Eightfold Classification of Hydrodynamic Dissipation

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We provide a complete characterization of hydrodynamic transport consistent with the second law of thermodynamics at arbitrary orders in the gradient expansion. A key ingredient in facilitating this analysis is the notion of adiabatic hydrodynamics, which enables isolation of the genuinely dissipative parts of transport. We demonstrate that most transport is adiabatic. Furthermore, in the dissipative part, only terms at the leading order in gradient expansion are constrained to be sign definite by the second law (as has been derived before).

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Introduction.—Hydrodynamics is the universal low energy description at sufficiently high temperatures of quantum systems near thermal equilibrium. The dynamical fields are the intensive parameters that describe the near thermal density matrix, viz., temperature T, chemical potential μ, along with the fluid velocity ($u^\mu, u^\mu u_\mu = -1$), which sets the local frame in which the state appears thermal. The background sources are the metric $g_{\mu\nu}$ and the flavor sources A_μ. The hydrodynamic state in a given background is then completely characterized by a "thermal vector" $\boldsymbol{\beta}^\mu$ and "thermal twist" $\Lambda_{\boldsymbol{\beta}}$ defined by

$$\mathcal{B} \equiv \left\{ \boldsymbol{\beta}^\mu = \frac{u^\mu}{T}, \Lambda_{\boldsymbol{\beta}} = \frac{\mu}{T} - \boldsymbol{\beta}^\sigma A_\sigma \right\}. \tag{1}$$

The response to the background sources is encoded in the energy momentum tensor ($T^{\mu\nu}$) and charge current (J^μ) of the theory given in terms of the hydrodynamic fields. The dynamical equations are the statements of conservation. In the presence of external sources and quantum anomalies (incorporated by the inflow Hall currents $T_H^{\mu\perp}$ and $J_H^\perp$), one has with $D_\mu = \nabla_\mu + [A_\mu, \cdot]$

$$\nabla_\nu T^{\mu\nu} = J_\nu \cdot F^{\mu\nu} + T_H^{\mu\perp}, \qquad D_\nu J^\nu = J_H^\perp. \tag{2}$$

Phenomenologically, a hydrodynamicist finds constitutive relations that express the currents in terms of the fields. The operators are tensors built out of $\mathcal{B}$, the background sources $\{g_{\mu\nu}, A_\mu\}$, and their gradients, multiplied by transport coefficients which are arbitrary scalar functions of T, μ. *A priori* this "current algebra" formulation appears simple, since classifying such unrestricted tensors is a straightforward exercise in representation theory.

However, hydrodynamic currents should satisfy a further constraint [1]—the second law of thermodynamics has to hold for arbitrary configurations of the low energy dynamics. In practice, one demands the existence of an entropy current J_S^μ with non-negative definite divergence $\nabla_\mu J_S^\mu \geq 0$.

At low orders in the gradient expansion it is not too hard to implement the constraints by hand and check what the second law implies; e.g., at one derivative order one finds that viscosities and conductivities need to be non-negative $\eta, \zeta, \sigma \geq 0$, which is physically intuitive. To date, no complete classification has been obtained at higher orders, though the impressive analyses of Refs. [2–4] come quite close.

From a (Wilsonian) effective field theorist's perspective, this phenomenological current algebralike approach is unsatisfactory. Not only is the entropy current not associated with any underlying microscopic principle, but also the origin of dynamics as conservation is obscure. *A priori* a Wilsonian description for density matrices should involve working with doubled microscopic degrees of freedom, in the manner of the Schwinger-Keldysh or Martin-Siggia-Rose-Janssen-deDominicis methods. But one has yet to understand the couplings between the two copies (influence functionals) allowed by the second law, which ought to encode information about dissipation (and, curiously, also anomalies [5]).

In this Letter we describe a new framework for hydrodynamic effective field theories and provide a complete classification of transport. In particular, hydrodynamic transport admits a natural decomposition into adiabatic and dissipative components: the latter contribute to entropy production, while the former do not. At low orders, terms such as viscosities are dissipative; a major surprise is that most higher order transport is adiabatic.

Adiabatic transport can be captured by an effective action with not only Schwinger-Keldysh doubling of the sources, but also a new gauge principle, $U(1)_{\mathsf{T}}$ Kubo-Martin-Schwinger (KMS) gauge invariance, with a gauge field $\mathsf{A}^{(\mathsf{T})}$. This symmetry implies adiabaticity, i.e., off-shell entropy conservation, providing thereby a rationale for J_S^μ (dissipative dynamics arise in the Higgs phase). We use this to prove an eightfold classification of adiabatic transport. Together with a key theorem from Ref. [3], we further argue that dissipative hydrodynamic transport is constrained by the second law only at leading order in gradients. In the following, we will sketch the essential features of our construction; details will appear in companion papers [6].

Adiabatic hydrodynamics and the eightfold classification.—The key ingredient of our analysis that enables the classification scheme is the notion of adiabaticity. The main complications in hydrodynamics arise from attempting to implement the second law of thermodynamics on shell. Significant simplification can be achieved by taking the constraints off shell. One natural way to do this is to extend the inequality $\nabla_\mu J_S^\mu \geq 0$ to an off-shell statement by the addition of the dynamical equations of motion with Lagrange multipliers [7]. Choosing the Lagrange multipliers for the energy momentum and charge conservation to be the hydrodynamic fields implies that

$$\nabla_\mu J_S^\mu + \boldsymbol{\beta}_\mu(\nabla_\nu T^{\mu\nu} - J_\nu \cdot F^{\mu\nu} - T_H^{\mu\perp}) + (\Lambda_{\boldsymbol{\beta}} + \boldsymbol{\beta}^\lambda A_\lambda) \cdot (D_\nu J^\nu - J_H^\perp) = \Delta \geq 0, \tag{3}$$

with Δ capturing the dissipation and the dot operator denotes flavor index contraction.

While taking the second-law inequality off shell allows us to ignore on-shell dynamics, one can obtain the most stringent conditions by examining the boundary of the domain where we marginally satisfy the constraint. We define an adiabatic fluid as one where the off-shell entropy production is compensated for precisely by energy momentum and charge transport. We thus motivate the study of the adiabaticity equation obtained from Eq. (3) by setting $\Delta = 0$. We will refer to the set of functionals $\{J_S^\mu, T^{\mu\nu}, J^\mu\}$ that satisfy $\Delta = 0$ as the adiabatic constitutive relations.

Implications of adiabaticity were first studied in the context of anomalous transport in Ref. [8] and are explored in greater detail in Ref. [6]. In the following, we will provide some of the salient results of our analysis and explain how it helps with the taxonomy.

Intuitively, the notion of adiabaticity is an off-shell generalization of nondissipativeness; imposing Eq. (2) we learn that the entropy current has to be conserved on shell. Moreover, apart from quantum anomalies encoded by the Hall currents, the contributions at each order in the gradient expansion can be decoupled. It is quite remarkable that this corner of the hydrodynamic constitutive relations is sufficient to delineate all the constraints on transport. We will first outline different classes of solutions to the adiabaticity equation (3), and then in the next section we explain how it can be utilized for taxonomic purposes.

The adiabatic transport finds a natural classification into eight primary classes; see Fig. 1. We emphasize that adiabatic constitutive relations encode those transport coefficients that never appear in the expression for entropy production. Together with class D (dissipative), we exhaust all forms of transport.

To understand the nomenclature and taxonomy, let us start with class A, which is consists of transport fixed by the quantum anomalies of the quantum field theory (QFT). Such anomalous transport gives a particular solution to the adiabaticity equation (3), cf. Ref. [8]—the anomalous Hall currents can be viewed as inhomogeneous source terms. This allows us to dispense with them once and for all and focus thence on the nonanomalous adiabaticity equation.

The simplest solutions to Eq. (3) can be obtained by restricting to hydrostatic equilibrium (class H). One subjects the fluid to arbitrary slowly varying, time-independent external sources $\{g_{\mu\nu}, A_\mu\}$. The background time independence implies the existence of a Killing vector and gauge transformation, $\mathcal{K} = \{K^\mu, \Lambda_K\}$, with $\delta_\mathcal{K} g_{\mu\nu} = \delta_\mathcal{K} A_\mu = 0$. Identifying the hydrodynamic fields with these background isometries $\boldsymbol{\beta}^\mu = K^\mu$, $\Lambda_{\boldsymbol{\beta}} = \Lambda_K$ solves Eq. (3). This information can equivalently be encoded in a hydrostatic partition function [9,10], which is the generating functional of (Euclidean) current correlators. Varying this partition function, we can then obtain a class of constitutive relations that solve Eq. (3).

The partition function has two distinct components: hydrostatic scalars H_S and vectors H_V. The transformation properties refer to the transverse spatial manifold obtained by reducing along the (timelike) isometry direction. The scalars H_S are terms one is most familiar with, e.g., the pressure p as a functional of intensive parameters (which now are determined by the background Killing fields). The vectors P^σ in H_V are both transverse to the Killing field and

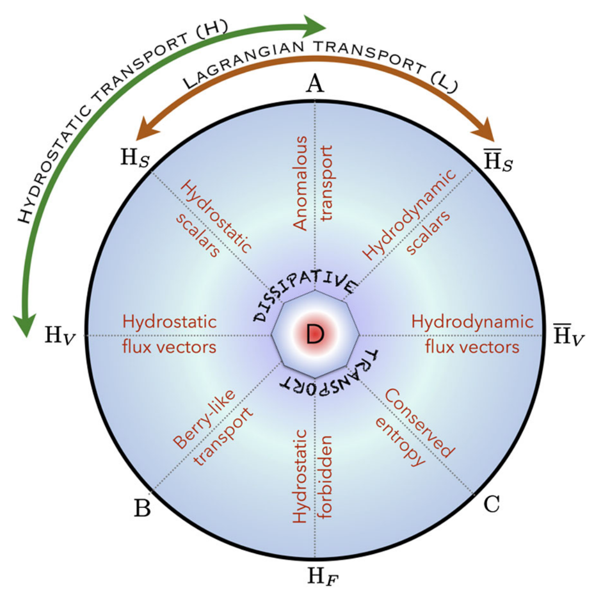

FIG. 1 (color online). The eightfold classification of hydrodynamic transport.

conserved on the codimension one achronal slice, i.e., $K_\sigma P^\sigma = \nabla_\sigma P^\sigma = 0$.

Hydrostatics fixes a part of the constitutive relations by imposing relations between *a priori* independent transport coefficients [9]. These relations (class H_F) capture the fact that nonvanishing hydrostatic currents expressed as independent tensor structures in equilibrium arise from a single partition function. More importantly, dangerous terms that can produce sign-indefinite divergence of entropy current are eliminated in class H_F.

The second set of solutions of Eq. (3) is generated by generalizing the scalar part of the partition function to time-dependent configurations, similar to the Landau-Ginzburg method. We call these Lagrangian (class L) solutions, since one can find a local Lagrangian (or Landau-Ginzburg free energy) of the hydrodynamic fields and sources, $\mathcal{L}[\boldsymbol{\beta}^\mu, \Lambda_{\boldsymbol{\beta}}, g_{\mu\nu}, A_\mu]$. The currents are defined through standard variational calculus, which can be expressed after suitable integrations by parts as

$$\frac{1}{\sqrt{-g}}\delta(\sqrt{-g}\mathcal{L}) = \frac{1}{2}T^{\mu\nu}\delta g_{\mu\nu} + J^\mu\cdot\delta A_\mu + T\mathfrak{h}_\sigma\delta\boldsymbol{\beta}^\sigma + T\mathfrak{n}\cdot(\delta\Lambda_{\boldsymbol{\beta}} + A_\sigma\delta\boldsymbol{\beta}^\sigma) + \text{boundary terms}, \tag{4}$$

while the entropy density is defined as (note that $J_S^\mu = su^\mu$)

$$s \equiv \left(\frac{1}{\sqrt{-g}}\frac{\delta}{\delta T}\int\sqrt{-g}\mathcal{L}[\boldsymbol{\Psi}]\right)\Bigg|_{\{u^\sigma,\mu,g_{\alpha\beta},A_\alpha\}=\text{fixed}},$$

with $\boldsymbol{\Psi} \equiv \{\boldsymbol{\beta}^\mu, \Lambda_{\boldsymbol{\beta}}, g_{\mu\nu}, A_\mu\}$. Diffeomorphism and gauge invariance of $\mathcal{L}$ together imply a set of Bianchi identities, which together with the definition of J_S^μ suffices to show that Eq. (3) is satisfied [11]. In the above equation, one can interpret $\{\mathfrak{h}_\sigma, \mathfrak{n}\}$ as characterizing the adiabatic heat current and adiabatic charge density which satisfy a relation of the form $Ts + \mu\cdot\mathfrak{n} = -u^\sigma\mathfrak{h}_\sigma$.

It is intuitively clear that by restricting class L solutions to hydrostatics, we recover the partition function scalars H_S. As a result one can write $L = H_S \cup \bar{H}_S$, with $\bar{H}_S$ denoting scalar invariants that vanish identically in hydrostatics; hence, hydrostatic scalars take values in a coset manifold $L/\bar{H}_S$.

There are two other adiabatic constitutive relations which are nonhydrostatic but nondissipative. One class of adiabatic constitutive relations describes Berry-like transport (class B), which can be parametrized as

$$\begin{aligned}(T^{\mu\nu})_B &\equiv -\frac{1}{2}\mathcal{N}^{(\mu\nu)(\alpha\beta)}\delta_{\mathcal{B}}g_{\alpha\beta} + \mathcal{X}^{(\mu\nu)\alpha}\cdot\delta_{\mathcal{B}}A_\alpha,\\ (J^\alpha)_B &\equiv -\frac{1}{2}\mathcal{X}^{(\mu\nu)\alpha}\delta_{\mathcal{B}}g_{\mu\nu} - \mathcal{S}^{[\alpha\beta]}\cdot\delta_{\mathcal{B}}A_\beta.\end{aligned} \tag{5}$$

Here $\mathcal{N}^{(\mu\nu)(\alpha\beta)} = -\mathcal{N}^{(\alpha\beta)(\mu\nu)}$, $\mathcal{X}^{\mu\nu\alpha}$, and $\mathcal{S}^{\alpha\beta}$ are arbitrary local functionals of $\boldsymbol{\Psi}$ with indicated (anti)symmetry properties, such that, along with $J_S^\mu = -\boldsymbol{\beta}_\nu T^{\mu\nu} - (\mu/T)J^\mu$, the adiabaticity equation is satisfied [6,14]. A prime example for structures of the type (5) is the parity odd shear tensor in three dimensions which contributes to Hall viscosity (class B). Thus, the tensors $\mathcal{N}^{\mu\nu\alpha\beta}$, $\mathcal{X}^{\mu\nu\alpha}$, and $\mathcal{S}^{\alpha\beta}$ can be thought of as a generalization of the notion of odd viscosities and conductivities.

We will denote the other class as class $\bar{H}_V$, which can be parametrized as

$$\begin{aligned}(T^{\mu\nu})_{\bar{H}_V} &\equiv \frac{1}{2}[D_\rho\mathfrak{C}_{\mathcal{N}}^{\rho(\mu\nu)(\alpha\beta)}\delta_{\mathcal{B}}g_{\alpha\beta} + 2\mathfrak{C}_{\mathcal{N}}^{\rho(\mu\nu)(\alpha\beta)}D_\rho\delta_{\mathcal{B}}g_{\alpha\beta}]\\ &\quad + D_\rho\mathfrak{C}_{\mathcal{X}}^{\rho(\mu\nu)\alpha}\cdot\delta_{\mathcal{B}}A_\alpha + 2\mathfrak{C}_{\mathcal{X}}^{\rho(\mu\nu)\alpha}\cdot D_\rho\delta_{\mathcal{B}}A_\alpha,\\ (J^\alpha)_{\bar{H}_V} &\equiv \frac{1}{2}[D_\rho\mathfrak{C}_{\mathcal{X}}^{\rho(\mu\nu)\alpha}\delta_{\mathcal{B}}g_{\mu\nu} + 2\mathfrak{C}_{\mathcal{X}}^{\rho(\mu\nu)\alpha}D_\rho\delta_{\mathcal{B}}g_{\mu\nu}]\\ &\quad + D_\rho\mathfrak{C}_{\mathcal{S}}^{\rho(\alpha\beta)}\cdot\delta_{\mathcal{B}}A_\beta + 2\mathfrak{C}_{\mathcal{S}}^{\rho(\alpha\beta)}\cdot D_\rho\delta_{\mathcal{B}}A_\beta,\end{aligned} \tag{6}$$

where $\mathfrak{C}_{\mathcal{N}}^{\rho(\mu\nu)(\alpha\beta)} = \mathfrak{C}_{\mathcal{N}}^{\rho(\alpha\beta)(\mu\nu)}$. The entropy current has a similar form as in class B along with an additional contribution which is quadratic in $\delta_{\mathcal{B}}g_{\mu\nu}$ and $\delta_{\mathcal{B}}A_\mu$. Finally, we have exactly conserved vectors (class C) that can be added to the entropy current without modification of the constitutive relations. They describe possible topological states which transport entropy but no charge or energy. We claim that the above classification is exhaustive.

Theorem.—The eight classes of adiabatic hydrodynamic transport can be obtained from a scalar Lagrangian density $\mathcal{L}_T[\boldsymbol{\beta}^\mu, \Lambda_{\boldsymbol{\beta}}, g_{\mu\nu}, A_\mu, \bar{g}_{\mu\nu}, \bar{A}_\mu, \mathrm{A}^{(\mathsf{T})}{}_\mu]$:

$$\mathcal{L}_T = \frac{1}{2}T^{\mu\nu}\bar{g}_{\mu\nu} + J^\mu\cdot\bar{A}_\mu + [J_S^\sigma + \boldsymbol{\beta}_\nu T^{\nu\sigma} + (\Lambda_{\boldsymbol{\beta}} + \boldsymbol{\beta}^\nu A_\nu)\cdot J^\sigma]\mathrm{A}^{(\mathsf{T})}{}_\sigma. \tag{7}$$

As indicated, the Lagrangian density depends not only on the hydrodynamic fields and the background sources, but also on the Schwinger-Keldysh partners of the sources $\{\bar{g}_{\mu\nu}, \bar{A}_\mu\}$ and a new KMS gauge field $\mathrm{A}^{(\mathsf{T})}{}_\mu$. This Lagrangian is invariant under diffeomorphisms and gauge transformations [15] and under $\mathrm{U}(1)_\mathsf{T}$, which acts only on the sources as a diffeomorphism or gauge transformation along $\mathcal{B}$. The $\mathrm{U}(1)_\mathsf{T}$ gauge invariance implies a Bianchi identity, which is nothing but the adiabaticity equation (3). Furthermore, a constrained variational principle for the fields $\{\boldsymbol{\beta}^\mu, \Lambda_{\boldsymbol{\beta}}\}$ ensures that the dynamics of the theory is simply given by conservation. We anticipate that the KMS gauge field plays a crucial role in implementing nonequilibrium fluctuation-dissipation relations which follow from the KMS condition; its significance both in hydrodynamic effective field theories as well as in holography will be discussed in a future work [17].

Route to dissipation.—Having classified solutions to the adiabaticity equation, let us now turn to the characterization of hydrodynamic transport including dissipative terms (class D). We will do so by first systematically eliminating all of the adiabatic transport by the following algorithm.

(1) Enumerate the total number of transport coefficients, $\mathrm{tot}_{k\partial}$, at the kth order in the derivative expansion. This can be done by either working in a preferred fluid frame, or more generally by classifying frame-invariant scalar, vector, and tensor data.

(2) Find the particular solution to the anomaly-induced transport (if any); this fixes all terms in class A.

(3) Restrict to hydrostatic equilibrium. The (independent) nonvanishing scalar fields and transverse conserved vectors determine H_S and H_V, respectively (after factoring out terms which are related up to total derivatives), which parametrize the (Euclidean) partition function [9,10].

(4) Classify the number of tensor structures entering constitutive relations that survive the hydrostatic limit. Since they are to be determined from H_S and H_V, respectively, we should have a number of hydrostatic relations H_F. In general, the hydrostatic constrained transport coefficients are given as linear differential combinations of unconstrained ones.

(5) Determine the class L scalars that vanish in hydrostatic equilibrium $\bar{H}_S$ from the list of frame invariant scalars after throwing out terms in H_S (and those related by total derivatives).

(6) Find all solutions to class B and $\bar{H}_V$ terms at the desired order in the gradient expansion by classifying potential tensor structures $\{\mathcal{N}, \mathcal{X}, \mathcal{S}\}$ and $\{\mathfrak{C}_{\mathcal{N}}, \mathfrak{C}_{\mathcal{X}}, \mathfrak{C}_{\mathcal{S}}\}$, respectively. We have now solved for the adiabatic part of hydrodynamics.

(7) The remainder of transport is dissipative and contributes to $\Delta \neq 0$. Class D is subdivided into two classes: terms constrained by the second law lie in class D_v, while those in class D_s contribute subdominantly to entropy production and are arbitrary. The goal at this stage is to isolate the D_v terms. Fortunately, they only show up at the leading order in the gradient expansion ($k = 1$); for $k \geq 1$, all dissipative terms are in class D_s (cf. Refs. [3,4]).

(8) Finally, class D_s can be written in terms of dissipative tensor structures using the same formalism employed for class B, except now we pick a different symmetry structure to ensure $\Delta \neq 0$.

Steps (1)–(6) can be implemented straightforwardly in the $\mathrm{U}(1)_\mathsf{T}$ invariant $\mathcal{L}_T$, but we will exemplify this algorithm by a more pedestrian approach below. In Table I, we provide a classification of transport for few hydrodynamic systems up to second order in gradient expansion.

Example: Weyl invariant neutral fluid.—To illustrate our construction consider a (parity-even) Weyl invariant neutral fluid which has been studied extensively in the holographic context [18–20]. Weyl invariance implies that the stress tensor must be traceless and built out of Weyl covariant tensors. Our classification suggests the following constitutive relation written in a basis adapted to the eightfold classification [21]:

TABLE I. Transport taxonomy for some simple (parity-even) fluid systems in $d \geq 4$. The fluid type refers to whether we describe pure energy-momentum transport (neutral) or transport with a single global symmetry (charged). We have indicated the derivative order at which we are working by $k\partial$.

Fluid Type	Total	H_S	$\bar{H}_S$	H_F	H_V	A	B	$\bar{H}_V$	D
Neutral 1∂	2	0	0	0	0	0	0	0	2
Neutral 2∂	15	3	2	5	0	0	2	0	3
Weyl neutral 2∂	5	2	1	0	0	0	1	0	1
Charged 1∂	5	0	0	2	0	0	0	0	3
Charged 2∂	51	7	5	17	0	0	11	2	9

$$\begin{aligned} T^{\mu\nu} &= p(du^\mu u^\nu + g^{\mu\nu}) - 2\eta\sigma^{\mu\nu} \\ &\quad + (\lambda_1 - \kappa)\sigma^{<\mu\alpha}\sigma_\alpha^{\nu>} + (\lambda_2 + 2\tau - 2\kappa)\sigma^{<\mu\alpha}\omega_\alpha^{\nu>} \\ &\quad + \tau(u^\alpha \mathcal{D}_\alpha^{\mathcal{W}}\sigma^{\mu\nu} - 2\sigma^{<\mu\alpha}\omega_\alpha^{\nu>}) + \lambda_3\omega^{<\mu\alpha}\omega_\alpha^{\nu>} \\ &\quad + \kappa(C^{\mu\alpha\nu\beta}u_\alpha u_\beta + \sigma^{<\mu\alpha}\sigma_\alpha^{\nu>} + 2\sigma^{<\mu\alpha}\omega_\alpha^{\nu>}). \end{aligned} \tag{8}$$

To obtain this, note that for a neutral fluid there are no anomalies, so $A = 0$. At first order there is only a class D term $\eta\sigma_{\mu\nu}$, which contributes to $\Delta = 2\eta\sigma^2$, leading to $\eta \geq 0$ (shear viscosity is non-negative). At second order we have two hydrostatic scalars $\omega_{\mu\nu}\omega^{\nu\mu}$ and ${}^{\mathcal{W}}R$; hence, $H_S = 2$ corresponding to λ_3 and κ terms. As $\sigma_{\mu\nu}$ vanishes in hydrostatics only two tensors survive the limit; thus, there are no constraints, $H_F = 0$. There are no transverse vectors and so $H_V = \bar{H}_V = 0$. Surprisingly, $(\lambda_2 + 2\tau - 2\kappa)\sigma^{\langle\mu\alpha}\omega_\alpha^{\nu\rangle}$ is a class B term—it can be obtained from $\mathcal{N}^{[(\mu\nu)|(\alpha\beta)]} \sim (\lambda_2 + 2\tau - 2\kappa)(\omega^{\mu\alpha}P^{\nu\beta} + \text{permutations})$. There is one nonhydrostatic scalar σ^2, which is in $\bar{H}_S$ corresponding to the τ term above. This leaves us with one class D term which can be inferred to be $(\lambda_1 - \kappa)\sigma^{\langle\mu\alpha}\sigma_\alpha^{\nu\rangle}$. Its contribution to entropy production is $\nabla_\mu J_S^\mu \sim (\lambda_1 - \kappa)\sigma_{\alpha\nu}\sigma^{\nu\beta}\sigma^\alpha{}_\beta$. This being subdominant to the leading order $\eta\sigma^2$ entropy production, it follows that $(\lambda_1 - \kappa)$ belongs to class D_s.

While this completes the classification, we note one rather intriguing fact. For holographic fluids dual to two derivative gravity, the second order constitutive relations (cf. Ref. [20]) can be derived from a class L Lagrangian:

$$\begin{aligned} \mathcal{L}^{\mathcal{W}} &= -\frac{1}{16\pi G_{d+1}}\left(\frac{4\pi T}{d}\right)^{d-2} \\ &\quad \times \left[\frac{{}^{\mathcal{W}}R}{(d-2)} + \frac{1}{2}\omega^2 + \frac{1}{d}\mathrm{Har}\left(\frac{2}{d} - 1\right)\sigma^2\right], \end{aligned} \tag{9}$$

where $\mathrm{Har}(x) = \gamma_E + \Gamma'(x)/\Gamma(x)$ is the harmonic number function (γ_E is Euler's constant). The first two terms are in H_S while $\sigma^2 \in \bar{H}_S$, and they give contributions to each of the five second order transport coefficients. We therefore have two relations:

$$\lambda_2 + 2\tau - 2\kappa = 0, \qquad \lambda_1 - \kappa = 0. \tag{10}$$

Eliminating κ, we have $\tau = \lambda_1 - \frac{1}{2}\lambda_2$, which was argued to, in fact, be a universal property of two derivative gravity theories [23]. Curiously, the first relation is also obeyed in kinetic theory to the orders in which computations are available [24]. We advance this as the evidence that our eightfold classification explains various hitherto unexplained coincidences in both perturbative transport calculations and nonperturbative results from the AdS/CFT correspondence.

The second relation in Eq. (10) suggests that the subleading entropy production from $(\lambda_1 - \kappa)$ is absent in AdS black holes [25]. Inspired by earlier observations regarding the lower bound of shear viscosity $\eta/s \geq 1/4\pi$ [26], we conjecture that holographic fluids obtained in the long-wavelength limit of strongly interacting quantum systems obey a principle of minimal dissipation. The fluid-gravity correspondence provides the shortest path in the eightfold classification: AdS black holes scramble fast to thermalize, but are slow to dissipate.

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