Orbital Angular Momentum and Spectral Flow in Two-Dimensional Chiral Superfluids

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(Received 8 October 2014; published 13 May 2015)

We study the orbital angular momentum (OAM) L_z in two-dimensional chiral $(p_x + ip_y)^\nu$-wave superfluids (SFs) of N fermions on a disk at zero temperature, in terms of spectral asymmetry and spectral flow. It is shown that $L_z = \nu N/2$ for any integer ν, in the Bose-Einstein condensation regime. In contrast, in the BCS limit, while the OAM is $L_z = N/2$ for the $p + ip$-wave SF, for chiral SFs with $\nu \geq 2$, the OAM is remarkably suppressed as $L_z = N \times O(\Delta_0/\varepsilon_F) \ll N$, where Δ_0 is the gap amplitude and ε_F is the Fermi energy. We demonstrate that the difference between the $p + ip$-wave SF and the other chiral SFs in the BCS regimes originates from the nature of edge modes and related depairing effects.

DOI: 10.1103/PhysRevLett.114.195301 PACS numbers: 67.30.H-, 74.20.-z

The orbital angular momentum (OAM) L_z of chiral superfluids (SFs) of fermions is a fundamental problem which has been under intense investigation over several decades [1–21]. The OAM is a direct manifestation of the broken chiral symmetry which was explicitly observed in the A phase of liquid ^{3}He [22]. It is also closely related to the edge current, which has not been experimentally observed so far in Sr_2RuO_4 despite the expectation from the $p + ip$-wave SF picture [23–25].

Most of the existing studies on the OAM have focused on $p + ip$-wave SFs. However, higher-order chiral SFs such as $d + id$, $f + if$, …-wave ones are also of interest [26,27] and have potential applications to candidates for chiral superconductors, such as UPt_3 [28], URu_2Si_2 [29], and SrPtAs [30]. We focus on fundamental chiral SFs with the pairing symmetry $\sim(p_x + ip_y)^\nu$ which can be classified by the integer angular momentum ν of each Cooper pair: $\nu = 1$ corresponds to $p + ip$, $\nu = 2$ to $d + id$, and so on. In fact, as we will demonstrate in this Letter, there is an unexpected fundamental difference between the $p + ip$-wave and higher-order chiral SFs with respect to the OAM; thus, it is essential to consider the higher-order ones as well, for a complete understanding of the problem.

The main issue with the OAM is that different viewpoints lead to different predictions for it, resulting in an apparent paradox [18]. One argument is that, since each Cooper pair has the OAM ν, $L_z = \nu N/2$ where N is the total number of fermions. There is, however, a different argument starting from the normal (nonsuperconducting) Fermi liquid, which has $L_z = 0$: since only the low-energy fermions near the original Fermi surface would be affected, L_z should be suppressed as $(\Delta_0/\varepsilon_F)^\gamma N/2$ with $\gamma > 0$, where Δ_0 is the pairing gap amplitude and ε_F is the Fermi energy.

Of course, the analysis did not stop at the hand-waving arguments, and many calculations have been carried out based on various schemes, leading to different results. We note that, in the limit of strong pairing of fermions, the superfluid phase may be understood as a result of Bose-Einstein condensation (BEC) of bosonic molecules. In this limit, it would be natural to expect that $L_z = \nu N/2$, since each bosonic molecule carries the OAM ν. However, this does not necessarily imply that the same value of L_z persists in the regime where the superfluid is described by Bardeen-Cooper-Schrieffer (BCS) theory. In fact, the "weak-pairing" chiral SF in the BCS regime is a topological superfluid with gapless edge states, while the "strong-pairing" chiral SF in the BEC regime is nontopological [31]. Thus, they are distinct superfluid phases and could have very different values of L_z. Besides, experimental investigation of the problem is difficult and there have been very few reports so far [32,33]. Therefore, the long-standing paradox is not yet resolved even for the $p + ip$-wave SF, let alone for the higher-order ones with $\nu \geq 2$.

In this study, we investigate the problem in the simplest ideal setting: two-dimensional (2D) chiral SFs confined on a completely circular disk with a specular wall at zero temperature, in the framework of a Bogoliubov–de Gennes (BdG) Hamiltonian. For simplicity, we assume that the d vector is $\boldsymbol{d} = (0, 0, d_z)$ for the triplet states so that both the singlet states and the triplet states can be discussed in a parallel way; our analysis is also applicable to the spinless fermions with slight modifications. We consider the Hamiltonian $\hat{H} = \int d^2x \psi_\sigma^\dagger[(p_x^2 + p_y^2)/2m_0 + V - \mu]\psi_\sigma + \int d^2x \psi_\uparrow^\dagger \Delta(p_x + ip_y)^\nu \psi_\downarrow^\dagger + (\text{H.c.})$, where $p_j = -i\partial/\partial x_j$, m_0 is the fermion mass, and μ is the chemical potential. $V(r)$ describes the wall of the container and is chosen to be $V(r < R) = 0$ and $V(r > R) = \infty$ with a radius R. The pairing gap amplitude for this Hamiltonian is given as $\Delta_0 = k_F{}^\nu \Delta$, where k_F is the Fermi momentum. There is no texture in this model.

The field operator is expanded in terms of a single particle basis as $\psi_\sigma(\boldsymbol{r}) = \sum_{nl} c_{nl\sigma}\varphi_{nl}(\boldsymbol{r})$ where φ satisfies $[(p_x^2+p_y^2)/2m_0 + V(r) - \mu]\varphi_{nl} = \varepsilon_{nl}\varphi_{nl}$ [8,10]. Then, the Hamiltonian becomes

$$\hat{H} = \sum_l \sum_{nn'} \begin{bmatrix} c^\dagger_{n,l+\nu,\uparrow} \\ c_{n,-l,\downarrow} \end{bmatrix}^T \begin{bmatrix} \varepsilon_{n,l+\nu}\delta_{nn'} & \Delta^{(l)}_{nn'} \\ \Delta^{(l)*}_{n'n} & -\varepsilon_{n,-l}\delta_{nn'} \end{bmatrix} \begin{bmatrix} c_{n',l+\nu,\uparrow} \\ c^\dagger_{n',-l,\downarrow} \end{bmatrix}, \tag{1}$$

where $\Delta^{(l)}_{nn'} = \int \varphi^*_{n,l+\nu}\Delta(p_x+ip_y)^\nu \varphi^*_{n',-l}$, with an appropriate high-energy regularization. We denote the above matrix as $(H^{(l)}_{\rm BdG})_{nn'}$. The particle-hole symmetry connects different l sectors as $PH^{(l)}_{\rm BdG}P^{-1} = -H^{(-l-\nu)*}_{\rm BdG}$ where $P = \sigma_x(i\sigma_y)$ in the Nambu space for odd (even) ν, which implies that, although eigenvalues come in pairs, each of them lies in different l sectors.

The OAM L_z corresponds to the operator $\hat{L}_z = \int d^2x \psi^\dagger_\sigma (xp_y - yp_x)\psi_\sigma$, while the total particle number operator is given as $\hat{N} = \int d^2x\psi^\dagger_\sigma\psi_\sigma$. These operators are clearly defined for the present model and include all the possible contributions. Neither $\hat{L}_z$ nor $\hat{N}$ commutes with the BdG Hamiltonian (1) owing to the pairing term in the Hamiltonian and is not conserved. Nevertheless, as pointed out in Refs. [5,6], the combination

$$\hat{\mathcal{L}}_z \equiv \hat{L}_z - \frac{\nu}{2}\hat{N} = \sum_{nl\sigma}\left(l - \frac{\nu}{2}\right) c^\dagger_{nl\sigma}c_{nl\sigma} \tag{2}$$

commutes with the Hamiltonian (1) and thus is a conserved quantity. Physically, $\hat{\mathcal{L}}_z$ represents the correction to the OAM with respect to its "full" value $\nu N/2$. If the ground state belongs to the zero eigenvalue sector of $\hat{\mathcal{L}}_z = 0$, it follows that $L_z = \nu N/2$. However, $\hat{\mathcal{L}}_z$ could take different eigenvalues in the ground state, as it is clear by considering the limit of $\Delta \to 0$, where $L_z = 0$ and $\mathcal{L}_z = -\nu N/2$ hold.

Thus, the eigenvalue $\mathcal{L}_z$ of $\hat{\mathcal{L}}_z$ in the ground state is a nontrivial quantity. In fact, it can still be calculated exactly for the Hamiltonian (1). After the Bogoliubov transformation, the ground state $|{\rm GS}\rangle$ is simply the vacuum with respect to all the positive energy quasiparticles. The eigenvalue $\mathcal{L}_z$ for $|{\rm GS}\rangle$ can be obtained explicitly as

$$\mathcal{L}_z = -\frac{1}{2}\sum_l \left(l + \frac{\nu}{2}\right)\eta_l, \qquad \eta_l = \sum_m {\rm sgn} E^{(l)}_m, \tag{3}$$

where $\{E^{(l)}_m\}_{m\in\mathbb{N}}$ are eigenvalues of $H^{(l)}_{\rm BdG}$ and η_l is called the spectral asymmetry [6,34–36]. From this formula, it is clear that $\mathcal{L}_z$ can change only when there is a spectral flow; namely, some of the eigenvalues of $H^{(l)}_{\rm BdG}$ cross 0 as model parameters are varied.

We first discuss the $p + ip$-wave states ($\nu = 1$) for which the spectrum is particle-hole symmetric about $l = -1/2$:

$$\left\{E^{(l)}_m\right\}_{m\in\mathbb{N}} = \left\{-E^{(-l-1)}_m\right\}_{m\in\mathbb{N}}. \tag{4}$$

For simplicity, we treat the two parameters μ and Δ independently as in Refs. [8,10,31] for a discussion of the spectral flow. In the BEC regime, by numerically diagonalizing $H^{(l)}_{\rm BdG}$, we obtain a fully gapped spectrum as shown in Fig. 1(a) and find that $\eta_l = 0$ for all l. As a consequence, $\mathcal{L}_z = 0$ and thus follows $L_z = N/2$, as expected. However, the spectrum becomes less trivial if the system is in the BCS regime. There, a single edge mode with $E^{(l)}_{\rm edge} \propto -(l + 1/2)$ appears as seen in Fig. 1(c), reflecting the topological nature of the phase [6,26,27,31]. This edge mode is particle-hole symmetric by itself $E^{(l)}_{\rm edge} \sim -(l+1/2) \sim -E^{(-l-1)}_{\rm edge}$ and is hereafter called a particle-hole-symmetric (PHS) edge mode. We have numerically confirmed that $\eta_l = 0$ for all values of l, for a sufficiently large system. Namely, even when Δ_0 is arbitrarily small (but nonzero), we find $\mathcal{L}_z = 0$ in the thermodynamic limit $R \to \infty$, implying no reduction of the OAM: $L_z = N/2$ holds exactly.

A natural question arising here is why $\mathcal{L}_z$ remains 0 in the BCS regime, despite the quantum phase transition separating it from the BEC regime. Let us first consider the limit $\mu = -\infty$ with a fixed $\Delta > 0$, where $L_z = N = 0$ holds trivially. Thus, we find $\mathcal{L}_z = 0$ in this limit. Increasing μ, L_z and N acquire nonzero values. However, as long as there is no gap closing, $\mathcal{L}_z = 0$ and thus $L_z = N/2$ still hold. This gives a proof for the physical expectation that $L_z = N/2$ holds throughout the BEC regime, which belongs to the nontopological strong-pairing phase.

The value of $\mathcal{L}_z$ in the BCS regime (weak-pairing phase) is more subtle, due to the presence of the quantum phase

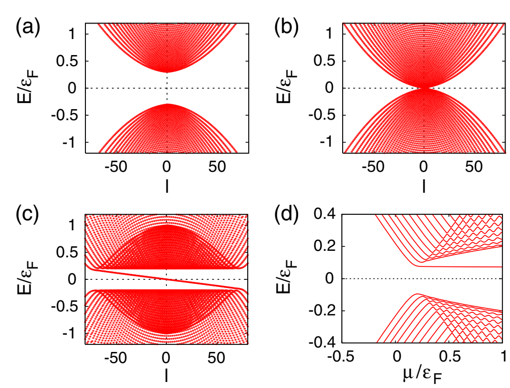

FIG. 1 (color online). Examples of spectra in the $p + ip$-wave SF when $k_F R = 80$, $k_F\Delta = 0.2\varepsilon_F$ for (a) $\mu = -0.3\varepsilon_F$ (BEC regime), (b) $\mu = 0$, and (c) $\mu = \varepsilon_F$ (BCS regime). (d) Evolution of the spectrum for fixed $l = -30$ as μ is changed.

transition at $\mu = 0$ in the thermodynamic limit. At the quantum phase transition, a gap closing is expected. However, a careful examination reveals that every eigenvalue keeps its sign when μ is continuously varied from $\mu = -\infty$ to $\mu = \varepsilon_F > 0$. In fact, for the system defined on a finite disk, the gap never closes. The gap does approach 0 at the quantum critical point and also inside the BCS regime giving rise to the gapless chiral edge mode but only in the thermodynamic limit. The change of the eigenvalues when μ is varied is shown in Fig. 1(d). The edge mode appears in the BCS regime as a set of eigenstates separated from bulk states. Although it converges to the linear gapless dispersion, all the eigenvalues corresponding to the edge mode for $l \le -1$ come off from the upper continuum of bulk eigenstates and remain positive. Likewise, all the edge mode eigenvalues for $l \ge 0$ come from the lower continuum and remain negative. Since all the eigenvalues depend continuously on μ, this is the only possible evolution to generate the PHS edge mode, under the particle-hole symmetry (4).

Therefore, although each of L_z and N changes from the trivial values $L_z = N = 0$, $\mathcal{L}_z = L_z - N/2 = 0$ always holds; any correction factor like $(\Delta_0/\varepsilon_F)^\gamma$ mentioned in the introduction cannot arise. We note that this is also true for a physical process where μ and Δ are simultaneously tuned to keep N constant. Our argument only relies on the formation of the PHS edge mode separated from the bulk eigenstates, which is valid for a sufficiently large system size. The conclusion of our analysis largely agrees with the recent related calculations on $p + ip$-wave SFs [10–13] but clarifies why L_z is exactly given by $N/2$ even for small Δ_0/ε_F.

Next, we move to the $d + id$-wave states for which the spectrum is particle-hole symmetric about $l = -1$. We have numerically confirmed that $\eta_l = 0$ for all l in the BEC regime and obtain $\mathcal{L}_z = 0$, i.e., $L_z = N$. On the other hand, a $d + id$-wave BCS state is known to have two nondegenerate edge modes at one boundary as visualized in Fig. 2(a) [37]. Each edge mode is particle-hole symmetric with the other branch but not symmetric by itself; we call them non-PHS edge modes. Their dispersion relations are given as $E^{(l)}_{\mathrm{edge}1,2} \propto -(l - l_{1,2})$, where $l_{1,2} \neq -1 (l_1 < l_2)$ are the "Fermi angular momenta" where the edge modes cross

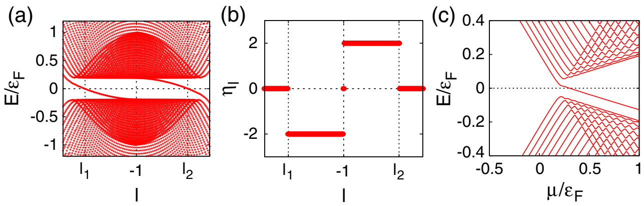

FIG. 2 (color online). Examples of (a) a spectrum in the $d + id$-wave SF and (b) the spectral asymmetry η_l for $k_F R = 80$, $k_F^2 \Delta = 0.2\varepsilon_F$, $\mu = \varepsilon_F$ (BCS regime). (c) Spectral flow for fixed $l = -30 (> l_1 = -56)$ as μ is changed.

the zero energy. The particle-hole symmetry requires $l_1 + 1 = -(l_2 + 1)$. Interestingly, for this spectrum, we find that some of the η_l's become nonvanishing. This can be understood in terms of the spectral flow staring from the BEC regime where $\eta_l = 0$. As in the case of the $p + ip$-wave SF, $\eta_l = 0$ remains valid up to the critical point, since the gap remains open in the entire BEC regime. As we move into the BCS regime, two non-PHS edge modes develop as in Fig. 2(a). During this evolution, the Fermi angular momenta evolve from $l = -1$ to nonvanishing values $l_{1,2}$. This induces spectral flows for the angular momenta l in the range $l_1 < l < l_2$, except at $l = -1$. An example of the spectral flow at a fixed value of l is shown in Fig. 2(c): η_l changes sign exactly when a Fermi angular momentum passes through this l.

This picture can also be confirmed in an analytic expression of the edge mode dispersions as functions of $\mu > 0$, in the limit of a large disk radius R [38]. In this limit, the edge of the disk corresponds to the boundary of a semi-infinite plane, with the momentum parallel to the boundary $k_\parallel$ related to the angular momentum l by $k_\parallel \simeq l/R$. In the BCS regime $0 < \mu$, the dispersion of the edge modes is given as $E^2_{\mathrm{edge}} = [\Delta_0^2 \varepsilon_F^2/(\varepsilon_F^2 + \Delta_0^2)](2k_\parallel^2/k_F^2 - \mu/\varepsilon_F)^2$, by solving the BdG equation [38]. The condition $E_{\mathrm{edge}} = 0$ then determines the Fermi wave vector $k_{F\parallel}$ of the edge modes as $k_{F\parallel}^2 = (k_F/\sqrt{2})^2 \mu/\varepsilon_F$. This indeed demonstrates that the Fermi wave vector $k_{F\parallel}$ evolves from 0 to a nonvanishing value, as μ is increased into the BCS regime $\mu > 0$. This confirms the spectral flow for $l_1 < l < l_2$ as found numerically for $\nu = 2$.

In fact, spectral flows, and $\eta_l \neq 0$ as a consequence, are common properties of the higher-order pairing states with $\nu \ge 2$ in the BCS regime. There, nonvanishing η_l are found numerically and can be understood generally in terms of edge modes: η_l changes by ± 2 when a non-PHS edge mode branch crosses zero energy, while a PHS edge mode does not contribute to η_l.

In the BCS regime for $\nu \ge 2$, nonvanishing η_l implies $\mathcal{L}_z \neq 0$. As a consequence, L_z is in fact strongly suppressed from the "full" value $\nu N/2$, in the BCS limit $\Delta_0 \ll \varepsilon_F$. To see this, we evaluate the actual value of $\mathcal{L}_z$ using Eq. (3), with the observations made above. By considering the limit of a large disk, the Fermi angular momenta l_j can be written in terms of the Fermi wave number parallel to the boundary $k_{F\parallel}^{(j)}$ as $l_j \simeq R k_{F\parallel}^{(j)}$. Within the quasiclassical formulation, which is legitimate for the BCS limit, we find $\sum_{j=1}^{\nu} (k_{F\parallel}^{(j)})^2 = \nu k_F^2/2$ [38]. Thus, in the leading order in N and Δ_0/ε_F, we obtain

$$\mathcal{L}_z \simeq -\frac{1}{2}\sum_{j=1}^{\nu} l_j{}^2 = -\frac{1}{2}\sum_{j=1}^{\nu} (R k_{F\parallel}^{(j)})^2 = -\frac{\nu N}{2}. \qquad (5)$$

Since $\mathcal{L}_z = L_z - \nu N/2$, the OAM is evaluated to be $L_z = N \times O(\Delta_0/\varepsilon_F)$ in the BCS limit for $\nu \ge 2$, where the $O(\Delta_0/\varepsilon_F)$ term represents possible additional contributions which are beyond the quasiclassical approximation. Indeed, numerical calculations of the OAM give $L_z/N \sim o(0.01)$ when $N \sim O(1000)$ for $\nu = 2, 3, 4$ in an extended range in the BCS regimes with $\Delta_0/\varepsilon_F \lesssim 0.2$, supporting the above quasiclassical analysis. Therefore, the naive evaluation $L_z = \nu N/2$ fails for the chiral SFs in the BCS regime with $\nu \geq 2$, even though it gives the correct value for the $p + ip$-wave states. That is, for $\nu \geq 2$, L_z is strongly suppressed as if in the naive weak-pairing picture where fermions only near the Fermi surface at $\Delta = 0$ contribute to L_z. However, our findings, in particular, the stark difference between the $p + ip$ and higher-order ($\nu \geq 2$) pairing cases, make it clear that the suppression cannot be understood by any of the arguments found in existing works. Our analysis is based on the robustness of the spectral asymmetry η_l and does not rely on assumptions and approximations used in the earlier papers, such as derivative expansions, which might fail to describe the correct physics, especially around boundaries where the edge modes are located [11]. We emphasize that the well-known topological protection of the existence of ν edge modes is not sufficient for determining the OAM, which depends on more detailed structures of the edge modes. In this sense, the OAM L_z is a surface-dependent quantity in the BCS regimes.

Finally, let us discuss why the OAM is suppressed for $\nu \geq 2$ but not for $\nu = 1$, in terms of the ground-state wave function. A general expression [38,39] for the ground state of a BdG Hamiltonian is given as $|\mathrm{GS}\rangle = \mathcal{N} \otimes_l |\mathrm{GS}\rangle_l$, where

$$|\mathrm{GS}\rangle_l = \left(\prod_{j=1}^{n_\uparrow^{(l)}} \tilde{c}^\dagger_{j,l+\nu,\uparrow}\right)\left(\prod_{j=1}^{n_\downarrow^{(l)}} \tilde{c}^\dagger_{j,-l,\downarrow}\right) \times \exp\left(\sum_{j>n_\uparrow^{(l)}}\sum_{j'>n_\downarrow^{(l)}} \tilde{c}^\dagger_{j,l+\nu,\uparrow} F^{(l)}_{jj'} \tilde{c}^\dagger_{j',-l,\downarrow}\right)|0\rangle. \quad (6)$$

Here, $|0\rangle$ is the vacuum for $c_{nl\sigma}$ and $\mathcal{N}$ is a normalization constant, $\tilde{c}_{jl\sigma}$ is a linear superposition of $\{c_{nl\sigma}\}_n$, and $n_\uparrow^{(l)}$, $n_\downarrow^{(l)}$ are non-negative integers. The ground state of a BdG Hamiltonian is often assumed to have a pure exponential form [$n_\sigma^{(l)} = 0$ in Eq. (6)], which implies that all the fermions are paired and thus $L_z = \nu N/2$. For $\nu = 1$, the ground state (of a sufficiently large system) is indeed reduced to the pure exponential form, implying the full OAM $L_z = N/2$. However, the ground state of a BdG Hamiltonian generally takes the form of Eq. (6). A nonvanishing $n_\sigma^{(l)}$ signals the existence of *unpaired* fermions, which contribute to the reduction of the OAM.

In fact, we can derive [38] the identity $\eta_l = 2(n_\downarrow^{(l)} - n_\uparrow^{(l)})$, which explicitly shows that the unpaired fermions are necessary for the spectral asymmetry and hence for the reduction of the OAM. We can determine the numbers of unpaired fermions explicitly [38]. For $\nu = 1$, there are no unpaired fermions ($n_\uparrow^{(l)} = n_\downarrow^{(l)} = 0$) as mentioned earlier. For $\nu \geq 2$, $n_\uparrow^{(l)} > 0$, $n_\downarrow^{(l)} = 0$ for $l_1 < l < -\nu/2$, and $n_\uparrow^{(l)} = 0$, $n_\downarrow^{(l)} > 0$ for $-\nu/2 < l < l_\nu$, where l_1 and l_ν are, respectively, the smallest and largest Fermi angular momenta. Therefore, the unpaired fermions generally carry angular momenta *opposite* to the given chirality, leading to the reduction of L_z from the full value $\nu N/2$. This depairing effect is associated with the formation of the non-PHS edge modes, signifying the fundamental difference between the $\nu = 1$ and $\nu \geq 2$ cases.

We thank Y. Maeno for introducing the problem of the OAM in chiral superfluids to us. We are also grateful to E. Demler, S. Fujimoto, A. Furusaki, J. Goryo, O. Ishikawa, N. Kawakami, Y. B. Kim, T. Kita, C.-K. Lu, T. Mizushima, S.-J. Mao, E.-G. Moon, Y. Nishida, M. Sato, K. Shiozaki, A. Shitade, M. Sigrist, H. Sumiyoshi, W.-F. Tsai, Y. Tsutsumi, K. Ueda, and S.-K. Yip for valuable discussions. This work was supported by the "Topological Quantum Phenomena" (No. 25103706) Grant-in-Aid for Scientific Research on Innovative Areas from the Ministry of Education, Culture, Sports, Science, and Technology (MEXT) of Japan.

Note added.—Recently, two related papers appeared on the arXiv. Volovik [43] confirmed our results on the OAM of $p + ip$-wave and higher-order chiral superfluids. In addition, he introduced a "parity violated boundary condition" with a phase factor, as a generalization of our hard-wall boundary condition. He showed that, under such a boundary condition, the OAM would be changed from $L_z = N/2$ even for a $p + ip$-wave chiral superfluid. Huang, Taylor, and Kallin [44] discussed the edge current in chiral superfluids and found a vanishing of the edge current, which leads to the suppression of the OAM, for $\nu \geq 2$, also in agreement with our results where they overlap.

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