New Class of Consistent Scalar-Tensor Theories

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We introduce a new class of scalar-tensor theories of gravity that extend Horndeski, or "generalized Galileon," models. Despite possessing equations of motion of higher order in derivatives, we show that the true propagating degrees of freedom obey well-behaved second-order equations and are thus free from Ostrogradski instabilities, in contrast to standard lore. Remarkably, the covariant versions of the original Galileon Lagrangians—obtained by direct replacement of derivatives with covariant derivatives—belong to this class of theories. These extensions of Horndeski theories exhibit an uncommon, interesting phenomenology: The scalar degree of freedom affects the speed of sound of matter, even when the latter is minimally coupled to gravity.

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The discovery of the present cosmological acceleration has spurred the exploration of gravitational theories that could account for this effect. Many extensions of general relativity (GR) are based on the inclusion of a scalar degree of freedom (DOF) in addition to the two tensor propagating modes of GR (see, e.g., [1] for a review). In this context, a recent important proposal is the so-called Galileon models [2], with Lagrangians that involve second-order derivatives of the scalar field and lead, nevertheless, to equations of motions of second order. Such a property guarantees the avoidance of Ostrogradski instabilities, i.e., of the ghostlike DOF that are usually associated with higher time derivatives (see, e.g., [3]).

Initially introduced in Minkowski spacetime, Galileons have then been generalized to curved spacetimes [4–6], where they turn out to be equivalent to a class of theories originally constructed by Horndeski 40 years ago [7]. Today, Horndeski theories, which include quintessence, k-essence, and $f(R)$ models, constitute the main theoretical framework for scalar-tensor theories, in which cosmological observations are interpreted. The purpose of this Letter is to show that this framework is not as exhaustive as generally believed and can in fact be extended to include new Lagrangians. Indeed, having equations of motion of second order in derivatives—while indeed sufficient—is not necessary to avoid Ostrogradski instabilities, as already pointed out in, e.g., Refs. [8,9]. The theories beyond Horndeski that we propose lead to distinct observational effects and are thus fully relevant for an extensive comparison of scalar-tensor theories with observations.

The model.—The theories that we consider here can be viewed as a broader generalization of the Galileons to curved spacetimes. They are described by linear combinations of the Lagrangians

$$L_2^\phi \equiv G_2(\phi, X), \tag{1}$$

$$L_3^\phi \equiv G_3(\phi, X)\Box\phi, \tag{2}$$

$$\begin{aligned} L_4^\phi \equiv\ & G_4(\phi, X)\,{}^{(4)}R - 2G_{4,X}(\phi, X)(\Box\phi^2 - \phi^{\mu\nu}\phi_{\mu\nu}) \\ & + F_4(\phi, X)\epsilon^{\mu\nu\rho}{}_\sigma \epsilon^{\mu'\nu'\rho'\sigma}\phi_\mu\phi_{\mu'}\phi_{\nu\nu'}\phi_{\rho\rho'}, \end{aligned} \tag{3}$$

$$\begin{aligned} L_5^\phi \equiv\ & G_5(\phi, X)\,{}^{(4)}G_{\mu\nu}\phi^{\mu\nu} \\ & + \frac{1}{3}G_{5,X}(\phi, X)(\Box\phi^3 - 3\Box\phi\phi_{\mu\nu}\phi^{\mu\nu} + 2\phi_{\mu\nu}\phi^{\mu\sigma}\phi^\nu{}_\sigma) \\ & + F_5(\phi, X)\epsilon^{\mu\nu\rho\sigma}\epsilon^{\mu'\nu'\rho'\sigma'}\phi_\mu\phi_{\mu'}\phi_{\nu\nu'}\phi_{\rho\rho'}\phi_{\sigma\sigma'}, \end{aligned} \tag{4}$$

which depend on a scalar field ϕ (and its derivatives $\phi_\mu \equiv \nabla_\mu\phi$, $\phi_{\mu\nu} \equiv \nabla_\nu\nabla_\mu\phi$), on $X \equiv g^{\mu\nu}\phi_\mu\phi_\nu$, and on a metric $g_{\mu\nu}$ with respect to which matter is assumed to be minimally coupled; $\epsilon_{\mu\nu\rho\sigma}$ is the totally antisymmetric Levi-Civita tensor, and a comma denotes a partial derivative with respect to the argument. Horndeski theories correspond to a subset of the above theories, subjected to the restricting conditions

$$F_4(\phi, X) = 0, \qquad F_5(\phi, X) = 0, \tag{5}$$

which ensure that the equations of motion (EOM) are second order. By contrast, we allow here arbitrary functions F_4 and F_5, which means that our theories contain two additional free functions with respect to the Horndeski ones.

The new terms proportional to F_4 and F_5 are, respectively, the covariant version of the original quartic and quintic Galileon Lagrangians proposed in Ref. [2]. This guarantees second-order dynamics for the scalar field in the absence of gravity. When the metric is dynamical, the EOM involve up to third-order derivatives in these extended theories, but this does not imply the presence of unwanted extra DOF, as we show below.

Arnowitt-Deser-Misner (ADM) formulation.—In cosmology, where the scalar field gradient is timelike, it is convenient to perform an ADM decomposition of spacetime, with metric

$$ds^2 = -N^2dt^2 + h_{ij}(dx^i + N^i dt)(dx^j + N^j dt), \quad (6)$$

by choosing the uniform scalar field ($\phi = \text{const}$) hypersurfaces as constant-time hypersurfaces. The above Lagrangians then have a very simple form in terms of the intrinsic and extrinsic 3D curvature tensors of the spatial slices, R_{ij} and K_{ij}, as well as the lapse function N. This reformulation uses the unit vector $n^\mu \equiv -\phi^\mu/\sqrt{-X}$ normal to the uniform ϕ hypersurfaces, in terms of which the extrinsic curvature is given by $K_{\mu\nu} \equiv (g^\sigma{}_\mu + n^\sigma n_\mu)\nabla_\sigma n_\nu$. We also make use of the Gauss-Codazzi relations to relate the 4D curvature to the 3D one.

After cumbersome but straightforward manipulations, one finds that any combination of the L_a^ϕ leads to an ADM Lagrangian density of the form $\mathcal{L} = \sqrt{-g}\sum_a L_a$, with

$$\begin{aligned} L_2 &= A_2, \\ L_3 &= A_3 K, \\ L_4 &= A_4\mathcal{K}_2 + B_4 R, \\ L_5 &= A_5\mathcal{K}_3 + B_5 K^{ij}[R_{ij} - h_{ij}R/2], \end{aligned} \quad (7)$$

where $K \equiv h^{ij}K_{ij}$, $R \equiv h^{ij}R_{ij}$, and the quantities $\mathcal{K}_2$ and $\mathcal{K}_3$ are, respectively, quadratic and cubic combinations of $K_{ij} \equiv (\dot{h}_{ij} - D_i N_j - D_j N_i)/(2N)$ (where D_i is the covariant derivative of h_{ij}), explicitly defined as

$$\mathcal{K}_2 \equiv K^2 - K_{ij}K^{ij}, \quad (8)$$

$$\mathcal{K}_3 \equiv K^3 - 3KK_{ij}K^{ij} + 2K_{ij}K^{ik}K^j{}_k. \quad (9)$$

The coefficients in Eq. (7) are related to the original functions in Eqs. (1)–(4) by

$$A_2 = G_2 - (-X)^{1/2}\int \frac{G_{3,\phi}}{2\sqrt{-X}}dX, \quad (10)$$

$$A_3 = -\int G_{3,X}\sqrt{-X}dX - 2\sqrt{-X}G_{4,\phi}, \quad (11)$$

$$A_4 = -G_4 + 2XG_{4,X} + \frac{X}{2}G_{5,\phi} - X^2F_4, \quad (12)$$

$$B_4 = G_4 + \sqrt{-X}\int \frac{G_{5,\phi}}{4\sqrt{-X}}dX, \quad (13)$$

$$A_5 = -\frac{(-X)^{3/2}}{3}G_{5,X} + (-X)^{5/2}F_5, \quad (14)$$

$$B_5 = -\int G_{5,X}\sqrt{-X}dX. \quad (15)$$

In this ADM formulation, these functions of ϕ and X can also be seen as functions of t and N via the relations $\phi = \phi_0(t)$ and $X = -\dot{\phi}_0^2(t)/N^2$.

By using Eqs. (12)–(15), the Horndeski conditions (5) translate into

$$A_4 = -B_4 + 2XB_{4,X}, \qquad A_5 = -XB_{5,X}/3. \quad (16)$$

Hamiltonian analysis.—In general, higher derivative theories are pathological, because they lead, according to Ostrogradski's theorem, to *extra* DOF that behave like ghosts. Here we show, by resorting to a simple counting of the number of DOF in the Hamiltonian formalism, that the theories (7) do not contain more than 3 degrees of freedom. Thus, there is no room for an extra DOF in addition to the scalar DOF initially built in and the two tensor modes similar to those of GR.

The Hamiltonian is obtained from the Lagrangian via a Legendre transform

$$H = \int d^3x[\pi^{ij}\dot{h}_{ij} - \mathcal{L}], \quad (17)$$

where the π^{ij} are the conjugate momenta associated with the h_{ij}, defined by

$$\pi^{ij} = \frac{\partial\mathcal{L}}{\partial\dot{h}_{ij}}. \quad (18)$$

Ignoring L_5 for simplicity, one can easily invert the above relation to express $\dot{h}_{ij}$ as a function of π^{ij} and obtain the explicit Hamiltonian, which can be written in the form

$$H = \int d^3x[N\mathcal{H}_0 + N^i\mathcal{H}_i], \quad (19)$$

with

$$\mathcal{H}_0 \equiv -\sqrt{h}\left[A_2 - \frac{3A_3^2}{8A_4} + \frac{A_3\pi}{2\sqrt{h}A_4} + B_4R + \frac{1}{2hA_4}(2\pi_{ij}\pi^{ij} - \pi^2)\right], \quad (20)$$

$$\mathcal{H}_i \equiv -2D_j\pi^j{}_i. \quad (21)$$

We leave aside the uninteresting case $A_4 = 0$, which does not contain propagating tensor DOF.

In GR, variation with respect to N and N^i yields, respectively, the Hamiltonian constraint $\mathcal{H}_0 = 0$ and the momentum constraints $\mathcal{H}_i = 0$. These constraints are, in Dirac's terminology, first class and eventually eliminate eight out of the initial ten degrees of freedom (see, e.g., [10]). In our case, the gauge invariance under *spatial* diffeomorphisms is preserved, leading to first-class constraints analogous to the momentum constraints of GR and eliminating six DOF (see [11] for details). However, variation with respect to N now gives the constraint $\tilde{\mathcal{H}}_0 \equiv \mathcal{H}_0 + N\partial\mathcal{H}_0/\partial N = 0$, which is in general second class, instead of first class. This can be understood as a consequence of the scalar field that fixes the preferred slicing and thus breaks the full spacetime diffeomorphism invariance. This entails the elimination of only one DOF (instead of two in GR). Note that this reasoning crucially depends on the absence of $\dot{N}$ from the Lagrangians (7), which is guaranteed by the specific form of the new terms proportional to F_4 and F_5 introduced in Eqs. (3) and (4). The final number of physical DOF is therefore three, which correspond to the two standard tensor modes plus a scalar mode, as will be clear from the linear analysis below.

When L_5 is included, the full Hamiltonian cannot be written in closed form, because one cannot invert explicitly the relation (18), even if the inversion is in general well defined locally [11]. For this reason, we have not been able to compute explicitly the constraint algebra in the full case. However, our counting depends only on the nature of the constraints. Since the full Hamiltonian is, by construction, invariant under spatial diffeomorphism, the associated constraints should remain first class and thus eliminate six DOF as before. Taking into account the other constraints, one thus expects at most three DOF and, therefore, the absence of any ghostly extra DOF. The counting is also similar if one includes matter, with the matter DOF adding to the three from the gravitational sector. Finally, note that our analysis could also be applied almost straightforwardly to general ADM Lagrangians invariant under spatial diffeomorphisms involving arbitrary combinations of the extrinsic and intrinsic curvature tensors and their spatial derivatives. However, such a wider set of possibilities is not necessarily a covariant extension of Galileons as Eqs. (1)–(4).

Covariant formulation.—The above Hamiltonian analysis is based on our ADM reformulation of the theories and requires the gradient of the scalar field to be timelike so that uniform scalar-field hypersurfaces are spacelike. Although this is the case in cosmology, which is the main motivation to study these models, one can wonder whether our findings are still valid for more general situations.

For simplicity, let us consider theories involving up to L_4^ϕ, but not L_5^ϕ. We have found that the analysis of their equations of motion can be greatly simplified via the use of disformal transformations. Indeed, the gravitational action with the Lagrangians (1)–(3) reexpressed in terms of ϕ and of the new metric

$$\tilde{g}_{\mu\nu} = g_{\mu\nu} + \Gamma(\phi, X)\partial_\mu\phi\partial_\nu\phi, \tag{22}$$

with

$$\Gamma = \int \frac{F_4}{G_4 - 2XG_{4X} + X^2F_4} dX, \tag{23}$$

turns out to belong to the Horndeski class. This means that the equations of motion obtained by varying the action with respect to the metric $\tilde{g}_{\mu\nu}$ are second order. By using this property and by combining the (third-order) equations of motion for ϕ and $g_{\mu\nu}$ derived from the full action (including that of matter minimally coupled to $g_{\mu\nu}$), one can explicitly replace higher-order time derivatives of ϕ by at most second-order time derivatives (see details in Ref. [11] and related ideas in Ref. [9]). This shows that the equations of motion can be reduced to second order in time derivatives and do not require additional initial conditions, thus extending the conclusions of our Hamiltonian analysis to general configurations. The same method applies to theories without L_4^ϕ, although one cannot simultaneously map L_4^ϕ and L_5^ϕ to Horndeski for general combinations of these Lagrangians.

Quadratic action.—The above arguments exclude the presence of extra DOF, but one still needs to check that the remaining scalar and tensor DOF are not themselves ghostlike, for which we need to calculate the quadratic action for perturbations of the propagating DOF and make sure that the kinetic terms have the right signs. We perform this calculation around a spatially flat Friedmann-Lemaître-Robertson-Walker metric and follow the general procedure developed in Ref. [12] for the specific Lagrangian L given by Eq. (7). Namely, we expand at second order the action $S = \int d^4x\sqrt{-g}L$, using ζ gauge, i.e., $h_{ij} = a^2(t)e^{2\zeta}(\delta_{ij} + \gamma_{ij})$, $\gamma_{ii} = 0 = \partial_i\gamma_{ij}$, and splitting the shift as $N^i = \partial_i\psi + N_V^i$, $\partial_i N_V^i = 0$. Because of the particular structure of the terms in Eqs. (8) and (9), the Lagrangian (7) satisfies the criteria obtained in Ref. [12] that ensure that the linear equations of motion contain no more than two spatial derivatives. In particular, terms proportional to $(\partial^2\psi)^2$ cancel up to a total derivative. By varying the action with respect to N^i, one obtains the momentum constraints, whose solution is $N_V^i = 0$ and

$$N = 1 + \mathcal{D}\dot{\zeta}, \qquad \mathcal{D} \equiv \frac{4\mathcal{A}_4}{2H(2\mathcal{A}_4 + \mathcal{A}_4') - \mathcal{A}_3'}. \tag{24}$$

Above and in the following, a dot and a prime, respectively, denote derivative with respect to t and N. Furthermore, we use the new functions

$$\mathcal{A}_2 \equiv A_2 + 3HA_3 + 6H^2 A_4 + 6H^3 A_5,$$
$$\mathcal{A}_3 \equiv A_3 + 6HA_4 + 12H^2 A_5,$$
$$\mathcal{A}_4 \equiv A_4 + 3HA_5,$$
$$\mathcal{B}_4 \equiv B_4 + \frac{1}{2N}\dot{B}_5|_{N=1} - (N-1)\frac{HB_5'}{2}. \quad (25)$$

After substitution of Eq. (24) into the action, all the terms containing ψ drop out, up to boundary terms [13]. After some manipulations, the quadratic action becomes $S^{(2)} = \int d^4x a^3 L^{(2)}$ with

$$L^{(2)} = \alpha\dot{\zeta}^2 - \beta\frac{(\partial_i\zeta)^2}{a^2} + \frac{1}{4}\left[-\mathcal{A}_4\dot{\gamma}_{ij}^2 - \mathcal{B}_4\frac{(\partial_k\gamma_{ij})^2}{a^2}\right], \quad (26)$$

where the functions α and β are defined as

$$\alpha \equiv \left[\frac{(N^2\mathcal{A}_2')'}{2} - 3H\mathcal{A}_3' + 6H^2(N\mathcal{A}_4)'\right]\mathcal{D}^2 - 6\mathcal{A}_4,$$
$$\beta \equiv -2\mathcal{B}_4 + \frac{2}{a}\frac{d}{dt}[a\mathcal{D}(N\mathcal{B}_4)'], \quad (27)$$

evaluated on the background ($N = 1$). As expected from the previous Hamiltonian analysis, the quadratic Lagrangian (26) does not contain higher time derivatives. Moreover, for $\alpha > 0$ and $-\mathcal{A}_4 > 0$ we ensure that the propagating DOF are not ghostlike. Gradient instabilities are avoided for $c_s^2 \equiv \beta/\alpha > 0$ and $c_\gamma^2 \equiv -\mathcal{B}_4/\mathcal{A}_4 > 0$.

Coupling with matter.—In cosmology, the power of gravity at large scales—and its irrelevance at short distances—is well illustrated by the Jeans phenomenon. A matter overdensity $\delta\rho_m$ of a given Fourier mode k evolves, schematically, as

$$(\partial_t^2 + c_m^2 k^2 - \text{gravity})\delta\rho_m = 0. \quad (28)$$

In the above, c_m^2 is the square of the speed of sound, proportional to the pressure perturbation: $c_m^2 = \delta p_m/\delta\rho_m$. For $c_m^2 > 0$, the positive sign in front of the k^2 term guarantees an oscillating solution at sufficiently short distances, where the overdensity is supported by its own pressure gradients. The last term in parentheses stands for k-independent contributions roughly of Hubble size $\sim H^2$. Only at distances larger than $\sim c_m H^{-1}$ do these terms dominate, leading to gravitational (Jeans) instability. This well-known feature of standard cosmological perturbation theory holds true at small scales also in most modified gravity models—say, for definiteness, in all Horndeski theories as long as matter fields are minimally coupled to the metric.

The extension of Horndeski theories that we are proposing provides a counterexample to such an apparently universal behavior, even when matter is minimally coupled to the metric tensor. Let us illustrate this with a matter scalar field σ (not to be confused with the dark energy field ϕ), described by the k-essence type action,

$$S_m = \int d^4x\sqrt{-g}P(Y,\sigma), \qquad Y \equiv g^{\mu\nu}\partial_\mu\sigma\partial_\nu\sigma, \quad (29)$$

with sound speed $c_m^2 \equiv P_{,Y}/(P_{,Y} - 2\dot{\sigma}_0^2 P_{,YY})$. One can then repeat the procedure discussed earlier in order to obtain the quadratic action for the scalar fluctuations expressed in terms of ζ, N, ψ, and the matter field perturbation $\delta\sigma$. Making use of the momentum constraints, the final Lagrangian expressed in terms of ζ and of the gauge-invariant variable $Q_\sigma \equiv \delta\sigma - (\dot{\sigma}_0/H)\zeta$ reads

$$L^{(2)} = \left(\alpha - \frac{c_m^2 g_t^2}{4P_{,Y}}\right)\dot{\zeta}^2 - \left(\beta + \frac{P_{,Y}\dot{\sigma}_0^2}{H^2} - \frac{\dot{\sigma}_0 g_s}{H}\right)\frac{(\partial_i\zeta)^2}{a^2}$$
$$- \frac{P_{,Y}}{c_m^2}\left(\dot{Q}_\sigma^2 - c_m^2\frac{(\partial_i Q_\sigma)^2}{a^2}\right) + g_t\dot{\zeta}\dot{Q}_\sigma + g_s\frac{\partial_i\zeta\partial_i Q_\sigma}{a^2}$$
$$+ \cdots, \quad (30)$$

where $g_s \equiv -c_m^2 g_t + 2\dot{\sigma}_0 P_{,Y}\Delta$, with

$$g_t \equiv \frac{2\dot{\sigma}_0 P_{,Y}}{c_m^2}\left(\mathcal{D} - \frac{1}{H}\right), \quad \Delta \equiv \mathcal{D}\left(1 + \frac{(N\mathcal{B}_4)'}{\mathcal{A}_4}\right), \quad (31)$$

and we have included only the terms quadratic in time or space derivatives, the other terms (in the ellipses) being irrelevant for the following discussion. The dispersion relations for the propagating DOF can be obtained by requiring that the determinant of the matrix of the kinetic and spatial gradient terms vanishes, which yields

$$(\omega^2 - c_m^2 k^2)(\omega^2 - \tilde{c}_s^2 k^2) = \frac{(\rho_m + p_m)}{2\alpha}\Delta^2\omega^2 k^2,$$
$$\tilde{c}_s^2 \equiv [\beta - (1/2)(\rho_m + p_m)(\mathcal{D} - \Delta)^2]/\alpha, \quad (32)$$

where we have used $2\dot{\sigma}_0^2 P_{,Y} = -(\rho_m + p_m)$. From this equation, one derives the two dispersion relations $\omega^2 = c_\pm^2 k^2$. In Horndeski theories, $\Delta \propto \mathcal{A}_4 + (N\mathcal{B}_4)' = 0$ because of Eq. (16), and we thus find that, despite the couplings in the action between the time and space derivative of ζ and Q_σ, the matter sound speed is unchanged as a consequence of the special relation $g_s = -c_m^2 g_t$. This is no longer the case in our non-Horndeski extensions, where $\Delta \neq 0$ and the two couplings are "detuned." This remarkable difference between Horndeski and non-Horndeski theories was not pointed out in the recent work in Ref. [14], which also extends our previous analysis [12] to compute the quadratic action of dark energy coupled to a scalar field.

This unusual behavior can also be seen by writing the perturbed EOM derived from the manifestly covariant Lagrangian for ϕ, together with Eq. (28). On sufficiently small scales, we find (see [11] for details)

$$(\partial_t^2 + \tilde{c}_s^2 k^2)\delta\phi - C_\phi \dot{\phi}\partial_t \delta\rho_m \approx 0, \tag{33}$$

$$(\partial_t^2 + c_m^2 k^2)\delta\rho_m - C_m k^2 \partial_t(\delta\phi/\dot{\phi}) \approx 0, \tag{34}$$

with

$$C_m \equiv \frac{\Delta(\rho_m + p_m)}{\Delta - \mathcal{D}}, \qquad C_\phi \equiv -\frac{\Delta(\Delta - \mathcal{D})}{2\alpha}, \tag{35}$$

which leads to the same dispersion relation as in Eq. (32). This clearly shows that, in contrast to the standard Jeans lore, the gravitational scalar mode $\delta\phi$ cannot be decoupled from matter by going at sufficiently short distances. The origin of the special coupling between matter and the scalar field in Eq. (33) can also be understood as follows. Taking the example of L_4 for simplicity, one can see that the variation of (3) with respect to ϕ yields a term of the form $\phi^\lambda(g^{\mu\nu} + n^\mu n^\nu)\nabla_\nu R_{\lambda\mu}$. Using Einstein's equations (this assumes to separate L_4 into a GR term and an effective additional term), one can express the Ricci tensor in terms of the matter energy-momentum tensor, which leads to the term $\dot{\phi}\partial_t\delta\rho_m$ in Eq. (33).

Conclusion.—We have introduced a novel class of scalar-tensor theories, which include and extend Horndeski theories. For configurations where the scalar field gradient is timelike, these theories can be formulated in a very simple form via an ADM description of spacetime based on uniform ϕ slicing. This formulation allows us to absorb the scalar degree of freedom in the spatial metric and makes it particularly transparent to show the absence of Ostrogradski instabilities. For generic configurations, one can use disformal transformations to relate subclasses of these theories to theories with manifest second-order equations of motion. However, this procedure cannot be simultaneously applied to the most general case that includes both L_4^ϕ and L_5^ϕ, which means that a complete understanding of the full covariant theory requires further investigation.

An important corollary of this work applies to the original Galileons proposed in Ref. [2]: Their direct covariantization, obtained by substituting ordinary derivatives with covariant ones, belongs to the class of theories considered here. Our work suggests that such theories are already free of ghosts instabilities and do not need the gravitational "counterterms" prescribed in Ref. [4].

We have also uncovered a remarkable phenomenological property of the non-Horndeski subclass of our theories: When *minimally* coupled to ordinary matter, they exhibit a kinetic-type coupling, leading to a mixing of the dark energy and matter sound speeds. It would be interesting to study further the phenomenology of these theories.

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