Precision Metrology Using Weak Measurements

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Weak values and measurements have been proposed as a means to achieve dramatic enhancements in metrology based on the greatly increased range of possible measurement outcomes. Unfortunately, the very large values of measurement outcomes occur with highly suppressed probabilities. This raises three vital questions in weak-measurement-based metrology. Namely, (Q1) Does postselection enhance the measurement precision? (Q2) Does weak measurement offer better precision than strong measurement? (Q3) Is it possible to beat the standard quantum limit or to achieve the Heisenberg limit with weak measurement using only classical resources? We analyze these questions for two prototypical, and generic, measurement protocols and show that while the answers to the first two questions are negative for both protocols, the answer to the last is affirmative for measurements with phase-space interactions, and negative for configuration space interactions. Our results, particularly the ability of weak measurements to perform at par with strong measurements in some cases, are instructive for the design of weak-measurement-based protocols for quantum metrology.

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Weak measurements reveal partial information about a quantum state without "collapsing" it. This is done by coupling a measurement apparatus (MA) feebly to a test quantum system (QS), the dynamics of which is of interest. A procedure involves probing the QS at an intermediate stage between a preselected prepared state and a postselected state which typically has little overlap with the prepared state [1]. A subsequent projective measurement on the MA yields an outcome known as the "weak value." The fact that the weak value may lie outside the spectrum of the measurement operator leads to some interesting results. This phenomena has been used to study numerous quantum effects [2–16] as well as to reconstruct the wave functions of quantum states [17–20].

Weak values may dramatically amplify the small perturbations of the meter state arising from the coupling between the QS and MA [21–23]. This amplification makes weak measurements potentially useful in estimating the coupling strength with enhanced precision [24–30]. Yet, the amplification effect of weak measurement comes at the cost of a reduced rate at which data can be acquired due to the requirement to select almost orthogonal pre- and postselected states of the QS. This leads to a majority of trials being "lost." Thus, the central question is whether the amplification effect of a weak measurement can overcome the corresponding reduction in the occurrence of such events to provide an estimation at a precision surpassing conventional techniques. This issue has garnered substantial interest recently [31,32], in particular the amplification of information [33–36] and its role in alleviating technical imperfections [28,37–39]. However, an unequivocal agreement as to the ultimate efficacy of weak measurements in precision metrology is still lacking. Our endeavor here is to provide such an answer in the ideal scenario (i.e., without technical imperfections).

In this Letter, we show that postselection does not enhance the precision of estimation, that weak measurements do not offer better precision relative to strong measurements, and that it is possible to beat the standard quantum limit and to achieve the Heisenberg limit of quantum metrology with weak measurements using only classical resources. These apparently contradictory conclusions arise from a complete consideration of where the maximum information resides in the weak-measurement protocol. Our results are valid both for single-particle MA states, in which the QS couples to a continuous degree of freedom of the MA, and for multiparticle states of a bosonic MA. Although in both cases the MA may have similar mathematical representations, the degrees of freedom involved are different and, therefore, the scaling of the precision is different and in consequence analyzed separately. Our analysis properly counts the resources involved in the measurement process, enabling us to compare the precision of different measurement strategies and strengths using tools of classical and quantum Fisher information. Weak measurements have a rich structure, and offer some prospects for novel strategies for quantum-enhanced metrology. Nonetheless, we show that a new approach is required to harness this potential.

Framework.—Our aim is to estimate a parameter associated with the interaction between two systems. We focus on the situation that one of them, the QS, is a two-state system with eigenstates $|-1\rangle, |+1\rangle$ of an observable $\hat{S}$ with corresponding eigenvalues -1 and 1. The initial (preselected) state of the QS is prepared as $|\psi_i\rangle = \cos(\theta_i/2)|-1\rangle + \sin(\theta_i/2)e^{i\phi_i}|+1\rangle$. The initial state of the other system, the MA, is $|\Phi_i\rangle$. The coupling strength g which is to be estimated appears in the Hamiltonian $H = -g\delta(t-t_0)\hat{S}\hat{M}$ coupling MA to QS, where $\hat{M}$ is an observable of the MA. After this interaction, the joint state of the MA and the QS is

$$|\Psi_j\rangle = \cos\frac{\theta_i}{2}|-1\rangle|\Phi_{-g}\rangle + \sin\frac{\theta_i}{2}e^{i\phi_i}|+1\rangle|\Phi_{+g}\rangle, \quad (1)$$

where $|\Phi_{\pm g}\rangle = \exp(\mp ig\hat{M})|\Phi_i\rangle$. Postselecting the QS in state $|\psi_f\rangle = \cos(\theta_f/2)|-1\rangle + \sin(\theta_f/2)e^{i\phi_f}|+1\rangle$ leads to the MA state $|\Phi_d\rangle = (\gamma_d^-|\Phi_{-g}\rangle + \gamma_d^+|\Phi_{+g}\rangle)/\sqrt{p_d}$, with $\gamma_d^- = \cos(\theta_i/2)\cos(\theta_f/2)$, $\gamma_d^+ = \sin(\theta_i/2)\sin(\theta_f/2)\exp(i\phi_0)$ and $\phi_0 = \phi_i - \phi_f$. The probability of successful postselection, i.e., of obtaining $|\Phi_d\rangle$ is p_d. When the postselection fails (with probability $p_r = 1 - p_d$), the MA state, which is not considered in the original protocol and is often ignored in experiments, is $|\Phi_r\rangle = (\gamma_r^-|\Phi_{-g}\rangle + \gamma_r^+|\Phi_{+g}\rangle)/\sqrt{1-p_d}$, where $\gamma_r^- = \cos(\theta_i/2)\sin(\theta_f/2)$, $\gamma_r^+ = -\sin(\theta_i/2)\cos(\theta_f/2)\times\exp(i\phi_0)$. Repeating the preselection-coupling-postselection process N times yields Np_d copies of $|\Phi_d\rangle$ and $N(1-p_d)$ copies of $|\Phi_r\rangle$. The best attainable precision in estimating g is given by the Cramér-Rao bound $\Delta^2 g \geq 1/(NF_{\mathrm{tot}})$ [40], where F_{tot} is the sum total of the classical and quantum Fisher information (FI) contained at different stages of the preselection-coupling-postselection process. Note that the single-parameter Cramér-Rao bound, both quantum and classical, can always be attained asymptotically for large N with a maximum-likelihood estimation.

Depending on the estimation protocol, F_{tot} may have different values. To date, almost all applications of the weak measurement to precision metrology focus on the amplification effect of weak values, which corresponds to considering the information about g contained in $|\Phi_d\rangle$. In this situation, $F_{\mathrm{tot}} = p_d Q_d$, where Q_d is the quantum FI (QFI) of $|\Phi_d\rangle$, i.e., the maximum FI that can be achieved with the optimal measurement on $|\Phi_d\rangle$, which is a set of projection operators onto the eigenstates of the symmetric logarithmic derivative of $|\Phi_d\rangle$ [40]. $p_d Q_d$ can be viewed as the total information in the postselected meter state. In addition, one may also monitor the failure mode $|\Phi_r\rangle$ to achieve better precision in parameter estimation [41,42] and state tomography [20]. The maximum information in the failure mode is $(1-p_d)Q_r$, where Q_r is the QFI of $|\Phi_r\rangle$. Finally, the distribution $\{p_d, 1-p_d\}$ of the postselection process on QS also contains information about g. This distribution yields a classical FI F_p which we refer to as the information in the postselection process. If we account for all these contributions, we have (see Sec. I in the Supplemental Material [43] for a proof)

$$F_{\mathrm{tot}} = p_d Q_d + (1-p_d)Q_r + F_p. \quad (2)$$

The whole process (postselection plus measurements on the MA state) is a special case of the global measurement on the joint state $|\Psi_j\rangle$ [43]; therefore, F_{tot} is no larger than the QFI Q_j of $|\Psi_j\rangle$; i.e., postselection cannot increase the precision in estimating g. This seemingly straightforward result provides important insight about the relation between the amplification effect and measurement precision, and allows us to access the rich structures of weak measurement and evaluate their quantum advantages. In particular, we note that Q_d or Q_r alone may be larger than Q_j due to the amplification effect of weak values. Nevertheless, this apparent gain of information is completely canceled by the small probability of successful postselection. Moreover, the postselection process may contain important information $F_p \geq 0$. This analysis is different from previous studies [33] by considering all the contributions to the total information, and thus provides a complete answer to Q1 posed in the abstract. We note that a similar conclusion was reached in Ref. [35]. In the following sections, we provide answers to Q2 and Q3 in both configuration and phase-space interactions.

Configuration space interactions.—We begin with the most widely used scenario in weak measurement [1,9,10,21,24–27,29,30], where both the QS and MA are single-particle states, possibly in different degrees of freedom of the same particle. In this situation, the MA is normally prepared in a Gaussian superposition state of two conjugate variables [44],

$$|\Phi\rangle = \int dq \frac{1}{(2\pi\sigma^2)^{1/4}}\exp\left(-\frac{q^2}{4\sigma^2}\right)|q\rangle$$
$$= \int dp \frac{(2\sigma^2)^{1/4}}{\pi^{1/4}}\exp(-\sigma^2 p^2)|p\rangle, \quad (3)$$

where p and q are, e.g., momentum and position or time and frequency. The two representations are related via a Fourier transform. The interaction Hamiltonian between the QS and MA is chosen as $H = -g\delta(t-t_0)\hat{S}\,\hat{q}$. Note that this interaction Hamiltonian entangles the QS with an *external* degree of freedom of the MA. It does not change the particle-number distribution in the state of the MA. After the interaction and postselection, the MA state becomes $|\Phi_k\rangle = \int dp\phi_k(g,p)|p\rangle$ $(k = d, r)$, with

$$\phi_k(g,p) = \frac{(2\sigma^2)^{1/4}}{\pi^{1/4}\sqrt{p_k}}[\gamma_k^- e^{-\sigma^2(p+g)^2} + \gamma_k^+ e^{-\sigma^2(p-g)^2}]. \quad (4)$$

The probability of successful postselection is

$$p_d = \frac{1 + \cos\theta_i\cos\theta_f + \sin\theta_i\sin\theta_f\cos\phi_0 e^{-2s^2}}{2}, \quad (5)$$

with $s = g\sigma$ characterizing the measurement strength. With Eqs. (4), (5), we can estimate Q_d, Q_r, and F_p (using the Supplemental Material [43]),

$$Q_d = \frac{4\sigma^2}{p_d}\left[p_d + S(2s^2-1) - \frac{1}{p_d}S^2 s^2\right],$$
$$Q_r = \frac{4\sigma^2}{1-p_d}\left[1 - p_d - S(2s^2-1) - \frac{1}{1-p_d}S^2 s^2\right],$$
$$F_p = \frac{4\sigma^2 s^2 S^2}{p_d(1-p_d)}, \tag{6}$$

where $S = e^{-2s^2}\sin\theta_i \sin\theta_f \cos\phi_0$. Furthermore, the QFI of the joint meter-system state before postselection is $Q_j = 4\sigma^2$. We can now calculate $F_{\rm tot}$ for different estimation strategies. In particular, if we take into account all of the contributions in Eq. (2), we have $F_{\rm tot} = Q_j$; i.e., we achieve the maximal precision. A commonly employed strategy retains only the information in the successfully postselected meter state. In this case, the complicated functional form of $F_{\rm tot} = p_d Q_d$ demands numerical maximization over ψ_i and ψ_f. Nonetheless, some limits that may be obtained analytically allow us to answer Q2. In the weak measurement limit, defined as $s \to 0$,

$$p_d Q_d = 2\sigma^2(1 + \cos\theta_i\cos\theta_f - \sin\theta_i\sin\theta_f\cos\phi_0), \tag{7}$$

the maximum value of which is $4\sigma^2$, attained when either $\theta_i = -\theta_f$ and $\phi_0 = 0$ or $\theta_i = \theta_f$ and $\phi_0 = \pi$. Interestingly, this does not coincide in general with the situation when the weak value is the largest, which requires $p_d = |\langle\psi_i|\psi_f\rangle|^2 \to 0$ [32]. In the limit of strong measurement, when $s \gg 1$,

$$p_d Q_d = 2\sigma^2(1 + \cos\theta_i\cos\theta_f), \tag{8}$$

which also attains the maximum of $4\sigma^2$, but for the situation that both pre- and postselected states are $|+1\rangle$ or $|-1\rangle$. In both these limits, $p_d Q_d = Q_j$, $F_p = 0$, and $Q_r = 0$.

More generally, non-Gaussian MA states also achieve this precision (see Sec. II in the supplemental Material [43] for proof). This may be relevant to recent experiments that exploit this resource [45,46]. The conclusion is that, when the uncertainty of the meter state σ is fixed, the precision in the weak measurement limit, that is, to estimate a small parameter g through preselection-coupling-postselection, is no better than that in the strong measurement limit, that is, when the coupling parameter is large. However, if the parameter to be estimated is fixed, the precision is always better if we use a meter state with larger σ, as is evident in Eqs. (7), (8) and $F_{\rm tot}$ since the FIs are proportional to σ^2. This answers Q2 for the configuration-space-interaction scenario.

This analysis focuses on the effect of the uncertainty in the external degrees of freedom of the MA as in the previous works [21,24–27,29–32,41], showing that weak measurements may or may not offer an overhead advantage. In quantum metrology, the relevant measure of the resource required to effect a measurement is the average number of photons (n) in the MA state. The scaling of the precision of estimation with respect to n is the signature of whether the system is capable of operating beyond the standard quantum limit (in which the FI scales linearly in n) and offering genuine quantum advantages. Since the interaction Hamiltonian does not change particle-number distributions, for QS and MA prepared in (multimode) coherent states with amplitude α, postselected meter states are also multimode coherent states, and the FIs in Eqs. (7), (8) pick up an additional factor of $n = |\alpha|^2$. Thus, the scalings are at the standard quantum limit. This is the answer to Q3 for the configuration-space-interaction scenario.

Phase-space interactions.—We now consider a scenario that can change the particle-number distribution. The initial state of the QS $|\psi_i\rangle$ is the same as before, while the MA is prepared in a coherent state $|\alpha\rangle$. A state-dependent interaction with $\hat{M} = \hat{n}$, where $\hat{n}$ is the particle number operator, leads to [47]

$$|\Psi\rangle = \cos\frac{\theta_i}{2}|-1\rangle|\alpha\rangle + \sin\frac{\theta_i}{2}e^{i\phi_i}|+1\rangle|\alpha e^{i2g}\rangle. \tag{9}$$

or [48]

$$|\Psi\rangle = \cos\frac{\theta_i}{2}|-1\rangle|\alpha e^{-ig}\rangle + \sin\frac{\theta_i}{2}e^{i\phi_i}|+1\rangle|\alpha e^{ig}\rangle. \tag{10}$$

Both states have the same precision in estimating g when n is large. In the following, we focus on the symmetric form in Eq. (10). The meter states after postselection are $(k = d, r)$ $|\Phi_k\rangle = (\gamma_k^-|\alpha e^{-ig}\rangle + \gamma_k^+|\alpha e^{ig}\rangle)/\sqrt{p_k}$. The probability of obtaining this state and the FIs are all given in Sec. IV in the Supplemental Material [43]. Again, the QFIs are attainable with the optimal measurement on $|\Phi_k\rangle$.

The QFI of the system-meter state in Eq. (10) is (Sec. III in [43]) $Q_j = 4n^2\sin^2\theta_i + 4n$, where $n = |\alpha|^2$ is again the mean photon number (or energy) of the meter state {similarly, the QFI of the state in Eq. (9) is $Q_j = 4n^2\sin^2\theta_i + 4n[4\sin^2(\theta_i/2)]$}. Q_j is the maximum amount of information, and can exhibit quantum scaling ($\sim n^2$) depending on the initial system state. The expression for Q_j immediately suggests that $\theta_i = 0, \pi$ will never provide a better-than-classical scaling. These are the two cases when the initial state is an eigenstate of $\hat{S}$, so that no entanglement is generated between the QS and MA. Indeed, for $|\psi_i\rangle = |\pm 1\rangle$, $p_d Q_d = 2n(1 \pm \cos\theta_f)$, $(1-p_d)Q_r = 2n(1 \mp \cos\theta_f)$, and $F_p = 0$. Thus, $F_{\rm tot} = 4n$, but the information may be equally shared between the successful and the failed postselection mode. This is important since the failed postselection mode is generally discarded completely [21,24–30].

In contrast, the maximal Q_j is found for $\theta_i = \pi/2$. We immediately find that $\theta_f = 0, \pi$ provides no better than classical scalings either. Thus, we set $\theta_f = \pi/2$ as well, and find that as $g \to 0$, it leads to

$$F_p = 4n^2. \tag{11}$$

This result shows that quantum-enhanced scaling can be attained in the sensing of the coupling parameter g in a weak measurement setup. On the other hand, in this same situation, the QFIs for both the successful and failed postselection mode scale classically; $p_d Q_d = 4n \sin^2(\phi_0/2)$ and $(1-p_d)Q_r = 4n\cos^2(\phi_0/2)$, where $\phi_0 = \phi_i - \phi_f$. This shows that $p_d Q_d$ achieves its maximum when $\phi_0 \to \pi$; i.e., ψ_i and ψ_f are orthogonal. Note also that if we take into account all of the contributions, we have $F_{\text{tot}} = Q_j$. This is a particularly interesting situation since most, if not all, earlier experiments considered only the information Q_d contained in the successfully postselected MA state. Yet, as our calculation shows, the postselection process has much more information, and indeed scales at the Heisenberg limit. The parameter g can be estimated with the precision derived in Eq. (11) from the statistics of the success or failure of the postselection, using a maximum likelihood estimator.

For interaction strengths $g > 0$, the contributions of the different terms in F_{tot} change. In Fig. 1, we plot the FI and QFIs contributing to F_{tot} for $\phi_0 = \pi$. Exploiting a symmetry of our model, we only plot the results in $g = \{0, \pi/2\}$. As shown earlier, for $g \to 0$, the main contribution comes from F_p, the classical FI in the postselection distribution. As g increases, F_p falls, and the information in the postselected states for both successful and failed QS measurement outcomes rises. For $g = \pi/2$, we plot the contributions in greater detail in Fig. 2 for $\phi_0 = \pi$. For this case, $F_p = 0$, while $(1-p_d)Q_r$, $p_d Q_d$ are almost equal. Indeed, the difference in the QFIs decreases with n, as $p_d Q_d - (1-p_d)Q_r = -4n(n-1)\exp(-2n)$. For $n \gg 1$, up to a small exponential correction, there is thus as much information in the successful postselection mode as in the failed mode, and both of them scale better than the classical scaling. In all cases, the total F_{tot} still matches the maximum QFI attainable, that is Q_j. These results provide answers to Q2 and Q3 for the conditional-phase-shift scenario.

Discussion and conclusions.—It is perhaps unsurprising that the Heisenberg limit for estimating the coupling parameter g in the conditional-phase-shift interaction can be attained when the system-meter coupling is strong, since in that case, the postselected MA states are Schrödinger-cat states. That is, the measurement protocol produces highly nonclassical states in the joint system. In the case of weak coupling ($g \to 0$), however, the postselected MA states are classical, and the Heisenberg scaling arises only in the postselection process itself. How this conditioning step using a classical MA state achieves a precision beyond the standard quantum limit is therefore an interesting open question.

Our calculations show that not only the failed postselection mode but the postselection process itself contains useful information. The analysis provides answers to three long-standing questions in the study of weak measurement posed in the abstract: (A1) postselection cannot enhance the measurement precision even when all the contributions are taken into account; (A2) for equal resources, weak measurement does not give improved precision over strong measurement, when both measurements are optimized. In particular, this result applies to all previous experiments that have explored weak-measurement enhancements to precision metrology.

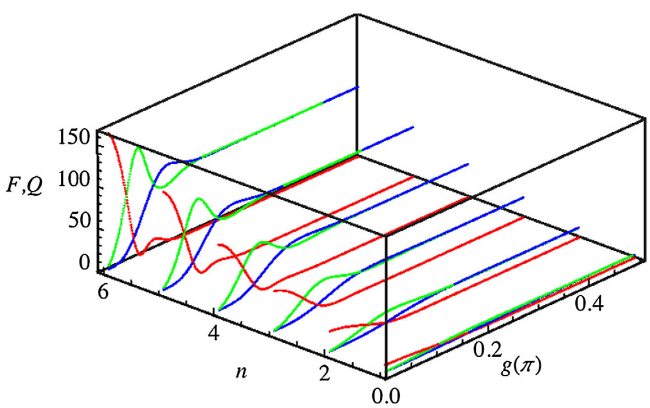

FIG. 1 (color online). Contributions to the total information from the three constituents in the conditional-phase-rotation scenario with pre- and postselected QS state $\psi_i = (|-1\rangle + |+1\rangle)/\sqrt{2}$ and $\psi_f = (|-1\rangle - |+1\rangle)/\sqrt{2}$, and initial MA state $|\alpha\rangle$. Red curves, F_p. Green curves, $p_d Q_d$. Blue curves, $(1-p_d)Q_r$. The sum of three quantities F_{tot} equals Q_j, the total QFI of the joint system-meter state which is $4n^2 + 4n$ and $n = |\alpha|^2$.

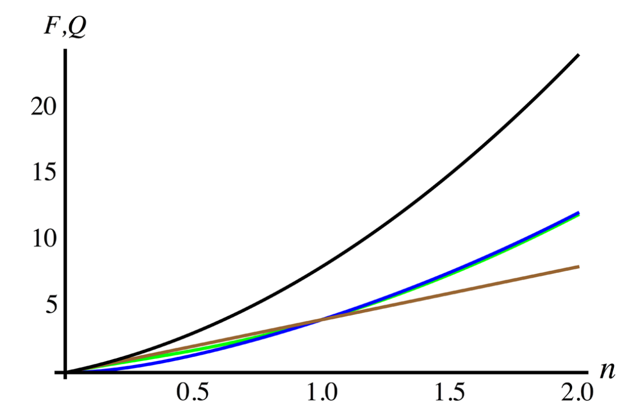

FIG. 2 (color online). Classical and quantum FIs for $g = \pi/2$. in the conditional phase-rotation scenario with $\psi_i = (|-1\rangle + |+1\rangle)/\sqrt{2}$ and $\psi_f = (|-1\rangle - |+1\rangle)/\sqrt{2}$, and initial MA state $|\alpha\rangle$. Green curve, $p_d Q_d$. Blue curve, $(1-p_d)Q_r$. Black curve, $Q_j = 4n^2 + 4n$, and Brown curve, classical scaling of $4n$. F_p is not shown since it is 0. The green and blue lines add up to the black line.

(A3) Weak measurement that modifies the particle-number distribution of the meter state can yield quantum-enhanced precision, though no nonclassical states need be involved. These results highlight the rich structure of the weak measurement and shed new light on both the understanding of quantum measurement and the development of new technologies for practical quantum metrology.

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