PRL **114,** 197001 (2015) PHYSICAL REVIEW LETTERS week ending
15 MAY 2015

Coexistence of Charge-Density-Wave and Pair-Density-Wave Orders in Underdoped Cuprates

Yuxuan Wang,[1] Daniel F. Agterberg,[2] and Andrey Chubukov[3]
[1]*Department of Physics, University of Wisconsin, Madison, Wisconsin 53706, USA*
[2]*Department of Physics, University of Wisconsin-Milwaukee, Milwaukee, Wisconsin 53211, USA*
[3]*William I. Fine Theoretical Physics Institute and School of Physics and Astronomy, University of Minnesota, Minneapolis, Minnesota 55455, USA*

(Received 27 January 2015; published 15 May 2015)

We analyze incommensurate charge-density-wave (CDW) and pair-density-wave (PDW) orders with transferred momenta $(\pm Q, 0)/(0, \pm Q)$ in underdoped cuprates within the spin-fermion model. Both orders appear due to an exchange of spin fluctuations before magnetic order develops. We argue that the ordered state with the lowest energy has nonzero CDW and PDW components with the same momentum. Such a state breaks C_4 lattice rotational symmetry, time-reversal symmetry, and mirror symmetries. We argue that the feedback from CDW/PDW order on fermionic dispersion is consistent with ARPES data. We discuss the interplay between the CDW/PDW order and $d_{x^2-y^2}$ superconductivity and make specific predictions for experiments.

DOI: 10.1103/PhysRevLett.114.197001 PACS numbers: 74.20.-z, 74.25.Jb, 74.62.Dh, 74.72.-h

Introduction.—The search for competitors to $d_{x^2-y^2}$ superconductivity (d-SC) in underdoped cuprates has gained strength over the last few years due to mounting experimental evidence that some form of electronic charge order spontaneously emerges below a certain doping and competes with d-SC (Refs. [1–16]) The two most frequently discussed candidates for electronic order are incommensurate charge-density-wave (CDW) order (Refs. [17–32]) and incommensurate pair-density-wave (PDW) order, which is a SC order with a finite Cooper pair momentum $\mathbf{Q}$ (Refs. [33–38]). Other potential candidates are loop current order [39] and CDW order with momentum near (π, π) (Ref. [40]).

CDW order in underdoped cuprates was proposed some time ago [17] and has been analyzed in detail by several groups in the last few years within the spin-fluctuation formalism [19,20,22–24,26–28] and within the $t - J$ model [18,21]. The initial discussion was focused on near equivalence between d-SC and the d-wave charge bond order (BO) with momenta (Q, Q) along the zone diagonal [19,20,27], but charge order of this type has not been observed in experiments. It was later found [22,23,26,28] that the same magnetic model also displays a CDW order with momenta $(Q, 0)$ or $(0, Q)$, which is consistent with the range of CDW wave vectors extracted from experiments [1–6,9,10,41]. Such a CDW order is also consistent with experiments that detect the breaking of discrete rotational and time-reversal symmetries in a (T, x) range where competing order develops [11–16]. In particular, when spin-fermion coupling is strong enough, the CDW order develops in the form of a stripe and breaks C_4 lattice rotational symmetry. A stripe CDW order with $(Q, 0)/(0, Q)$ in turn gives rise to modulations in both charge density and charge current and breaks time-reversal and mirror symmetries [23,24,28,31].

The agreement with the data is encouraging, but two fundamental issues with the CDW order remain. First, within the mean-field approximation, T_{cdw} is smaller than the superconducting T_c [and also the onset temperature for the (Q, Q) order]. It has been conjectured that T_{cdw} may be enhanced by adding, e.g., phonons [17] or a nearest-neighbor Coulomb interaction [42], or by assuming that the CDW emerges from an already preexisting pseudogap [26,29]. T_{cdw} is also enhanced by fluctuations beyond mean field [23,24], but whether such enhancements are strong enough to make T_{cdw} larger than T_c remains to be seen. Second, the stripe CDW order cannot explain qualitative features of the ARPES data away from zone boundaries [36].

It has been argued [36] that ARPES experiments for all momentum cuts can be explained by assuming that the competing order is PDW rather than CDW. The PDW order was initially analyzed for doped Mott insulators [33,37,38], but it also emerges in the spin-fermion model [28] with the same momentum $(Q, 0)/(0, Q)$ as the CDW order, and its onset temperature T_{pdw} is close to T_{cdw} (the two become equivalent if one neglects the curvature of the fermionic dispersion at the hot spots [27,28]). Given that the PDW order explains the ARPES experiments, it seems logical to consider it as a candidate for competing order. Just like CDW, the PDW order develops in the form of a stripe and breaks C_4 lattice rotational symmetry [28,34] if, again, the coupling is strong enough. However, it does not naturally break time-reversal and mirror symmetries [35] (although it does so for a particular Fermi surface geometry [34]), and the mean-field T_{pdw} is also smaller than T_c for d-SC.

In this Letter we build on the results of the generic Ginzburg-Landau analysis [28] and propose how to resolve the partial disagreement with experiments for pure CDW or PDW orders. We first reiterate that pure CDW/PDW orders emerge in the forms of stripes only if the spin-fermion interaction g is strong enough. In practice, g has to be at least comparable to the upper energy cutoff of the spin-fermion model Λ (see details below). For smaller couplings the system develops a checkerboard order for which C_4 symmetry is preserved [43]. The spin-fermion model is a low-energy model and it is rigorously defined only when the coupling g is smaller than Λ. In this respect, stripe CDW or PDW orders emerge, only at the edge of the applicability of the model. Here we consider the spin-fermion model at smaller couplings, well within its applicability range, and allow both the CDW and the PDW order to develop. We show that the system develops a mixed CDW/PDW order in which a CDW component develops between hot fermions separated along, say, the Y direction and a PDW component develops between fermions separated along the X direction (see Fig. 1). Because the momentum carried by an order parameter is the transferred momentum for CDW and the total momentum for PDW, the CDW order along Y and the PDW order along X actually carry the same momentum $(0, Q)$. We argue that such a state further lowers its free energy by developing (via an emerging triple coupling) secondary homogeneous superconducting orders [28]. This effect favors the mixed CDW/PDW state over the pure checkerboard CDW or PDW states, which would otherwise all be degenerate. The mixed CDW/PDW state breaks C_4 symmetry because both orders carry either momentum $(Q, 0)$ or momentum $(0, Q)$, but not both, and it also breaks time-reversal and mirror symmetries as the pure stripe CDW order with $(Q, 0)$ or $(0, Q)$ does.

The presence of the PDW component is relevant for the interpretation of the ARPES data. Without it, the fermionic spectrum in the CDW phase would contain the lower energy branch, which never crosses Fermi level, and the upper energy branch, which would approach the Fermi level *from above* as the momentum cuts enter the arc region. As discussed in Ref. [36], this is inconsistent with the data [9] which show that the dispersion approaches the Fermi level *from below*. We show that the presence of the PDW component changes the structure of fermionic dispersion in such a way that now the lower branch crosses the Fermi level in the arc region (see Fig. 2), in full agreement with the ARPES experiments.

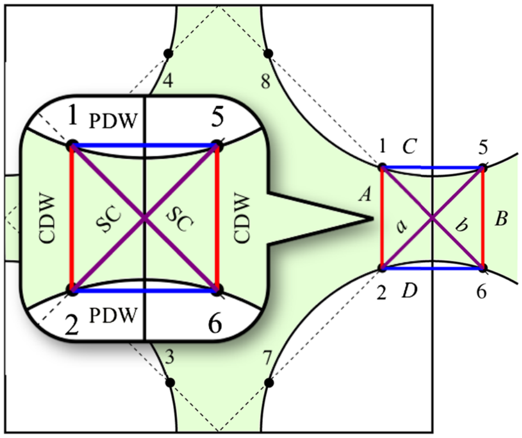

FIG. 1 (color online). The Brillouin zone, the Fermi surface, and the hot spots. We label bonds connecting hot spots as A, B, C, D, a, and b. (Inset) The structure of the mixed CDW/PDW state in one of the hot regions.

We also consider the interplay between the CDW/PDW order and d-SC and present the phase diagram in Fig. 3. The reduction of the superconducting T_c in the coexistence region with CDW/PDW is the obvious consequence of competition for the Fermi surface. A small drop (of order g/Λ) of T_c upon entering the coexistence region is the result of a weak first-order CDW/PDW transition. There exists, however, a more subtle feature of the phase diagram. Namely, a secondary SC order is generated by the CDW/PDW order which preserves the same sign of the gap along each quadrant of the Fermi surface. Below T_c for d-SC, this secondary superconducting order couples with the $d_{x^2-y^2}$ order, and the net result is the removal or shifting of the gap nodes. Simultaneously, the CDW order acquires an extra

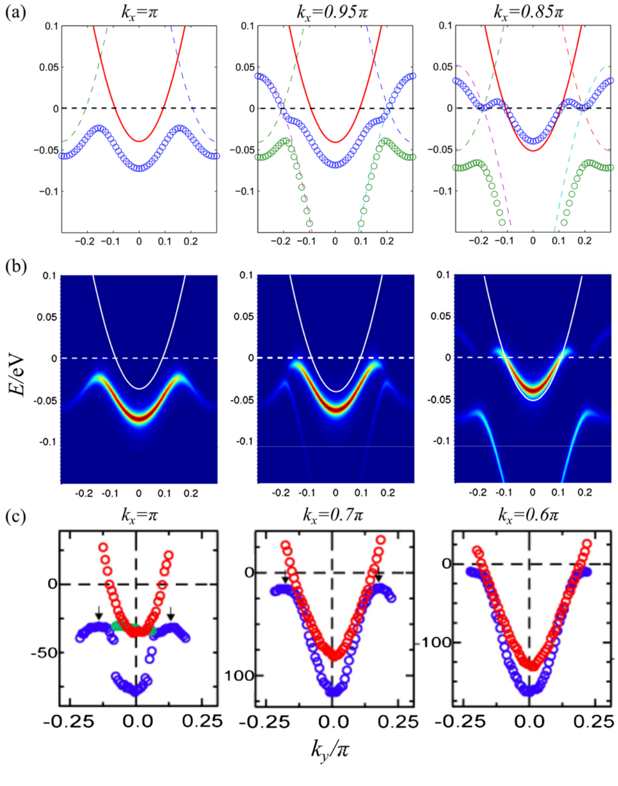

FIG. 2 (color online). Fermionic dispersion in the antinodal region in the presence of the mixed CDW/PDW order. (Panel (a)) The dispersion in the presence of the CDW/PDW order for various k_x's ($k_x = \pi$ corresponds to the cut along the Brillouin zone boundary). (Panel (b)) The spectral function. Thin line on both panels is the bare dispersion. (Panel (c)) Experimental data from Ref. [9] for various k_x's, for comparison. The experimental data have been taken below T_c and show a gapped dispersion in a wider range of $\pi - k_x$.

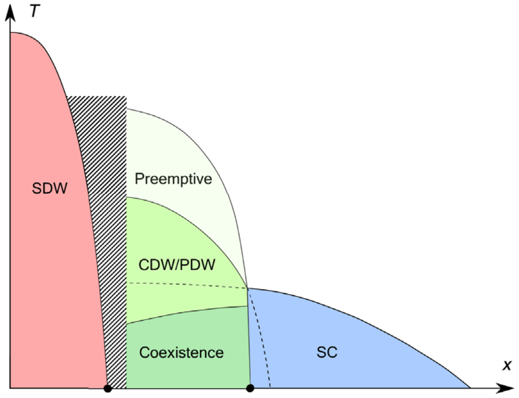

FIG. 3 (color online). The phase diagram. The transition into the CDW/PDW state is weakly first order and the superconducting T_c drops by a finite amount upon entering into the coexistence region. In the region labeled as "preemptive," discrete C_4 and time-reversal or mirror symmetries are broken but continuous U(1) translational symmetry (associated with the locking of the common phases of ρ_A and ρ_B) remains unbroken [23]. In the shaded region, Mott physics develops and the onset temperature of charge ordering shrinks.

component with an s-form factor; i.e., the magnitude of its s-wave portion increases. We propose to verify these through experiments.

The model.—We follow previous works [19,20,23,28] and consider an emerging charge order within the spin-fermion model [44]. This model describes interactions between itinerant electrons and their near-critical antiferromagnetic collective spin excitations in two spatial dimensions. Eight "hot" spots, defined as points on the Fermi surface separated by antiferromagnetic ordering momentum (π, π) (points 1–8 in Fig. 1), are the most relevant for destruction of a normal Fermi liquid state. The known instabilities of the spin-fermion model include d-SC (e.g., $\langle c_1 c_6 \rangle$; see Fig. 1) [19,45,46], charge BO with momenta $(\pm Q, \pm Q)$ (e.g., $\langle c_1^\dagger c_6 \rangle$) [19,20,27], CDW order with momenta $(0, \pm Q)$ and $(\pm Q, 0)$ (e.g., $\langle c_1^\dagger c_2 \rangle$) [23,26,30], and PDW order with momenta $(0, \pm Q)$ and $(\pm Q, 0)$ (e.g., $\langle c_1 c_2 \rangle$) [27,28]. The model has an approximate SU(2) particle-hole symmetry [19,20,27,28,32] which becomes exact once one linearizes the fermionic dispersion in the vicinity of the hot spots. This gives rise to near degeneracy between d-SC and the BO and between the CDW and the PDW.

The Ginzburg-Landau analysis.—We introduce four order parameters: Ψ for SC, Φ for the BO, ψ for the PDW, and ρ for the CDW respectively. SC and BO order parameters connect hot spots along diagonal bonds, which we label as a and b in Fig. 1, while the PDW and the CDW connect hot spots along vertical and horizontal bonds, which we label as A, B, C, and D. We define the CDW order parameter residing on bond A as $\rho_A \sim \langle c_1^\dagger c_2 \rangle$ and use analogous notations for other order parameters. The effective action is the sum of three terms:

$$\mathcal{S}_{\text{eff}} = \mathcal{S}_{\text{cdw/pdw}}[\rho, \psi] + \mathcal{S}_{\text{sc/bo}}[\Psi, \Phi] + \mathcal{S}_{\text{int}} \tag{1}$$

The $\mathcal{S}_{\text{cdw/pdw}}[\rho, \psi]$ term is our primary interest. Keeping the SU(2) symmetry exact, we follow Ref. [28] and combine the PDW and CDW orders on a given bond (say, bond A) into a 2×2 matrix order parameter

$$\Delta_A^{\mu\nu} \equiv \begin{pmatrix} \psi_A & \rho_A^* \\ -\rho_A & \psi_A^* \end{pmatrix} \equiv \sqrt{|\rho_A|^2 + |\psi_A|^2}\, U_A, \tag{2}$$

where $\rho_A \sim c_1^\dagger c_2$, $\psi_A \sim c_1 c_2$, and U_A is an SU(2) matrix "phase." The order parameters $\Delta_{B,C,D}$ and phases $U_{B,C,D}$ are similarly defined [see the Supplemental Material (SM) [47] for details]. Minimizing the free energy, we obtain $\Gamma \equiv \text{Tr}(U_A U_C^\dagger U_B U_D) = -2$, $\sqrt{|\rho_A|^2 + |\psi_A|^2} = \sqrt{|\rho_B|^2 + |\psi_B|^2} \equiv |\Delta_y|$, and $\sqrt{|\rho_C|^2 + |\psi_C|^2} = \sqrt{|\rho_D|^2 + |\psi_D|^2} \equiv |\Delta_x|$. Under these conditions, the CDW/PDW action becomes

$$\mathcal{S}_{\text{cdw/pdw}} = \frac{\alpha}{2}(|\Delta_x|^2 + |\Delta_y|^2) + \beta(|\Delta_x|^4 + |\Delta_y|^4) + (\tilde{\beta} - \bar{\beta})|\Delta_x|^2|\Delta_y|^2 + O(\Delta^6), \tag{3}$$

where $\alpha \sim \Lambda/v_F^2 \times (T - T_{\text{cdw}})/T_{\text{cdw}}$ and $T_{\text{cdw}} = T_{\text{pdw}} \sim g$ (Ref. [23]). The prefactors β, $\tilde{\beta}$, and $\bar{\beta}$ are determined by different convolutions of four fermionic propagators (the square diagrams [23,28,30]). At $g \ll \Lambda$ we have $\beta \sim 1/(v_F^2 \Lambda)$, $\tilde{\beta} \sim \log(\Lambda/g)/(v_F^2 \Lambda)$, and $\bar{\beta} \sim (\Lambda/g)/(v_F^2 \Lambda)$. We see that $\bar{\beta}$ is the largest term; hence, the action (3) is minimized when $|\Delta| \equiv |\Delta_x| = |\Delta_y|$. Because $\tilde{\beta} - \bar{\beta} < 0$, the action is unbounded, which implies that the transition is first order and sixth-order terms (coming from six-leg diagrams) have to be included to stabilize the order. Including these terms we obtain a first order into the CDW/PDW state at $T_{\text{cdw/pdw}} = T_{\text{cdw}}(1 + O(g/\Lambda))$. We emphasize that this temperature is higher than the one for a pure CDW (or PDW) transition.

The constraint $\Gamma \equiv \text{Tr}(U_A U_C^\dagger U_B U_D) = -2$ leaves the ground state hugely degenerate—the order parameter manifold is $\text{SO}(4) \times \text{SO}(4)$ (Ref. [28]). This manifold includes pure CDW and pure PDW checkerboard states and mixed CDW/PDW states. To select the actual ground state configuration, we note that, if the CDW and PDW orders have components which carry the same momentum $\mathbf{Q}$, the free energy is further lowered by creating a secondary order whose magnitude is a product of the CDW and PDW order parameters. This secondary order is a homogeneous SC with an equal sign of the gap along each quadrant of the Fermi surface (FS) [28]. One can straightforwardly check that the reduction of the free energy is maximal when, in a nominally checkerboard state, CDW occurs along vertical bonds and PDW occurs along horizontal bonds or vice versa; i.e., each order develops in the form of a stripe. This corresponds to either $\psi_{A,B} = \rho_{C,D} = 0$ (as in the inset of Fig. 1) or $\psi_{C,D} = \rho_{A,B} = 0$; the choice breaks C_4 lattice rotation symmetry. Furthermore, the stripe

CDW order parameters ρ_A and ρ_B and the PDW order parameters ψ_C and ψ_D get separately coupled by fermions away from hot spots, and the coupling between ρ_A and ρ_B locks the relative phase of ρ_A and ρ_B such that $\rho_B = \pm i\rho_A$ (Ref. [23]). The choice of the sign breaks time-reversal and mirror symmetries. The coupling between ψ_C and ψ_D does not lock their phases.

Feedback from CDW/PDW order on fermions.—We now show that the feedback from stripe CDW/stripe PDW order on the fermionic dispersion at $k \sim (\pi, 0)$, taken as a function of k_y for various $k_x = \pi - \delta k_x$, yields results in quite reasonable agreement with the ARPES data [9,10]. Previous studies have shown [23] that a pure CDW order can explain the ARPES spectrum for a cut along the Brillouin zone (BZ) boundary, but not for cuts that are closer to the BZ center (see Refs. [36,47]). To obtain the dispersion along various cuts in the presence of both the CDW and the PDW, we have extended our analysis of the CDW/PDW order to a finite momentum range away from the hot spots. We find that at the BZ boundary, the CDW order has a larger amplitude due to better FS nesting, but the PDW component increases as the cuts move towards the hot spots. We present the details in the SM and show the results in Fig. 2. There are three key features in our scenario that are qualitatively consistent with the experiments: (1) at the BZ boundary ($k_x = \pi$), the locus of minimum excitation energy shifts from k_F to a larger value $k_G \approx Q/2$, where Q is the CDW momentum, (2) as k_x decreases, the excitation approaches the Fermi level from below, and (3) at k_x when the Fermi arc emerges, the fermionic dispersion becomes flat for $|k_y| > k_F$. These features are also reproduced by the pure PDW order [36] and from a spatially homogeneous self-energy arising from a d-wave CDW order peaked at (π, π) [21]. However, neither of these scenarios immediately explains the observation of broken time-reversal symmetry or a CDW order with small incommensurate momentum. To obtain quantitative agreement with the experiments, we would need to know how the CDW and PDW order parameters depend on frequency. This would require one to model the bare dispersion far away from k_F and solve complex integral equations for frequency-dependent order parameters.

Interplay between CDW/PDW order and $d_{x^2-y^2}$ superconductivity.—We next consider other terms in the effective action in Eq. (1). The term $\mathcal{S}_{\mathrm{sc/bo}}$ has been analyzed in Refs. [20,27,30]. When SU(2) symmetry is exact, d-SC and BO are degenerate and the action has four Goldstone modes. Once SU(2) symmetry is broken by FS curvature, only d-SC order develops below T_c. We assume that this is the case and keep only the d-SC component Ψ in $\mathcal{S}_{\mathrm{sc/bo}}$; i.e., we reduce it to $\mathcal{S}_{\mathrm{sc/bo}} = \alpha_s|\Psi|^2 + \beta_s|\Psi|^4$, with $\alpha_s \sim \Lambda/v_F^2 \times (T - T_c)/T_c$, $T_c \sim g$, and $\beta_s \sim \Lambda/(v_F g)^2$. The coupling between the CDW/PDW and d-SC orders is again obtained by evaluating the square diagrams. The calculation yields $\mathcal{S}_{\mathrm{int}} = \beta'|\Delta|^2|\Psi|^2$ with $\beta' \sim 1/(v_F^2 g)$. Note that the magnitude of the coupling is phase sensitive; hence, the phase locking between ρ_A and ρ_B at $\pm\pi/2$ is important (see the SM for details).

The analysis of the full action is straightforward and we show the results in Fig. 3. The mean-field temperature $T_{\mathrm{cdw/pdw}} \geq T_{\mathrm{cdw}}$ is comparable to T_c near the SDW boundary but is enhanced by fluctuations [23,26,29]. We assume that this enhancement lifts $T_{\mathrm{cdw/pdw}}$ above T_c at large ξ. Because the CDW/PDW transition is first order, T_c jumps upon entering into the coexistence region, but the jump is again small in g/Λ. Similar behavior has been recently observed in Fe pnictides [51]. At small T, the CDW/PDW and d-SC orders coexist.

The phase diagram in Fig. 3 is similar to that for the pure CDW order [23], but there are some extra features. First, the combination of CDW/PDW orders induces a secondary SC order [28] with a nonzero gap along the zone diagonal (s wave or d_{xy}). In the coexistence region with d-SC, this order Ψ_s couples with the d-SC order Ψ and, as a result, gap nodes either get shifted (the $d + s$ state) or removed (the $d + e^{i\theta}s$ state). A similar coupling has been examined in the context of the Fe pnictides [52]. A finite gap along zone diagonals has been observed by ARPES at doping $x < 0.1$ (Ref. [53]) and has also been inferred from Raman spectroscopy [54]. Second, by the same logic, the d-SC and PDW orders induce a secondary s-wave CDW order with the same momentum as the primary one. We propose to search for the SC gap opening or node shifting, and to examine the s-component of the CDW order in the coexistence region.

Conclusions.—In this Letter we proposed a state with unidirectional CDW and PDW orders which carry the same momentum. We argued that this state is a member of the ground state manifold of the low-energy spin-fermion model and that its energy is further reduced by induction of a secondary SC order. We further argued that the CDW/PDW state has a number of features consistent with previous experiments: it breaks both C_4 and time-reversal symmetry and the feedback from the CDW/PDW order on fermions reproduces the ARPES data from the BZ boundary to the tip of the Fermi arc. The transition into the CDW/PDW state is weakly first order and occurs at a higher transition temperature than that for a pure unidirectional CDW or PDW order. We considered the interplay between the CDW/PDW order and d-SC and found that a SC gap becomes nonzero along zone diagonals. We proposed to search for this gap opening in the region where a charge order and d-SC coexist.

We acknowledge our useful discussions with W. A. Atkinson, D. Chowdhury, E. Fradkin, S. Kivelson, P. A. Lee, C. Pépin, and S. Sachdev. Our work was supported by DOE Grant No. DE-FG02-ER46900 (A. C. and Y. W.) and by NSF Grant No. DMR-1335215 (D. F. A.).

[1] T. Wu, H. Mayaffre, S. Krämer, M. Horvatić, C. Berthier, W. N. Hardy, R. Liang, D. A. Bonn, and M.-H. Julien, Nature (London) **477**, 191 (2011); T. Wu, H. Mayaffre, S. Krämer, M. Horvatić, C. Berthier, W. N. Hardy, R. Liang, D. A. Bonn, and M.-H Julien, Nat. Commun. **6**, 6438 (2015).
[2] G. Ghiringhelli *et al.*, Science **337**, 821 (2012).
[3] A. J. Achkar, R. Sutarto, X. Mao, F. He, A. Frano, S. Blanco-Canosa, M. Le Tacon, G. Ghiringhelli, L. Braicovich, M. Minola, M. Moretti Sala, C. Mazzoli, R. Liang, D. A. Bonn, W. N. Hardy, B. Keimer, G. A. Sawatzky, and D. G. Hawthorn, Phys. Rev. Lett. **109**, 167001 (2012).
[4] R. Comin, A. Frano, M. M. Yee, Y. Yoshida, H. Eisaki, E. Schierle, E. Weschke, R. Sutarto, F. He, A. Soumyanarayanan, Y. He, M. Le Tacon, I. S. Elfimov, J. E. Hoffman, G. A. Sawatzky, B. Keimer, and A. Damascelli, Science **343**, 390 (2014).
[5] E. H. da Silva Neto, P. Aynajian, A. Frano, R. Comin, E. Schierle, E. Weschke, A. Gyenis, J. Wen, J. Schneeloch, Z. Xu, S. Ono, G. Gu, M. Le Tacon, and A. Yazdani, Science **343**, 393 (2014).
[6] K. Fujita, M. H. Hamidian, S. D. Edkins, C. K. Kim, Y. Kohsaka, M. Azuma, M. Takano, H. Takagi, H. Eisaki, S. Uchida, A. Allais, M. J. Lawler, E.-A. Kim, S. Sachdev, and J. C. Séamus Davis, Proc. Natl. Acad. Sci. U.S.A. **111**, E3026 (2014).
[7] J. M. Tranquada, G. D. Gu, M. Hücker, Q. Jie, H.-J. Kang, R. Klingeler, Q. Li, N. Tristan, J. S. Wen, G. Y. Xu, Z. J. Xu, J. Zhou, and M.v. Zimmermann, Phys. Rev. B **78**, 174529 (2008).
[8] E. Berg, E. Fradkin, S. A. Kivelson, and J. M. Tranquada, New J. Phys. **11**, 115004 (2009).
[9] R.-H. He *et al.*, Science **331**, 1579 (2011).
[10] M. Hashimoto, R.-H. He, K. Tanaka, J. P. Testaud, W. Meevasana, R. G. Moore, D. Lu, H. Yao, Y. Yoshida, H. Eisaki, T. P. Devereaux, Z. Hussain, and Z. X. Shen, Nat. Phys. **6**, 414 (2010).
[11] J. Xia, E. Schemm, G. Deutscher, S. A. Kivelson, D. A. Bonn, W. N. Hardy, R. Liang, W. Siemons, G. Koster, M. M. Fejer, and A. Kapitulnik, Phys. Rev. Lett. **100**, 127002 (2008); H. Karapetyan, J. Xia, M. Hücker, G. D. Gu, J. M. Tranquada, M. M. Fejer, and A. Kapitulnik, Phys. Rev. Lett. **112**, 047003 (2014).
[12] Y. Sidis and P. Bourges, in *10th International Conference on Materials and Mechanisms of Superconductivity (M2 S-X)*, Journal of Physics: Conference Series Vol. 449 (Institute of Physics, Bristol, England, 2013); Y. Li, V. Balédent, G. Yu, N. Barišić, K. Hradil, R. A. Mole, Y. Sidis, P. Steffens, X. Zhao, P. Bourges, and M. Greven, Nature (London) **468**, 283 (2010).
[13] Y. Lubashevsky, L. D. Pan, T. Kirzhner, G. Koren, and N. P. Armitage, Phys. Rev. Lett. **112**, 147001 (2014).
[14] Y. Ando, K. Segawa, S. Komiya, and A. N. Lavrov, Phys. Rev. Lett. **88**, 137005 (2002).
[15] V. Hinkov, P. Bourges, S. Pailhès, Y. Sidis, A. Ivanov, C. D. Frost, T. G. Perring, C. T. Lin, D. P. Chen, and B. Keimer, Nat. Phys. **3**, 780 (2007).
[16] O. Cyr-Choinière, G. Grissonnanche, S. Dufour-Beauséjour, S. Badoux, B. Michon, J. Day, D. A. Bonn, W. N. Hardy, R. Liang, N. Doiron-Leyraud, and L. Taillefer (private communication).
[17] C. Castellani, C. Di Castro, and M. Grilli, Phys. Rev. Lett. **75**, 4650 (1995); A. Perali, C. Castellani, C. Di Castro, and M. Grilli, Phys. Rev. B **54**, 16216 (1996).
[18] S. Chakravarty, R. B. Laughlin, D. K. Morr, and C. Nayak, Phys. Rev. B **63**, 094503 (2001).
[19] M. A. Metlitski and S. Sachdev, Phys. Rev. B **82**, 075128 (2010).
[20] K. B. Efetov, H. Meier, and C. Pépin, Nat. Phys. **9**, 442 (2013); H. Meier, C. Pepin, M. Einenkel, and K. B. Efetov, Phys. Rev. B **89**, 195115 (2014); K. B. Efetov (private communication).
[21] A. Greco and M. Bejas, Phys. Rev. B **83**, 212503 (2011).
[22] S. Sachdev and R. La Placa, Phys. Rev. Lett. **111**, 027202 (2013).
[23] Y. Wang and A. V. Chubukov, Phys. Rev. B **90**, 035149 (2014).
[24] A. M. Tsvelik and A. V. Chubukov, Phys. Rev. B **89**, 184515 (2014).
[25] A. Melikyan and M. R. Norman, Phys. Rev. B **89**, 024507 (2014).
[26] D. Chowdhury and S. Sachdev, Phys. Rev. B **90**, 134516 (2014).
[27] C. Pépin, V. S. de Carvalho, T. Kloss, and X. Montiel, Phys. Rev. B **90**, 195207 (2014).
[28] Y. Wang, D. F. Agterberg, and A. V. Chubukov, Phys. Rev. B **91**, 115103 (2015).
[29] S. Bulut, W. A. Atkinson, and A. P. Kampf, Phys. Rev. B **88**, 155132 (2013); W. A. Atkinson, A. P. Kampf, and S. Bulut, New J. Phys. **17**, 013025 (2015).
[30] D. Chowdhury and S. Sachdev, Phys. Rev. B **90**, 245136 (2014).
[31] Y. Wang, A. V. Chubukov, and R. Nandkishore, Phys. Rev. B **90**, 205130 (2014).
[32] T. Kloss, X. Montiel, and C. Pépin, arXiv:1501.05324.
[33] E. Fradkin and S. A. Kivelson, Nat. Phys. **8**, 864 (2012); E. Fradkin, S. A. Kivelson, and J. M. Tranquada, arXiv:1407.4480 [Rev. Mod. Phys. (to be published)].
[34] D. F. Agterberg, D. S. Melchert, and M. K. Kashyap, Phys. Rev. B **91**, 054502 (2015).
[35] D. F. Agterberg and H. Tsunetsugu, Nat. Phys. **4**, 639 (2008).
[36] P. A. Lee, Phys. Rev. X **4**, 031017 (2014).
[37] T. Senthil and P. A. Lee, Phys. Rev. Lett. **103**, 076402 (2009).
[38] P. Corboz, T. M. Rice, and M. Troyer, Phys. Rev. Lett. **113**, 046402 (2014).
[39] Z. Wang, G. Kotliar, and X.-F. Wang, Phys. Rev. B **42**, 8690 (1990); C. M. Varma, Phys. Rev. B **55**, 14554 (1997).
[40] S. Chakravarty, R. B. Laughlin, D. K. Morr, and C. Nayak, Phys. Rev. B **63**, 094503 (2001).
[41] N. Doiron-Leyraud, S. Badoux, S. René de Cotret, S. Lepault, D. LeBoeuf, F. Laliberté, E. Hassinger, B. J. Ramshaw, D. A. Bonn, W. N. Hardy, R. Liang, J.-H. Park, D. Vignolles, B. Vignolle, L. Taillefer, and C. Proust, Nat. Commun. **6**, 6034 (2015).
[42] A. Allais, J. Bauer, and S. Sachdev, Phys. Rev. B **90**, 155114 (2014).
[43] More accurately, the system either develops a checkerboard order or a stripe order which preserves time-reversal and

mirror symmetries. The first necessarily develops in the hot spot model, while the second may develop due to contributions from fermions away from hot regions [22,23].

[44] Ar. Abanov, A. V. Chubukov, and M. A. Finkelstein, Europhys. Lett. **54**, 488 (2001); Ar. Abanov, A. V. Chubukov, and J. Schmalian, Adv. Phys. **52**, 119 (2003).

[45] Y. Wang and A. V. Chubukov, Phys. Rev. Lett. **110**, 127001 (2013).

[46] Y. Wang and A. V. Chubukov, Phys. Rev. B **88**, 024516 (2013).

[47] See Supplemental Material at http://link.aps.org/supplemental/10.1103/PhysRevLett.114.197001, which includes Refs. [48–50], for details of the calculation.

[48] D. Chowdhury and S. Sachdev, Phys. Rev. B **90**, 134516 (2014).

[49] L. E. Hayward, D. G. Hawthorn, R. G. Melko, and S. Sachdev, Science **343**, 1336 (2014).

[50] W. D. Wise, M. C. Boyer, K. Chatterjee, T. Kondo, T. Takeuchi, H. Ikuta, Y. Wang, and E. W. Hudson, Nat. Phys. **4**, 696 (2008).

[51] Note in passing that a similar situation holds in doped Fe pnictides; see A. E. Böhmer *et al.*, arXiv:1412.7038; J. Kang, X. Wang, A. V. Chubukov, and R. M. Fernandes, Phys. Rev. B **91**, 121104(R) (2015).

[52] A. Hinojosa, R. M. Fernandes, and A. V. Chubukov, Phys. Rev. Lett. **113**, 167001 (2014).

[53] I. M. Vishik, M. Hashimoto, R.-H. He, W.-S. Lee, F. Schmitt, D. Lu, R. G. Moore, C. Zhang, W. Meevasana, T. Sasagawa *et al.*, Proc. Natl. Acad. Sci. U.S.A. **109**, 18332 (2012); E. Razzoli, G. Drachuck, A. Keren, M. Radovic, N. C. Plumb, J. Chang, Y.-B. Huang, H. Ding, J. Mesot, and M. Shi, Phys. Rev. Lett. **110**, 047004 (2013); Y. Peng, J. Meng, D. Mou, J. He, L. Zhao, Y. Wu, G. Liu, X. Dong, S. He, J. Zhang *et al.*, Nat. Commun. **4**, 2459 (2013); Y.-M. Lu, T. Xiang, and D.-H. Lee, Nat. Phys. **10**, 634 (2014).

[54] S. Sakai, S. Blanc, M. Civelli, Y. Gallais, M. Cazayous, M.-A. Méasson, J. S. Wen, Z. J. Xu, G. D. Gu, G. Sangiovanni, Y. Motome, K. Held, A. Sacuto, A. Georges, and M. Imada, Phys. Rev. Lett. **111**, 107001 (2013).