## Loophole in $K \rightarrow \pi \nu \bar{\nu}$ Search and New Weak Leptonic Forces

Kaori Fuyuto,<sup>1</sup> Wei-Shu Hou,<sup>2,\*</sup> and Masaya Kohda<sup>3</sup>

<sup>1</sup>Department of Physics, Nagoya University, Nagoya 464-8602, Japan

<sup>2</sup>Department of Physics, National Taiwan University, Taipei 10617, Taiwan

<sup>3</sup>Department of Physics, Chung-Yuan Christian University, Chung-Li 32023, Taiwan

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Weakly interacting  $K \to \pi X^0$  emission with  $m_{X^0} \cong m_{\pi^0}$  is out of sight of the current  $K^+ \to \pi^+ \nu \bar{\nu}$  study, but it can be sensed by the  $K_L \to \pi^0 \nu \bar{\nu}$  search. This evades the usual Grossman-Nir bound of  $\mathcal{B}(K_L \to \pi^0 \nu \bar{\nu}) < 1.4 \times 10^{-9}$ ; thus, the KOTO experiment is already starting to probe new physics. An intriguing possibility is the Z' gauge boson of a weak leptonic force that couples to  $L_{\mu} - L_{\tau}$  (the difference between the muon and tauon numbers), which may explain the long-standing "muon g - 2" anomaly, but is constrained by  $\nu_{\mu}N \to \nu_{\mu}N\mu^+\mu^-$  scattering to  $m_{Z'} \leq 400$  MeV. An explicit model for  $K \to \pi Z'$  is given, which illustrates the link between rare kaon and  $B \to K\mu^+\mu^-$ ,  $K^{(*)}\nu\bar{\nu}$  decays. Complementary to these searches and future lepton experiments, the LHC might discover the scalar boson  $\phi$  responsible for light  $m_{Z'}$  generation via  $\phi \to Z'Z' \to 2(\mu^+\mu^-)$ .

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Introduction.—Despite discovering a 126 GeV scalar boson [1], there is anxiety at the Large Hadron Collider (LHC): no sign of new physics (NP) has so far emerged. But NP need not come from high energy. One longstanding hint [1] is the "muon g-2" anomaly, the discrepancy between precision experimental measurement and standard model (SM) calculations. A new experiment [2], Muon g-2, is under preparation that aims for a factor of 4 improvement in precision, with theory efforts to match [3]. One attractive NP possibility is a new force that couples to the muon, for example, gauging [4] the difference between the  $\mu$  and  $\tau$  numbers,  $L_{\mu} - L_{\tau}$  (much like gauging electric charge), with an associated gauge boson Z'. The scenario is well protected because, besides the muon, the Z' boson interacts with only  $\tau$ s and neutrinos.

The muon g-2 anomaly maps out a band in  $(m_{Z'}, g')$  space [5], where g' is the gauge coupling, and may also explain the so-called " $P'_5$  anomaly" [6] in  $B^0 \to K^{*0}\mu^+\mu^-$  angular variables. It was found [7], however, that the neutrino trident production or  $\nu_{\mu}N \to \nu_{\mu}N\mu^+\mu^-$  process constrains the Z' boson to be light,

$$m_{Z'} \lesssim 400 \text{ MeV},$$
 (1)

and g' is far weaker than the weak coupling. If this Z' boson couples to quarks in some way, then rare K decays might probe for the existence of this light Z' boson. While contemplating this link, we uncover a loophole in the usual Grossman-Nir (GN) bound [8]

$$\mathcal{B}(K_L \to \pi^0 \nu \bar{\nu}) < 1.4 \times 10^{-9},$$
 ("GN bound"). (2)

Kinematic selection in the  $K^+ \to \pi^+ \nu \bar{\nu}$  search allows  $K^+ \to \pi^+ Z'$  to go unnoticed, if  $m_{Z'} \sim m_{\pi^0}$ , but  $K_L \to \pi^0 Z'$ 

can be sensed by the  $K_L \rightarrow \pi^0 \nu \bar{\nu}$  search; thereby, the bound of Eq. (2) is evaded.

Besides pointing out this generic loophole, in this Letter we give an explicit model (see Fig. 1) that also shows how rare kaon and analogous rare *B* processes are interlinked. We point out further that the LHC could search for the scalar boson  $\phi$  behind  $m_{Z'}$  generation, via a pair of very light dimuons, i.e.,  $\phi \to Z'Z' \to 2[\mu^+\mu^-]$ .

 $K \rightarrow \pi \nu \bar{\nu}$  search.—The E787/949 experiment [9] has measured  $\mathcal{B}(K^+ \rightarrow \pi^+ \nu \bar{\nu}) = (1.73^{+1.15}_{-1.05}) \times 10^{-10}$ , which is consistent with SM expectations, and the NA62 [10] experiment aims at collecting  $\mathcal{O}(100)$  events in next three years. In a similar time frame, the KOTO experiment [11] aims at a  $3\sigma$  measurement of  $K_L \rightarrow \pi^0 \nu \bar{\nu}$  assuming the SM rate. KOTO has a better chance to uncover NP, because the  $K_L \rightarrow \pi^0 \nu \bar{\nu}$  decay is intrinsically *CP* violating (CPV), and the existing limit [12]

$$\mathcal{B}(K_L \to \pi^0 \nu \bar{\nu}) < 2.6 \times 10^{-8},$$
 (E391a) (3)



FIG. 1. Effective dsZ' (sbZ') coupling, with the Z' boson coupled to a vectorlike U quark that mixes with c, t ("×" flips chirality) and connects with external d-type quarks via a W boson loop.

is weaker. Equation (3) is, however, far above the bound of Eq. (2), which follows from inserting the E787/949 measurement into the relation [8]

$$\mathcal{B}(K_L \to \pi^0 \nu \bar{\nu}) \lesssim 4.3 \times \mathcal{B}(K^+ \to \pi^+ \nu \bar{\nu}), \qquad (4)$$

where the number 4.3 arises from isospin and  $\tau_{K_L}/\tau_{K^+}$  [8]. This is the origin of the usually perceived GN bound, that KOTO can only probe NP after Eq. (2) is reached. But KOTO has suffered a few inadvertent setbacks, and accumulated just 100 h of data in 2013. Though sensitivity comparable to Eq. (3) is reached [13], there is one event in the signal box, compared with zero events for the E391a [12] experiment; hence, KOTO appears to be still far from the bound of Eq. (2).

*Experimental loophole.*—The design of experiments has "accidental" features that are akin to the factor of 4.3 in Eq. (4) being not just a simple isospin factor. The E787/949 experiment observes the  $K^+$  decay at rest, and detects the emitted  $\pi^+$ , but nothing else. However, due to the "brightness" of  $\mathcal{B}(K^+ \to \pi^+ \pi^0) \simeq 21\%$ , the region around  $m_{\pi^0}$ , i.e., the range of  $p_{\pi^+}$  corresponding to  $116 \lesssim m_{\text{miss}} \lesssim 152$  MeV, is kinematically excluded. The region for  $m_{\text{miss}} > 261$  MeV is further excluded [14] due to the  $K^+ \to \pi^+ \pi \pi$  background. Although NA62 measures the  $K^+$  decay in flight, the regions of  $100 \lesssim m_{\text{miss}} \lesssim 165$  MeV and  $m_{\text{miss}} \gtrsim 260$  MeV are similarly excluded.

A  $K_L \rightarrow \pi^0 \nu \bar{\nu}$  experiment, however, cannot do kinematic reconstruction: besides detecting two photons (assumed to be  $\pi^0$ ), it measures "nothing to nothing." The  $K_L$  and " $\pi^0$ " momenta are not known. The approach is thus to veto everything, and to learn while pushing down the sensitivity. However, the  $\nu \bar{\nu}$  being the target, one cannot veto weakly interacting light particles. Thus, for  $K \rightarrow \pi X^0$  where  $X^0$  is any weakly interacting light particle that falls into the missing mass window, the  $K^+$  experiment would be oblivious, but the  $K_L$  experiment can have a blunt feel. Although the GN relation of Eq. (4) is in no way violated, the perceived GN bound of Eq. (2) does not apply. This is the main and rather simple point of this Letter, independent of the model discussion. The  $X^0$  need not be the leptonic force, as it simply goes undetected.

The E949 experiment performed a tagged search for  $\pi^0 \rightarrow \nu \bar{\nu}$  [15] inside the kinematically excluded window around  $\pi^0$ , giving the 90% C.L. bound [9]

$$\mathcal{B}(K^+ \to \pi^+ X^0) < 5.6 \times 10^{-8}, \qquad (m_{X^0} = m_{\pi^0}), \quad (5)$$

which is much weaker than their  $\mathcal{B}(K^+ \to \pi^+ \nu \bar{\nu})$  bound. Applying the analog of Eq. (4) would imply  $\mathcal{B}(K_L \to \pi^0 X^0) < 2.4 \times 10^{-7}$ , much weaker than the E391a bound of Eq. (3). Hence Eq. (3) provides a direct and more stringent bound on  $K_L \to \pi^0 X^0$  than implied by Eq. (5), which illustrates our main point. We now give an explicit Z' model to illustrate the potential impact of a  $K_L \rightarrow \pi^0 X^0$  discovery.

*Explicit model.*—We were interested in the  $t \rightarrow cZ'$  decay in the model of Ref. [5], where tree level sbZ' and ctZ' couplings are generated through the mixing of SM quarks with vectorlike doublet Q and singlet D, U quarks. With the Z' boson of the gauged  $L_{\mu} - L_{\tau}$  solution to the muon g - 2 anomaly constrained [7] by neutrino trident production to be light, Eq. (1), one is motivated to study the rare K decay, but the model can still be applied.

For  $s \rightarrow d$  transitions, mixing in the down-type sector would become too fine tuned; hence, setting them to zero is reasonable, and we consider only the mixing of up-type quarks with U, which is less constrained. This can be achieved, e.g., by introducing a  $Z_2$  symmetry under which Q and D are odd while U and other fields are even [16]. Diagrams like Fig. 1 can start from a U and t, c mixing core where the Z' boson is emitted, and dressed up with assistance from the SM into a loop-induced  $s \rightarrow dZ'$ (or  $b \rightarrow sZ'$ ) transition. The loop is finite because tree level down-type mixing is set to zero.

It is intriguing that with reasonable Uc and Ut mixing parameters (but with Uu mixing set to zero), loop diagrams as in Fig. 1 bring the  $s \rightarrow d$  transition into current experimental sensitivities. To introduce our subsequent notation, note that the vectorlike quark U in Fig. 1 carries the extra U(1)' charge and hence emits the Z' boson, while it mixes with up-type quarks i = c, t (right handed) through a "Yukawa coupling"  $Y_{Ui}$  to an exotic scalar field  $\Phi$  with U(1)' charge (and  $\langle \Phi \rangle = v_{\phi}/\sqrt{2}$  generates  $m_{Z'}$ ). For more details, see Ref. [17].

The effective  $\bar{d}_L \gamma^{\mu} s_L Z'_{\mu}$  coupling [17] has the coefficient

$$g_{ds} = \frac{g' v_{\phi}^2}{32\pi^2 v^2} [c_{cc} f_{cc} + (c_{tc} + c_{ct}) f_{ct} + c_{tt} f_{tt}], \qquad (6)$$

where  $c_{ij} = V_{is}V_{jd}^*Y_{Ui}Y_{Uj}^*m_im_j/m_U^2$  and

$$\begin{split} f_{ct} &= 1 + \log \frac{m_U^2}{m_t^2} + \frac{3m_W^2}{m_t^2 - m_W^2} \log \frac{m_t^2}{m_W^2}, \\ f_{tt} &= \frac{3m_W^2}{m_t^2 - m_W^2} \left( 1 - \frac{m_W^2}{m_t^2 - m_W^2} \log \frac{m_t^2}{m_W^2} \right) + \log \frac{m_U^2}{m_t^2} \end{split}$$

with  $f_{cc}$  obtainable from  $f_{tt}$  in the  $m_t^2 \ll m_W^2$  limit. These expressions are in the large  $m_U$  limit, though we use exact one-loop expressions (see Ref. [17]) in our numerics. Note that  $c_{ct} \neq c_{tc}$ , and  $c_{ij}$  are complex, even for real  $Y_{Ui}$ .

The branching ratio for  $K^+ \to \pi^+ Z'$  is given by

$$\mathcal{B}(K^+ \to \pi^+ Z') = \frac{m_{K^+}}{\Gamma_{K^+}} \frac{|g_{ds}|^2}{64\pi \hat{m}_{Z'}^2} \lambda^{3/2} (1, \hat{m}_{\pi^+}^2, \hat{m}_{Z'}^2) [f_+^{K\pi}(m_{Z'}^2)]^2, \quad (7)$$



FIG. 2 (color online). Left: for  $m_{Z'} = 135$  MeV ( $Z' \rightarrow \nu \bar{\nu} 100\%$ ), bounds for  $B(K^+ \rightarrow \pi^+ Z') < 5.6 \times 10^{-8}$  (dark gray exclusion region) and  $B(K_L \rightarrow \pi^0 Z') < 2.6 \times 10^{-8}$  (blue solid) on the  $Y_{Uc}$ - $Y_{Ut}$  plane. Right: for  $m_{Z'} = 285$  MeV ( $Z' \rightarrow \nu \bar{\nu} 54\%$ ), bounds for  $B(K^+ \rightarrow \pi^+ Z')B(Z' \rightarrow \mu^+ \mu^-) < 2.1 \times 10^{-9}$  (dark gray exclusion region) and  $B(B^+ \rightarrow K^+ Z')B(Z' \rightarrow \mu^+ \mu^-) < 2.0 \times 10^{-8}$  (pink allowed region) on the  $Y_{Uc}$ - $Y_{Ut}$  plane. In both panels, we give the usual GN bound of  $B(K_L \rightarrow \pi^0 Z')B(Z' \rightarrow \nu \bar{\nu}) < 1.4 \times 10^{-9}$  (red dashed) and  $2\sigma$  range for  $B(B^+ \rightarrow K^+ Z')B(Z' \rightarrow \nu \bar{\nu}) = (0.35^{+0.6}_{-0.15}) \times 10^{-5}$  (light green allowed region). The horizontal lines mark the reasonable  $Y_{Uc}$  range, and in the backdrop we plot  $B(t \rightarrow cZ')$  contours.

where  $\lambda(x,y,z) \equiv x^2 + y^2 + z^2 - 2(xy + yz + zx), \ \hat{m} \equiv m/m_{K^+},$ and  $f_{+}^{K\pi}$  is a form factor. The formula for  $K_L \to \pi^0 Z'$  is analogous, with  $|g_{ds}|$  replaced by Im $g_{ds}$ . Taking  $f_{\pm}^{K\pi}$  values from Ref. [18], we plot on the lhs of Fig. 2 the bound of Eq. (5) for  $K^+ \to \pi^+ Z'|_{m_{Z'}=m_{\pi^0}}$  in the  $Y_{Uc} - Y_{Ut}$  (treated as real) plane. We have taken  $g' \sim 10^{-3}$  as fixed [7] by the muon g-2 excess and neutrino trident bound, and  $m_U = 2$  TeV,  $v_{\phi} = 135$  GeV. We also plot  $K_L \rightarrow \pi^0 Z'$ assuming the E391a bound of Eq. (3), which turns out comparable. But if we apply Eq. (2) as a bound on  $K_L \rightarrow \pi^0 Z'$  ("GN" in Fig. 2), it would be much more stringent than the direct bound of Eq. (3). We have argued, however, that this application of the GN bound is incorrect for the present case. Hence, the region between Eqs. (3) and (2) is fair game for discovery. Note that  $K_L \rightarrow \pi^0 Z'$  is sensitive to the imaginary part of dsZ' coupling in Eq. (6), and hence probes also extra CPV phases arising from  $Y_{Uc}$ and  $Y_{IIt}$ . Other curves and regions on the lhs of Fig. 2 would then be explained shortly.

For the  $m_{\rm miss} > 260$  MeV exclusion zone for  $K^+ \rightarrow \pi^+ \nu \bar{\nu}$ , the  $Z' \rightarrow \mu^+ \mu^-$  decay is allowed. We find [17] that the  $K^+ \rightarrow \pi^+ \mu^+ \mu^-$  data by the NA48/2 experiment [19] permits a "best possible spike" at  $m_{\mu\mu} \approx 285$  MeV, with  $\delta \mathcal{B}(K^+ \rightarrow \pi^+ \mu \mu)$  up to  $2.1 \times 10^{-9}$  in strength. This is plotted (dark gray exclusion region) on the rhs of Fig. 2 and is as stringent as the GN bound of Eq. (2), hence much more stringent than Eq. (3). The model parameters are  $g' = 1.3 \times 10^{-3}$ ,  $m_U = 2$  TeV, and  $v_{\phi} \approx 219$  GeV.

We have shown that KOTO is already starting to probe NP. If a genuine excess appears above the perceived GN bound of Eq. (2), the likely explanation would be an unobserved recoil  $X^0$  particle in the " $\pi^0$  exclusion window" of the  $K^+ \rightarrow \pi^+ \nu \bar{\nu}$  search. Note that the bound of Eq. (2) cannot improve by much, even as NA62 accumulates data, unless  $\mathcal{B}(K^+ \rightarrow \pi^+ \nu \bar{\nu})$  is found to be

below the SM expectation. If KOTO pushes down to this bound of Eq. (2) without discovery, then NA62 should scan above 260 MeV for dimuon peaks. It could also push the bound on  $\pi^0 \rightarrow \nu \bar{\nu}$  [15] in the  $m_{\pi^0}$  exclusion window, and extend the study of E787/949 for  $K^+ \rightarrow \pi^+ X^0$  (see Fig. 18 of Ref. [9] and the discussion). With sufficient statistics, one might still uncover peaking events in  $m_{\text{miss}}$ .

We remark that, for both cases of discussion, we have checked that the benchmark parameters satisfy the kaon mixing constraint of Eq. (11) in Ref. [20].

Further model implications.—We have kept Uc and Ut mixings but set the mixing of heavy vectorlike quarks with down-type quarks (as well as u) to zero. But Fig. 1 generates sbZ' couplings alongside dsZ' couplings by W exchange in the loop. This brings in rare B decays, where the LHCb experiment has demonstrated its prowess recently, while Belle II is under construction. The formulas are analogous to Eqs. (6) and (7).

For the  $m_{Z'} = 285$  MeV case that we have just illustrated,  $Z' \rightarrow \mu^+ \mu^-$  and  $\nu \bar{\nu}$  rates are comparable, and the decay is prompt. Thus, it can show up in the  $B \rightarrow K^{(*)} \mu \mu$  decay with very low  $m_{\mu\mu}$ . The LHCb experiment has updated differential rates [21] for  $B \rightarrow K^{+,0} \mu \mu$  and  $K^{*+} \mu \mu$  decays to 3 fb<sup>-1</sup>, or the full run 1 data set. The  $B^0 \rightarrow K^{*0} \mu \mu$  decay, relevant for the  $P'_5$  anomaly, has yet to be updated from the 1 fb<sup>-1</sup> data [6]. But perhaps influenced by the latter, Ref. [21] starts at  $q^2 \equiv m_{\mu\mu}^2 > 0.1$  GeV<sup>2</sup>, or  $m_{\mu\mu} \gtrsim 316$  MeV, which covers only half the region of  $m_{\mu\mu}$  allowed by Eq. (1) above the dimuon threshold.

The 1 fb<sup>-1</sup> paper for  $B^+ \rightarrow K^+\mu\mu$  [22], however, does go down to  $q^2 = 0.05 \text{ GeV}^2$ , or  $m_{\mu\mu} = 224 \text{ MeV}$ , hence can be compared with our  $m_{Z'} = 285 \text{ MeV}$  case. Interestingly, in the lowest  $0.05 < q^2 < 2.00 \text{ GeV}^2$  bin, there is a mild excess above the mean for  $1.00 < q^2 < 6.00 \text{ GeV}^2$ . Treating experimental error at the  $2\sigma$  level, our estimate [17] for this excess is  $\sim 2 \times 10^{-8}$ . If we attribute this all to the presence of  $B^+ \rightarrow K^+ Z' [\rightarrow \mu^+ \mu^-]$  then, scaling by  $\mathcal{B}(Z' \rightarrow \mu^+ \mu^-) \approx 46\%$ , this implies  $B^+ \rightarrow K^+ Z'$  at  $4.4 \times 10^{-8}$  level. Using the form factors of Ref. [23], we plot this constraint on the rhs of Fig. 2, which is stronger than our estimate of the NA48/2 bound. Actually, there also seems to be some excess in the first 0.1 <  $q^2$  < 0.98 GeV<sup>2</sup> bin for  $B \rightarrow K^+ \mu \mu$  in the full 3 fb<sup>-1</sup> data set [6]; hence. the Z' boson could be above 316 MeV. We urge LHCb to refine their analysis, optimize binning to  $q^2$  resolution, and extend a spike search down to 0.045 GeV<sup>2</sup>.

The  $B^0 \to K^0 \mu \mu$  modes have less statistics, while the  $B \to K^* \mu \mu$  modes would have a low  $q^2$  photon peak, making interpretation more difficult. Note that our estimate based on LHCb data is stronger than that based on NA48/2 data, even though the former is only based on the 1 fb<sup>-1</sup> data set. However,  $s \to d$  and  $b \to s$  processes may or may not be correlated as in our model. So, when KOTO reaches the usual GN bound, NA62 should still conduct a spike search above  $m_{\mu\mu} > 260$  MeV. We note in passing that the Belle experiment has conducted a  $B^0 \to K^{*0}X^0$  search [24] for light  $X^0 \to \mu^+\mu^-$ , and the bound is roughly  $5 \times 10^{-8}$  for  $m_{X^0} \approx 285$  MeV. We suggest that Belle (and *BABAR*), however, conduct the search for  $B \to K + X^0 [\to \mu^+\mu^-]$  to avoid the photon peak.

Like our illustration on the lhs of Fig. 2, if  $m_{Z'}$  falls into the " $\pi^0$  blind spot," NA62 would be oblivious, and so would LHCb. Fortunately, because  $\mathcal{B}(B \to K\pi^0) \ll \mathcal{B}(K \to \pi\pi^0)$ , the (super-)B factories can cross-check in the  $B \to K^{(*)} \nu \bar{\nu}$ modes, where there is no photon peak. The BABAR experiment has led the way by conducting a binned  $m_{\nu\bar{\nu}}^2$  search [25], where the lowest  $s_B \equiv m_{\nu\bar{\nu}}^2/m_B^2 < 0.1$  bin for both the  $B^+ \to K^+ \nu \bar{\nu}$  and  $B^0 \to \bar{K^{*0}} \nu \bar{\nu}$  modes shows some excess. which drives a lower bound for the  $K^+\nu\bar{\nu}$  mode. From Fig. 6 of Ref. [25], we estimate  $\mathcal{B}(B^+ \to K^+ \nu \bar{\nu}) =$  $(0.35^{+0.6}_{-0.15}) \times 10^{-5}$  in this bin, and plot the  $2\sigma$  range on the lhs of Fig. 2. The result is stronger than the kaon modes, and the allowed region extends to the usual GN bound. On the other hand, for the  $m_{Z'} = 285$  MeV example where  $Z' \rightarrow \mu^+ \mu^-$  is also allowed, plotting the *BABAR* result on the rhs of Fig. 2 shows some tension with our LHCb 1 fb<sup>-1</sup> estimate for  $B^+ \to K^+ Z' [\to \mu^+ \mu^-]$ , with the latter most stringent. Our estimates are, however, rudimentary and for illustration only. It would be better done by the experiments.

In this vein, although Belle led the way in the  $B^+ \rightarrow K^+ \nu \bar{\nu}$  search [26], its follow-up paper [27] just added 40% more data but followed the same analysis, including a cut on high  $p_{K^+}$  for the sake of rejecting  $B \rightarrow K^* \gamma$ , which precisely cuts out the  $B \rightarrow K^{(*)}Z'$  possibility. We urge Belle to conduct a binned  $m_{\nu\bar{\nu}}^2$  study and optimize the binning according to resolution. It should also practice optimizing the  $m_{\nu\bar{\nu}}^2$  or recoil mass resolution with the full *B*-tag method, towards a future Belle II search.

Discussion and conclusion.—We have given the branching ratio  $\mathcal{B}(t \to cZ')$  in the backdrop of Fig. 2, and have drawn  $|Y_{Uc}| < 0.2$  (arbitrarily chosen) bands to indicate that  $|Y_{Uc}|$  should not be too large, while  $|Y_{Uc}| < |Y_{Ut}|$ should hold in general (further discussion is given in Ref. [17]). We find  $\mathcal{B}(t \to cZ') \lesssim 10^{-7}$  for  $|Y_{Uc}| < 0.2$ , but the rate can certainly be larger if one considers general  $|Y_{Uc}|$  values. Thus, given that  $Z' \to \mu^+\mu^-$  at ~50% for Z'above the dimuon threshold,  $t \to cZ'$  should be searched for at the LHC, while the rare  $t \to cZ'$  case could perhaps drive a 100 TeV pp collider study for a future "top factory."

With spontaneous  $L_{\mu} - L_{\tau}$  symmetry breaking but Z' light because of very weak gauge coupling, the  $v_{\phi}$  scale is not too different from v of the SM. The mass of the exotic scalar  $\phi$  is quite arbitrary as the self-coupling is unknown, but should be at the weak scale. However, the U quark mixes with the c and t quarks, which generates effective  $gg\phi$  coupling, while  $\phi$  predominantly decays via a Z'Z' pair. This motivates a search for the light Z' boson at the LHC, which can potentially uncover the associated  $\phi$ boson, independent of rare K and B studies.

Our investigation [17] shows that a  $\phi$  search is accessible at the LHC for the example of a 285 MeV Z' boson decay, where the signature is  $(gg \rightarrow)\phi \rightarrow Z'Z' \rightarrow [\mu^+\mu^-][\mu^+\mu^-]$ with the brackets indicating low dimuon mass. The Z' decay is prompt. Interestingly, the CMS experiment conducted a search [28] with 2012 data that can be applied to  $\phi \rightarrow Z'Z' \rightarrow (\mu^+\mu^-)(\mu^+\mu^-)$ , where one event was found at low dimuon pair mass. The two dimuon pairs have masses ~200 and 300 MeV, respectively, which is right on the spot. It is too early to tell, but with run 2 to start in 2015, this study should be carefully watched, and vigorously pursued. Note that the U quark, with a mass in the TeV range, can also be searched for.

For the original motivation, the muon g - 2 anomaly is pursued by the E989 or Muon g-2 experiment [2], while neutrino trident production can [7] be covered by the LBNE experiment [29]. Although the schedule is yet uncertain for these two pursuits at Fermilab, we have shown that the next few years could see major progress on related issues, ranging from rare kaon decays (KOTO/ NA62) to rare *B* decays (LHCb/Belle(II)), and perhaps at the LHC.

In conclusion, we point out a loophole in the experimental setup when comparing the  $K^+ \rightarrow \pi^+ \nu \bar{\nu}$  and  $K_L \rightarrow \pi^0 \nu \bar{\nu}$  searches, and find that the KOTO experiment is already starting to explore new physics territory, while the commonly perceived "Grossman-Nir bound" may not apply. Although the mass range for weakly interacting light particle emission is a bit restricted, our explicit model illustrates the potential wide-ranging impact of discovering  $\mathcal{B}(K_L \rightarrow \pi^0 \nu \bar{\nu}) \gtrsim 1.4 \times 10^{-9}$ . Conversely, many measurements at *B* factories and the LHC could uncover correlated phenomena, which could shed light on what may be behind the muon g - 2 anomaly. K. F. is supported by the Nagoya University Program for Leading Graduate Schools and the "Leadership Development Program for Space Exploration and Research" (N01) by JSPS. W.-S. H. is supported by the Academic Summit grant MOST 103-2745-M-002-001-ASP, as well as by Grant No. NTU-EPR-103R8915. M. K. is supported under NSC Grant No. 102-2112-M-033-007-MY3. W.-S. H. thanks T. Blake, P. Chang, K.-F. Chen, Y. B. Hsiung, M. Pepe-Altarelli, and T. Yamanaka for discussions, and K. F. thanks the NTUHEP group for hospitality during exchange visits.

<sup>\*</sup>Corresponding author.

wshou@phys.ntu.edu.tw

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