_\mu(q,p)$. Using this vertex, we can rewrite the correlator Eq. (5) at small q^2 as follows [see Fig. 1(a)]:

$$K_{\mu\nu}(q) = \frac{1}{(2\pi)^4 i}\int d^4p\Gamma_\mu(q,p)\frac{1}{p^2(q-p)^2}$$
$$\Gamma_\nu(q,p) = -\frac{\mu^4}{q^2}g_{\mu\nu}. \tag{9}$$

From Eq. (9) we find that

$$\Gamma_\mu(q,p)\Gamma_\nu(q,p) \propto -\frac{\mu^4}{p^2}g_{\mu\nu}, \quad \text{for } q \le p. \tag{10}$$

The vertices $\Gamma_\mu(q,p)$ describe the excitation of the ghost by gluons, and affect the gluon propagation at small virtualities.

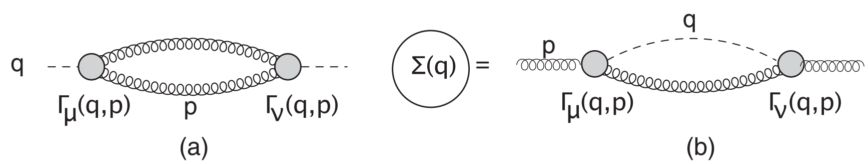

FIG. 1. (a) Equation (5) in momentum representation; helix lines represent gluons and the dashed line depicts the ghost. (b) The gluon dressed by the interactions with the ghost: a glost, see Eq. (11) for the corresponding self-energy expression.

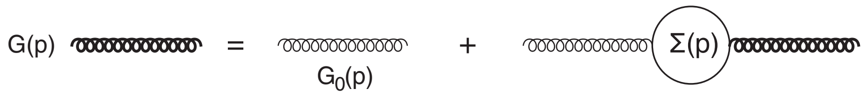

FIG. 2. The graphic form of the equation for the gluon propagator, Eq. (13). The full propagator is denoted by the bold helix line; thin helix stands for the perturbative gluon propagator $G_0(p)$, and the blob for self-energy $\Sigma(p)$ given by Eqs. (11) and (12).

Indeed, the gluon propagator is now the solution of the equation shown in Fig. 2 with $\Sigma(p)$ given by

$$\Sigma_{\mu\nu}(p) = \frac{1}{(2\pi)^4 i}\int d^4q\Gamma_\mu(q,p)\frac{1}{q^2(q-p)^2}\Gamma_\nu(q,p), \quad (11)$$

where $1/q^2$ is the propagator of the ghost. In evaluating the integral of Eq. (11) we assume that $q \ll p$ since Γ_μ's describe the nonperturbative effects concentrated at small momenta (long distances). Therefore,

$$\Sigma_{\mu\nu}(p) \simeq \frac{4\pi}{(2\pi)^4}\int_0^p dq^2 q^2 \Gamma_\mu(q,p)\frac{1}{q^2p^2}$$

$$\Gamma_\nu(q,p) = -g_{\mu\nu}\frac{\mu^4}{p^4}\int^{p^2} dq^2 = -g_{\mu\nu}\frac{\mu^4}{p^2}, \quad (12)$$

where we used Eq. (10); note that in Eq. (12) we made rotation to the pseudo-Euclidean space. We can now write down the Schwinger-Dyson equation for the gluon propagator [9] $G_{\mu\nu}(p) = g_{\mu\nu}G(p)$ in terms of $\Sigma_{\mu\nu}(p) = g_{\mu\nu}\Sigma(p)$, see Fig. 2:

$$G(p) = \frac{1}{p^2} + \frac{1}{p^2}\Sigma(p)G(p), \quad (13)$$

with the solution

$$G(p) = \frac{1}{p^2 - \Sigma(p)} = \frac{1}{p^2 + \frac{\mu^4}{p^2}}. \quad (14)$$

The propagator (14) has remarkable properties. First, $G(p)$ has no infrared singularities and no gluon pole in the physical region. Indeed, this propagator has only complex poles at $p^2 = \pm i\mu^2$. As a result, gluons cannot be observed as particles in detectors—in other words, they are confined. Second, the propagator of the type of Eq. (14) was proposed by Gribov [2] as a solution to the problem of gauge copies—multiple solutions to the gauge fixing condition, see [10,11] for reviews. Hence, we can state that introducing the coupling to the ghost (and thus taking account of the periodicity of the θ vacuum) solves the problem of Gribov copies and leads to the confinement of gluons. The dimensionful Gribov parameter acquires a well-defined meaning of topological susceptibility $\chi_{\rm top} = \mu^4$ related to the η' mass by the Witten-Veneziano relation [4,7]; since $\mu \simeq \Lambda_{\rm QCD}$, confinement emerges at distances of about 1 fm. Note that close to the deconfinement transition, the topological susceptibility vanishes reflecting the restoration of $U_A(1)$ symmetry [12,13], see [14] for a review. Since at $\mu \to 0$ the gluon propagator becomes perturbative, the restoration of $U_A(1)$ symmetry and deconfinement should occur at the same temperature as suggested by the lattice data [12]; however, close to T_c the nonperturbative interactions induced by $\mu \neq 0$ are important.

In our approach, the propagator Eq. (14) results from the admixture of the ghost to the perturbative gluon [see Fig. 1(b)], with an amplitude defined by the topological susceptibility μ^4. We thus propose the following name for the particle with propagator given by Eq. (14) that represents a coherent mixture of a gluon and a ghost—a glost. Unlike the ghost, the glost can be produced in a physical process, but unlike the perturbative gluon, it is confined and can propagate only at short distances $\sim\mu^{-1} \sim 1$ fm.

Let us now reconsider the asymptotic freedom of QCD [15,16] using the glost propagator Eq. (14). The gluon propagator $G_{\mu\nu}(p) = g_{\mu\nu}G(p)$ was introduced above in Feynman gauge, in accord with the prescription (9) for the correlation function of topological current. However, since the Lorentz structure of the glost propagator is identical to that of the perturbative gluon, we can use any gauge that is convenient for a specific computation replacing $G_0(p) = 1/p^2$ by (14). In our derivation we will compute the interaction energy of two heavy quarks in the Coulomb gauge [17,18] that is free from the Faddeev-Popov ghosts, so we can avoid dealing with two different types of ghosts. The dominant contribution responsible for the asymptotic freedom stems from the diagram of Fig. 3(a). The contribution of this diagram in perturbative QCD takes the form

$$\Pi^{(a)} = 3g^2C_2^2\int\frac{d^4k'}{(2\pi)^4 i}\frac{1}{(\boldsymbol{k}-\boldsymbol{k}')^2(k_0'^2-\boldsymbol{k}'^2)}\left(1-\frac{(\boldsymbol{k}\cdot\boldsymbol{k}')^2}{\boldsymbol{k}^2\boldsymbol{k}'^2}\right); \quad (15)$$

Π is related to Σ by $\Sigma = k^2\Pi$.

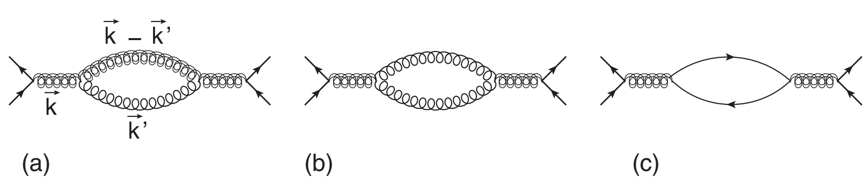

FIG. 3. The first order corrections to the Coulomb energy: (a) the Coulomb gluon dressed by the transverse gluon; (b) the transverse gluon loop; (c) the quark loop. Helix lines denote the transversely polarized gluon, the double helix lines show the longitudinally polarized gluon, and the solid arrow lines depict the quarks.

For the glost propagator (14), Eq. (15) changes and takes a different form:

$$\Pi^{(a)} = 3g^2C_2^2\int\frac{d^4k'}{(2\pi)^4 i}\frac{(\boldsymbol{k}-\boldsymbol{k}')^2}{[(\boldsymbol{k}-\boldsymbol{k}')^2]^2+\mu^4}\frac{(k_0'^2-\boldsymbol{k}'^2)}{[(k_0'^2-\boldsymbol{k}'^2)]^2+\mu^4}\left[1-\frac{(\boldsymbol{k}\cdot\boldsymbol{k}')^2}{\boldsymbol{k}^2\boldsymbol{k}'^2}\right]$$
$$= 3g^2C_2^2\int\frac{d^4k'}{(2\pi)^4 i}\mathrm{Re}\left\{\frac{1}{(\boldsymbol{k}-\boldsymbol{k}')^2+i\mu^2}\right\}\mathrm{Re}\left\{\frac{1}{k_0'^2-\boldsymbol{k}'^2+i\mu^2}\right\}\frac{1}{\boldsymbol{k}^2\boldsymbol{k}'^2}[\boldsymbol{k}^2\boldsymbol{k}'^2-(\boldsymbol{k}\cdot\boldsymbol{k}')^2]. \quad (16)$$

In Eq. (16) we have four terms obtained by choosing different signs of $i\mu^2$. To illustrate the procedure of calculation, let us evaluate one of them:

$$\Pi_1^{(a)} = 3g^2C_2^2\int\frac{d^4k'}{(2\pi)^4 i}\frac{\boldsymbol{k}^2\boldsymbol{k}'^2-(\boldsymbol{k}\cdot\boldsymbol{k}')^2}{[(\boldsymbol{k}-\boldsymbol{k}')^2+i\mu^2](k_0'^2-\boldsymbol{k}'^2+i\mu^2)\boldsymbol{k}^2\boldsymbol{k}'^2}. \quad (17)$$

Introducing Feynman parameters $\alpha_1+\alpha_2+\alpha_3=1$ we obtain ($\boldsymbol{P}=\boldsymbol{k}'-\alpha_1\boldsymbol{k}$ and $\alpha_S=g^2/4\pi$):

$$\Sigma_1^{(a)} = 3g^2C_2^2\frac{1}{\boldsymbol{k}^2}\int_0^1 d\alpha_1\int_0^{1-\alpha_1}d\alpha_2\int\frac{d^4k'}{(2\pi)^4 i}\frac{\boldsymbol{k}^2\boldsymbol{P}^2-(\boldsymbol{k}\cdot\boldsymbol{P})^2}{[\alpha_2k_0^2-\boldsymbol{P}^2-k^2\alpha_1(1-\alpha_1)+i\mu^2(\alpha_1+\alpha_2)]^3}$$
$$\xrightarrow{\text{integrating over } k_0'} g^2C_2^2\int_0^1\frac{d\alpha_2}{\sqrt{\alpha_2}}\int_0^{1-\alpha_2}d\alpha_1\int\frac{d^3\boldsymbol{P}}{(2\pi)^3}\frac{3}{16}\frac{\boldsymbol{P}^2}{[\boldsymbol{P}^2+k^2*\alpha_1(1-\alpha_1)+i\mu^2(\alpha_1+\alpha_2)]^{5/2}}$$
$$\xrightarrow{\text{integrating over } \boldsymbol{P}} \frac{3\alpha_S}{8\pi}C_2^2\int_0^1\frac{d\alpha_2}{\sqrt{\alpha_2}}\int_0^{1-\alpha_2}d\alpha_1\left\{-\frac{4}{3}+\ln\left(\frac{2L}{\sqrt{k^2\alpha_1(1-\alpha_1)+i\mu^2(\alpha_1+\alpha_2)}}\right)\right\}, \quad (18)$$

where L is an ultraviolet cutoff in the integration over momentum. The integrals over α_1 and α_2 can be taken analytically. The main features of Eq. (18) are the following: it has a logarithmic divergence at large k and is finite at $k=0$.

Summing all terms we find that we need to replace $\ln(L^2/k^2)$ of perturbative QCD in the diagram of Fig. 3(a) by the following function:

$$\ln(L^2/k^2)\longrightarrow\ln(L,k,\mu)$$
$$\equiv\frac{3}{8}\int_0^1\frac{d\alpha_1}{\sqrt{\alpha_1}}\int_0^{1-\alpha_1}d\alpha_2$$
$$\times\left\{-\frac{16}{3}+\ln\left(\frac{4L^2}{\sqrt{k^4\alpha_2^2(1-\alpha_2)^2+\mu^4(\alpha_1+\alpha_2)^2}}\right)\right.$$
$$\left.+\ln\left(\frac{4L^2}{\sqrt{k^4\alpha_2^2(1-\alpha_2)^2+\mu^4(\alpha_1-\alpha_2)^2}}\right)\right\}. \quad (19)$$

It turns out that the same substitution has to be done in the expression for Fig. 3(b) that in the perturbative approach gives a positive contribution to the β function. However, the contribution of the quark loop [Fig. 3(c)] remains the same as in perturbative QCD. The sign of this contribution to the β function is also positive, and it leads to Landau pole and a “Moscow zero” [19]. As a result, the QCD coupling in our approach tends to zero in the infrared region of $k\to 0$.

If we choose the renormalization point $k=\mu$, the running coupling takes the form

$$\alpha_S(k^2)$$
$$=\frac{\alpha_S(\mu)}{1+\alpha_S(\mu)\{\frac{11N_c}{12\pi}[\ln(L,k,\mu)-\ln(L,\mu,\mu)]-\frac{2N_f}{12\pi}\ln(k^2/\mu^2)\}}. \quad (20)$$

In Fig. 4 we plot the coupling α_S as a function of k for two cases: our model for QCD and pure gluodynamics [Fig. 4(a)], and the comparison of α_S in gluodynamics with perturbative QCD calculations in the leading order [Fig. 4(b)]. In both cases we choose the renormalization mass to be equal to the mass of the Z boson; we use the value of $\mu=0.18$ GeV from the original paper [4] where it has been determined from the mass of η' meson, and $N_c=N_f=3$.

One can see that replacing the gluon propagator by the propagator of the glost in gluodynamics removes the Landau pole and leads to the finite value of α_S at $k=0$. On the other hand, with the inclusion of quarks, the strong coupling vanishes at $k=0$. At short distances, the running coupling is dominated by the perturbative contribution and so is not modified. The ghost affects the running coupling in a way that is quite different from the effect of a single instanton, which has been shown to increase the effective coupling at distances on the order of the instanton size [20,21]. This may not be surprising as the ghost describes the effect of many instanton transitions throughout the θ vacuum. The screening effect of the ghost admixture is clearly a consequence of the fact that it is a spin-zero pseudoscalar “particle”.

It has been argued by Dokshitzer [22] that the experimental data indicate that in the IR region the QCD coupling remains effectively small:

$$\alpha_0=\frac{1}{\mu_I}\int^{\mu_I}dk\alpha_S(k)\approx 0.5 \quad \text{for } \mu_I=2\text{ GeV}. \quad (21)$$

In our approach we get $\alpha_0=0.59$ for renormalization point $k=M_Z$, in reasonable agreement with Eq. (21).

It is of fundamental interest to establish the microscopic dynamics responsible for the long-range correlations of topological current captured by the ghost. A recent study within the “deformed QCD" attributes these correlations to the topological order in the vacuum [23]. Our result

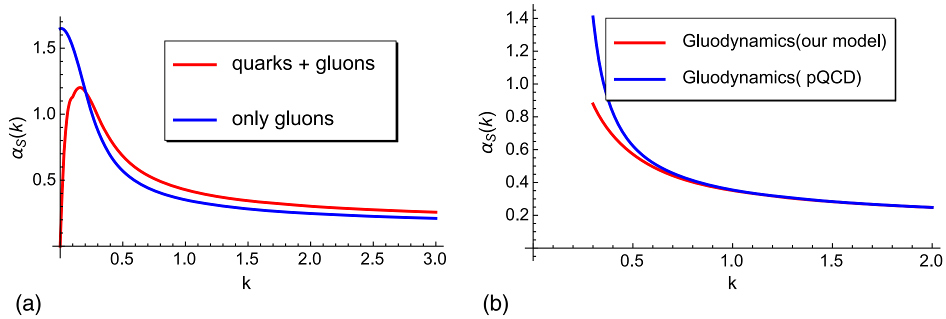

FIG. 4 (color online). The running constant α_S as a function of momentum k. Figure 4(a) shows the result for α_S in our model for QCD with light quarks (blue line that goes to zero at $k \to 0$) and gluodynamics (red line). Figure 4(b) shows the comparison of momentum dependence of α_S in our model for gluodynamics (blue line) and perturbative QCD (red line that is above the blue one at small k). The renormalization point is chosen at the mass of Z boson, $k = M_Z$.

suggests a link between the confinement and the long-range topological correlations in the QCD vacuum, and provides a practical way of computing power-suppressed corrections to QCD amplitudes, in particular the ones that are forbidden in the perturbative approach.

To summarize, we propose to modify the gluon propagator in perturbative QCD by taking account of the periodic structure of the QCD θ vacuum. Our prescription for the gluon propagator leads to the coupling of the gluons to the ghost saturating the anomalous Ward identity for topological current. The resulting glost propagator appears to have the functional form originally proposed by Gribov, in which the role of dimensionful parameter is played by the topological susceptibility $\chi_{\rm top} \equiv \mu^4$. Our approach thus removes the Gribov copies that usually plague perturbation theory, and describes confinement of gluons at distances $\sim\mu^{-1} \simeq 1$ fm. We also find that the running coupling in the IR freezes in pure gauge theory, or tends to zero in QCD with light quarks. Because the topological susceptibility vanishes above the deconfinement transition, the glosts become usual perturbative gluons in the deconfined phase at high temperatures. The glost propagator leads to the exponential falloff of the high-energy hadron scattering amplitude at large impact parameters needed to satisfy the Froissart bound; this can solve the long-standing problem of the perturbative approach in describing high energy scattering [24]. In QCD amplitudes the coupling to the ghost can give rise to spin asymmetries [25] that are different from the usual perturbative approach—it will be interesting to study the resulting implications for spin physics at colliders.

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[1] A. A. Belavin, A. M. Polyakov, A. S. Schwartz, and Y. S. Tyupkin, Phys. Lett. **59B**, 85 (1975).
[2] V. N. Gribov, Nucl. Phys. **B139**, 1 (1978).
[3] R. Jackiw, I. Muzinich, and C. Rebbi, Phys. Rev. D **17**, 1576 (1978).
[4] G. Veneziano, Nucl. Phys. **B159**, 213 (1979).
[5] J. B. Kogut and L. Susskind, Phys. Rev. D **11**, 3594 (1975).
[6] S. Weinberg, Phys. Rev. D **11**, 3583 (1975).
[7] E. Witten, Nucl. Phys. **B156**, 269 (1979).
[8] D. I. Dyakonov and M. I. Eides, Zh. Eksp. Teor. Fiz. **81**, 434 (1981).
[9] This is the expression in the Feynman gauge; since the self-energy $\Sigma_{\mu\nu}$ does not change the polarization of the gluon, the Dyson-Schwinger series can be resummed in the usual way.
[10] N. Vandersickel and D. Zwanziger, Phys. Rep. **520**, 175 (2012), and references therein.
[11] Y. L. Dokshitzer and D. E. Kharzeev, Annu. Rev. Nucl. Part. Sci. **54**, 487 (2004).
[12] C. Bonati, M. DElia, H. Panagopoulos, and E. Vicari, Phys. Rev. Lett. **110**, 252003 (2013).
[13] D. Kharzeev, R. D. Pisarski, and M. H. G. Tytgat, Phys. Rev. Lett. **81**, 512 (1998).
[14] D. E. Kharzeev, arXiv:1501.01336.
[15] D. J. Gross and F. Wilczek, Phys. Rev. Lett. **30**, 1343 (1973).
[16] H. D. Politzer, Phys. Rev. Lett. **30**, 1346 (1973).
[17] I. B. Khriplovich, Yad. Fiz. **10**, 409 (1969) [Sov. J. Nucl. Phys. **10**, 235 (1969)].
[18] V. N. Gribov, in *The Gribov Theory of Quark Confinement*, edited by J. Nyiri (World Scientific, Singapore, 2001), pp. 24–38.
[19] L. D. Landau, A. A. Abrikosov, and I. M. Khalatnikov, Dokl. Akad. Nauk SSSR **95**, 497 (1954); **95**, 773 (1954); **95**, 1177 (1954).
[20] C. G. Callan, Jr., R. F. Dashen, and D. J. Gross, Phys. Rev. D **17**, 2717 (1978).
[21] L. Randall, R. Rattazzi, and E. V. Shuryak, Phys. Rev. D **59**, 035005 (1999).
[22] Y. L. Dokshitzer, in *Proceedings of the 29th International Conference on High-Energy Physics, ICHEP'98, Vancouver, 1998* (World Scientific, Singapore, 1999), pp. 305–324; arXiv:hep-ph/9812252.
[23] A. R. Zhitnitsky, Ann. Phys. (Amsterdam) **336**, 462 (2013).
[24] A. Kovner and U. A. Wiedemann, Phys. Rev. D **66**, 051502 (2002); **66**, 034031 (2002); Phys. Lett. B **551**, 311 (2003).
[25] Z. B. Kang and D. E. Kharzeev, Phys. Rev. Lett. **106**, 042001 (2011).