Intrinsic Damping of Collective Spin Modes in a Two-Dimensional Fermi Liquid with Spin-Orbit Coupling

Saurabh Maiti[1,2] and Dmitrii L. Maslov[1]
[1]Department of Physics, University of Florida, Gainesville, Florida 32611, USA
[2]National High Magnetic Field Laboratory, Tallahassee, Florida 32310, USA
(Received 30 January 2015; published 17 April 2015)

A Fermi liquid with spin-orbit coupling (SOC) is expected to support a new set of collective modes: oscillations of magnetization in the absence of the magnetic field. We show that these modes are damped by the electron-electron interaction even in the limit of an infinitely long wavelength ($q = 0$). The linewidth of the collective mode is on the order of $\bar{\Delta}^2/E_F$, where $\bar{\Delta}$ is a characteristic spin-orbit energy splitting and E_F is the Fermi energy. Such damping is in stark contrast to known damping mechanisms of both charge and spin collective modes in the absence of SOC, all of which disappear at $q = 0$, and arises because none of the components of total spin is conserved in the presence of SOC.

DOI: 10.1103/PhysRevLett.114.156803 PACS numbers: 73.21.-b, 67.85.-d, 85.75.-d

Electron systems with spin-orbit coupling (SOC) exhibit rich physics, some of which may have technological applications; [1,2] equally rich is the physics of cold-atom systems with synthetic SOC [3,4]. Combining many-body interactions with broken SU(2) symmetry, one obtains a special—"chiral"—kind of Fermi liquid (FL) [5–7] that supports a new set of collective modes, the "chiral-spin waves"—oscillations of the spin density in zero magnetic field [8–11].

In the absence of SOC, electron-electron interaction (*eei*) [12] does not affect certain properties of an electron system given that some symmetries are preserved. For example, the conductivity and cyclotron-resonance frequency of a Galilean-invariant system are not affected by *eei*; the same is true for the de Haas–van Alphen (dHvA) frequency in an isotropic system [13] and for the Larmor frequency in the presence of an SU(2)-symmetric interaction. Being a relativistic effect, SOC breaks both Galilean (but not necessarily rotational) invariance and SU(2) symmetry and thus lifts the protection ensured by these symmetries. As a result, several physical quantities become dependent on *eei*, such as the optical conductivity [16], Drude weight [17], and frequencies of collective spin modes, which play the role of Larmor frequencies in zero magnetic field [8–11].

In this Letter, we discuss another fundamentally new effect induced solely by SOC: intrinsic damping of collective spin modes in the uniform ($q = 0$) limit. Interaction-induced damping of collective modes is not, by itself, a new effect. For example, plasmons in 3D [18], 2D [19], and 1D (Ref. [20]) electron systems, the Silin-Leggett (SL) mode [21,22] in a partially spin-polarized FL [23,24], and magnons in a ferromagnetic FL [24,25] are all damped by interaction processes involving excitations of multiple particle-hole pairs. (This mechanism is different from Landau damping which involves only a single particle-hole pair. Damping by multiple pairs occurs even outside the single-particle continuum; its effect on absorption was studied in Refs. [16] and [19] within the Fermi golden rule.) However, Galilean invariance, in the case of charge modes [26], and conservation of the total spin component along the field (S_3), in the case of spin modes [27], ensure that this kind of damping vanishes at $q = 0$. We show here that this is not the case for electron systems with SOC.

Two-dimensional (2D) electron systems with momentum-dependent SOC, e.g., of Rashba or Dresselhaus types, bear a certain similarity to a partially spin-polarized FL. The latter has a transverse SL mode in the spin sector (see Fig. 1, left) [21,22], while the former has three (two transverse and one longitudinal) chiral-spin modes ($\Omega_1 \ldots \Omega_3$ in Fig. 1, right) [8–11], which correspond to oscillations of the three components of magnetization. Indices 1–3 label the Cartesian system with the 3 axis along the normal to the plane of a 2D electron gas. (Although the SL mode has been studied previously in

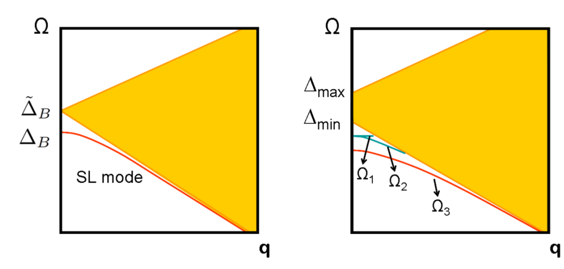

FIG. 1 (color online). Left: The Silin-Leggett mode in a partially spin-polarized FL. Right: The chiral-spin modes in a FL with Rashba spin-orbit coupling. The shaded regions denote the particle-hole continua, Δ_B is the Larmor frequency, $\tilde{\Delta}_B$ is the quasiparticle Zeeman energy, and $\Delta_{\min(\max)}$ is the lower (upper) boundary of the continuum at $q = 0$.

3D, the same mode should occur in 2D as well.) Conservation of S_3 ensures that the frequency of the SL mode at $q=0$ coincides with the Larmor frequency in the absence of *eei*. In the presence of SOC, none of the three spin components is conserved. As a result, one obtains three distinct modes with frequencies renormalized by *eei*. In addition, as we show here, these modes have finite linewidth which, in order of magnitude, is given by the inverse transport lifetime of a quasiparticle with energy equal to the spin-orbit splitting. That the SL mode at $q=0$ is not affected by *eei* follows already from the exact equations of motion for magnetization [23]. Diagrammatically, this occurs due to a cancellation between the self-energy and vertex graphs for the spin susceptibility [23]. Such a cancellation, however, does not occur in the presence of SOC.

The single-particle Hamiltonian of a 2D system with SOC can be written as (we set $\hbar=1$)

$$\mathcal{H}_k = \left(\frac{k^2}{2m}-\mu\right)\sigma_0 + \lambda\vec{\sigma}\cdot\vec{f}(\vec{k}), \tag{1}$$

where μ is the chemical potential, σ_0 is the 2×2 unit matrix, $\vec{\sigma}$ is the three-dimensional vector of Pauli matrices, λ is the SOC constant, and $\vec{f}(\vec{k}) = -\vec{f}(-\vec{k})$ is a 2D vector that depends on the details of SOC; e.g., $\vec{f}=(k_2,-k_1,0)$ for linear Rashba SOC. The single-particle Green's function is given by

$$G(K)=\sum_s \Omega_s(\vec{k})g_s(K), \qquad \Omega_s(\vec{k})=\frac{1}{2}[\sigma_0+s\hat{\eta}_{\vec{k}}], \tag{2}$$

where $g_s(K)=(ik_0-(k^2/2m)-s\Delta_{\vec{k}}/2+\mu)^{-1}$, $K\equiv(ik_0,\vec{k})$, $s=\pm 1$ labels either spin projection or chirality, $\hat{\eta}_{\vec{k}}\equiv \vec{\sigma}\cdot\vec{f}/|\vec{f}|$, and $\Delta_{\vec{k}}=2\lambda|\vec{f}|$ is the spin-orbit splitting which, in general, depends not only on the magnitude but also on the direction of $\vec{k}$. We will be primarily interested in the case of weak SOC, when $\Delta_{\vec{k}} \ll E_F$ for any $\vec{k}$. In what follows, we will be comparing the Ω_3 chiral-spin mode to the (2D) SL mode, as both modes are transverse to the SOC-induced or Zeeman magnetic field, correspondingly. The latter can be described by the same Hamiltonian with $\vec{f}=(0,0,\Delta_B/2\lambda)$, where $\Delta_B\equiv g\mu_B B$, g is the effective g factor, μ_B is the Bohr magneton, and $\vec{B}$ is the magnetic field chosen to be along the 3 direction. The orbital effect of the field is not considered here.

Within the random phase approximation (RPA), the spin susceptibility tensor is given by the ladder series in Fig. 2, where the boxed wavy line is a short-range interaction, U_x, which mimics the exchange interaction in the spin channel. As shown in Ref. [11], the frequencies of the collective modes correspond to the roots of the equation $\mathrm{Det}[\sigma_0\otimes\sigma_0+(U_x/2)\Pi^0]$, where the elements of the 4×4 spin-charge polarization matrix Π^0 are given by

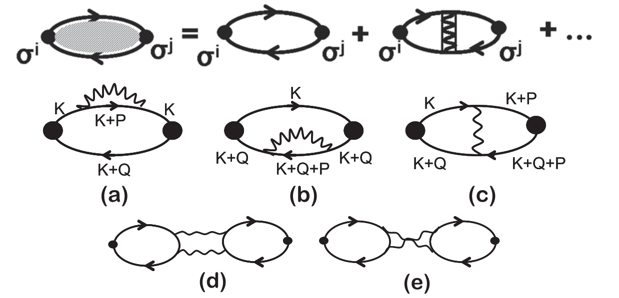

FIG. 2. Top: The ladder (RPA) series for the spin susceptibility. The boxed wavy line is the static effective interaction U_x. Bottom: Diagrams contributing to damping of the collective modes. The wavy line in diagrams (a)–(e) denotes a dynamic interaction, $V_{\mathrm{eff}}(P)$.

$$\Pi^0_{ij}(Q)=\int_K \mathrm{Tr}[\sigma_i G(K)\sigma_j G(K+Q)]. \tag{3}$$

Here, $\int_K \equiv T\sum_{k_0}\int(d^2k/(2\pi)^2)$, $i,j\in 0,1,2,3$, and 0 corresponds to the charge component. At $q=0$, all mixed spin-charge susceptibilities, Π^0_{0j} with $j\neq 0$, vanish by charge conservation, while the matrix of spin susceptibilities can always be transformed to a diagonal form. In general, there are three spin modes, whose frequencies are found from the equations $1+\delta_j U_x \Pi^0_{ii}=0$, with $\delta_1=\delta_2=1$ and $\delta_3=1/2$. The Green's functions in Π^0_{ij} contain the self-energy parts. However, for the special case of $U_x=$ constant, they drop out (see below) and, after analytic continuation ($iq_0\to\Omega+i\delta$), one obtains

$$\Pi^0_{33}(\Omega)=2\nu\left\langle\frac{\Delta^2_{\vec{k}_F}}{(\Omega+i\delta)^2-\Delta^2_{\vec{k}_F}}\right\rangle_{\mathrm{FS}}, \tag{4}$$

where we have already assumed that SOC is weak in the sense specified above, ν is the density of states per spin projection, $\langle\cdots\rangle$ denotes averaging over the Fermi surface (FS), $\vec{k}_F=k_F\vec{k}/k$, and k_F is the Fermi momentum in the absence of SOC. In general, $\Delta_{\vec{k}}$ varies from $\Delta_{\min}$ to $\Delta_{\max}$ along the FS. The continuum of inter-subband particle-hole excitations, where $\mathrm{Im}\Pi^0_{33}\neq 0$, is confined to the interval $\Delta_{\min}\leq\Omega\leq\Delta_{\max}$. At the boundaries of the continuum, $\mathrm{Re}\Pi^0_{33}$ has square root singularities [28], which guarantee a solution of the eigenmode equation $1+U_x\Pi^0_{33}/2=0$ for $\Omega<\Delta_{\min}$ even at weak coupling. If SOC is isotropic, $\Delta_{\min}=\Delta_{\max}\equiv\Delta$, the two square-root singularities merge into a single pole at $\Omega=\Delta$, the inter-subband continuum shrinks to a single point, and the mode frequency is given by $\Omega_3 = \Delta\sqrt{1-u}$, where $u \equiv U_x\nu$[11].

Renormalization of the mode frequency by *eei* in the SOC case and the lack of it thereof in the SL case is an important difference, which we discuss now as it will help us to understand the differences in damping later on. This difference occurs because the Greens' functions in the RPA series in Fig. 1 include chirality- or spin-dependent shifts in the chemical potential, which are given by the momentum- and frequency-independent parts of the self-energy. Each rung of the ladder diagram (Π^0_{ij}) contains a difference of the self-energies

$$\delta\Sigma_s = \Sigma_s - \Sigma_{-s} = -U_x\sum_{s'}\int_P (B_{s,s'} - B_{-s,s'})g_{s'}(P), \quad (5)$$

where $B_{s,s'}$ is the matrix element for the transition $s \to s'$. Within a given rung, $\delta\Sigma_s$ renormalizes the spin-splitting perturbation, be it SOC or the magnetic field. Since $s = s'$ in the SL case, the self-energy of an electron with given spin is proportional to the number density of electrons with the same spin. Hence, $\delta\Sigma_s$ is proportional to magnetization $[\delta\Sigma_s = su\Delta_B/(1-u)]$, and each rung of the diagram contains the renormalized Zeeman energy of a quasiparticle, $\tilde{\Delta}_B = \Delta_B/(1-u)$. The boundary of the continuum at $q = 0$ is shifted from Δ_B to $\tilde{\Delta}_B$ (cf. Fig. 1, left) as can be seen, e.g., from the 11 component of the rung: $\Pi^0_{11}(\Omega) = \nu[2\tilde{\Delta}_B^2/(\Omega + i\delta)^2 - \tilde{\Delta}_B^2]$. The Larmor theorem is effected via a cancellation between the self-energy and vertex contributions [23]: when the rung is substituted into the eigenmode equation, the factor of $1-u$ cancels out and the frequency of the mode coincides with bare Δ_B.

The SOC case is different in that chirality, in contrast to spin, is not conserved by *eei*, and the sum over s' in Eq. (5) contains both the $s = s'$ and $s' = -s$ terms. For $U_x = \text{const}$, this implies that the self-energy of an electron with given chirality is proportional to the total number density, and thus $\delta\Sigma_s = 0$ [6]. The vertex part is, however, nonzero. Therefore, there is no cancellation between the self-energy and vertex contributions, and the frequencies of the modes are renormalized by *eei*. If $U_x \neq \text{const}$, one can show [29,30] that $\delta\Sigma_s$ does not contain the zeroth angular harmonic of the interaction (which is why $\delta\Sigma_s = 0$ for $U_x = \text{const}$), whereas the vertex contribution does. Thus, there is no cancellation between the two contributions in the general case as well.

Damping of collective modes in the region of frequencies and momenta outside the single-particle continuum occurs via generation of multiple particle-hole pairs, which requires a dynamic interaction, e.g., a dynamically screened Coulomb potential. The self-energy of collective modes is depicted by diagrams (a)–(e) in Fig. 2. The same set of diagrams has been encountered in the analysis of various two-particle correlation functions in the case of an RPA-type interaction [31–34]. Although the Aslamazov-Larkin (AL) diagrams (d) and (e) contain two wavy lines, they are of the same order in the bare coupling constant of the theory (the electron charge in our case) as in diagrams (a)–(c). However, the contribution of the AL diagrams to damping vanishes within the approximations made in this work, i.e., weak SOC and long-range interaction (see below) [35,36].

In the case of the SL mode, renormalization of the transverse spin susceptibility ($\chi_\perp = \langle S^- S^+\rangle$) by diagrams (a)–(c) is given by $\delta\chi_\perp(Q) = -\mu_B^2(\Pi_a + \Pi_b + \Pi_c)$, where

$$\begin{aligned}
\Pi_a &= \int_K [g_-^2(K)g_+(K+Q)\Sigma_-(K) + (\Delta_{\vec{k}_F} \to -\Delta_{\vec{k}_F})],\\
\Pi_b &= \int_K [g_-^2(K+Q)g_+(K)\Sigma_-(K+Q)\\
&\quad + (\Delta_{\vec{k}_F} \to -\Delta_{\vec{k}_F})],\\
\Pi_c &= \int_K \bigg(\frac{g_-(K+Q)g_+(K)}{iq_0 + \Delta_B}[\Sigma_+(K) - \Sigma_-(K+Q)]\\
&\quad + [\Delta_{\vec{k}_F} \to -\Delta_{\vec{k}_F}]\bigg),
\end{aligned} \quad (6)$$

$+(-)$ denotes up (down) spins, $Q = (iq_0, 0)$, $\Sigma_\pm(K) = -\int_P g_\pm(K+P)V_{\text{eff}}(P)$, and $V_{\text{eff}}(P)$ is some dynamic interaction. In the last line of Eq. (6), we used the identity

$$g_\pm(K+P+Q)g_\mp(K+P) = \frac{g_\pm(K+P) - g_\mp(K+P+Q)}{iq_0 \pm \Delta_B} \quad (7)$$

and integrated over P. Because the denominator in Eq. (7) does not depend on P, this last step produced the same self-energies, $\Sigma_\pm$, as in diagrams (a) and (b). Adding up the three lines of Eq. (6), we arrive at

$$\Pi_a + \Pi_b + \Pi_c = \frac{2\Delta_B}{q_0^2 + \Delta_B^2}(A_+ - A_-), \quad (8)$$

where $A_\pm \equiv \int_K g_\pm^2(K)\Sigma_\pm(K)$. Recalling that $V_{\text{eff}}(P)$ is real on the Matsubara axis and changing the variables as $k_0 \to -k_0$ and $p_0 \to -p_0$, we find that $A_\pm = A_\pm^*$. Thus the frequency-independent prefactor in Eq. (8), $A_+ - A_-$, is real. Continuing iq_0 to the real axis, we see that the imaginary part of $\delta\chi_\perp$ comes only from a resonance at the bare Larmor frequency, $\Omega = \Delta_B$, which coincides with the pole of $\chi_\perp$ in the RPA. The only effect of the interaction processes represented by diagrams (a)–(c) is thus to renormalize the amplitude of the SL mode without either shifting its frequency or smearing it.

For the case of SOC, it is convenient to consider renormalization of the out-of-plane spin susceptibility, $\delta\chi_{33}(Q) = -\mu_B^2(\Pi_a + \Pi_b + \Pi_c)$, where now

$$\Pi_a = \int_K \frac{1}{2}[g_-^2(K)g_+(K+Q)\{\Sigma_{+-}(K)+\Sigma_{-+}(K)\} + g_+^2(K)g_-(K+Q)\{\Sigma_{++}(K)+\Sigma_{--}(K)\}],$$
$$\Pi_b = \int_K \frac{1}{2}[g_-^2(K+Q)g_+(K)\{\Sigma_{+-}(K+Q)+\Sigma_{-+}(K+Q)\} + g_+^2(K+Q)g_-(K)\{\Sigma_{++}(K+Q)+\Sigma_{--}(K+Q)\}],$$
$$\Pi_c = -\int_K\int_P \frac{1}{2}V_{\mathrm{eff}}(P)\left(\frac{\mathcal{N}_{-+}\mathcal{M}_{+--}}{iq_0+\Delta_{\vec{k}+\vec{p}}} + \frac{\mathcal{N}_{+-}\mathcal{M}_{+-+}}{iq_0+\Delta_{\vec{k}+\vec{p}}} + \frac{\mathcal{N}_{-+}\mathcal{M}_{-++}}{iq_0-\Delta_{\vec{k}+\vec{p}}} + \frac{\mathcal{N}_{+-}\mathcal{M}_{-+-}}{iq_0-\Delta_{\vec{k}+\vec{p}}}\right),$$
$$\mathcal{M}_{rts} = [g_r(K+P) - g_t(K+P+Q)][1+s\cos(\phi_{\vec{k}}-\phi_{\vec{k}+\vec{p}})], \qquad \mathcal{N}_{rt} = g_r(K)g_t(K+Q). \tag{9}$$

Here, $r, t, s = \pm$label the spin-split bands and $\phi_{\vec{k}}$ depends on the azimuthal angle $\theta_{\vec{k}}$ of $\vec{k}$ (and is equal to $\theta_{\vec{k}}$ for linear Rashba SOC) [37]. The partial self-energies are defined as $\Sigma_{rt}(K) = -\int_P g_r(K+P)V_{\mathrm{eff}}(P)[1+t\cos(\phi_{\vec{k}}-\phi_{\vec{k}+\vec{p}})]$, such that the total self-energies of the Rashba subbands are $\Sigma_+ = \Sigma_{++}+\Sigma_{--}$ and $\Sigma_- = \Sigma_{+-}+\Sigma_{-+}$. In the last line of Eq. (9), we only used the identity [Eq. (7)]. In contrast to the SL case, however, the spin-orbit splitting in the denominators of Π_c depend on $\vec{k}+\vec{p}$, and thus integration over P does not, in general, produce the self-energies. This already tells us that, in general, the imaginary part of $\delta\chi_{33}$ cannot cancel out between the self-energy and vertex diagrams.

However, there are two realistic approximations, namely, of a long-range interaction ($p \ll k_F$) and of weak SOC ($\Delta_{\vec{k}} \ll E_F$), within which the momentum dependence of $\Delta_{\vec{k}+\vec{p}}$ can be neglected. Assuming that these two conditions are satisfied, the vertex part can again be rewritten in terms of the partial self-energies. Even in this limit, however, there is no complete cancellation between the self-energy and vertex diagrams. Namely, we find that $\Pi_a+\Pi_b+\Pi_c$ can be rewritten as $\Pi_R+\Pi_D$, where $\Pi_R = \int_K(\Delta_{\vec{k}_F}/q_0^2 + \Delta_{\vec{k}_F}^2)\{\Sigma_+(K)g_+^2(K) - \Sigma_-(K)g_-^2(K)\}$ and

$$\Pi_D = -\int_K \frac{\Delta_{\vec{k}_F}}{q_0^2+\Delta_{\vec{k}_F}^2}\{[\Sigma_{--}(K+Q) - \Sigma_{+-}(K)]g_-(K)g_+(K+Q) - (\Delta_{\vec{k}_F}\to -\Delta_{\vec{k}_F})\}. \tag{10}$$

The first term, Π_R, has the same structure as in Eq. (8); using the same arguments as before, we conclude that Π_R does not contribute to damping. In contrast, the second term, Π_D, does have, in general, an imaginary part at all frequencies, indicating presence of damping. To calculate Π_D explicitly, one needs to specify the interaction, which we choose to be in the form of a dynamically screened Coulomb potential. Deferring the computational details to Sec. IV of the Supplemental Material [37], we quote here only the final result for χ_{33}: near the resonance at $\Omega = \Omega_3$,

$$\chi_{33}^{-1}(\Omega) = (2\nu\mu_B^2)^{-1}A[\Omega_3^2 - (\Omega + i\Gamma/2)^2]$$
$$A = \left\langle\left(\frac{\Delta_{\vec{k}_F}\xi_{\vec{k}_F}}{\Delta_{\vec{k}_F}^2-\Omega_3^2}\right)^2\right\rangle_{\mathrm{FS}}\left\langle\frac{\Delta_{\vec{k}_F}^2\xi_{\vec{k}_F}^2}{\Delta_{\vec{k}_F}^2-\Omega_3^2}\right\rangle_{\mathrm{FS}}^{-2};$$
$$\Gamma = \frac{\omega_{\mathrm{C}}^2}{2E_F}\left(\frac{\Delta_{\vec{k}_F}}{E_F}\right)^2, \tag{11}$$

where $\omega_{\mathrm{C}}^2 = r_s^2E_F^2\ln r_s^{-1}/12\pi$, $r_s = \sqrt{2}e^2/v_F$ is the coupling constant of the Coulomb interaction, and $\xi_{\vec{k}_F}^2$ is a dimensionless form factor which depends on the details of SOC; for isotropic SOC, $\xi_{\vec{k}_F}^2 = 1$.

The damping rate Γ has an expected FL form. Notice though that the *quasiparticle* damping rate in 2D scales as $\Gamma_{\mathrm{qp}} \propto \Omega^2\ln\Omega$, as opposed to just Ω^2, with a prefactor which does not depend on r_s [38]. Being a gauge-invariant quantity, Γ contains the *differences* of the single-particle self-energies [see Eq. (10)], while Γ_{qp} is related to the self-energy itself. The infrared singularity in the self-energy, which gives rise to the $\ln\Omega$ factor in Γ_{qp}, cancels out in Γ. As a result, Γ is on the order of the *transport* decay rate, which is smaller than Γ_{qp}. The FL nature of the result for Γ indicates that it would not change substantially if, instead of a FL with Coulomb interaction, we consider a FL of neutral particles with short-range interaction. The only change would be in the prefactor ω_{C}^2 which, for the case of a contact interaction with coupling U, should be replaced by $\sim(U\nu E_F)^2$.

Since the frequencies of chiral-spin modes are proportional to the spin-orbit splitting, one might be tempted to conclude that it is better to look for these modes in materials with strong SOC. Our result in Eq. (11) shows that the advantage of strong SOC has its limits. Indeed, the ratio of the linewidth to the mode frequency, $\gamma \equiv \bar{\Gamma}/\bar{\Delta}$, scales as $C\bar{\Delta}/E_F$, where $\bar{\Gamma}$ and $\bar{\Delta}$ are the appropriate angular averages of Γ and $\Delta_{\vec{k}_F}$, correspondingly, and C a dimensionless prefactor. In a material with sufficiently strong *eei*, one should expect that $C\sim 1$. If, in addition, SOC is also strong ($\bar{\Delta}\sim E_F$), then $\gamma\sim 1$ and the mode is overdamped. We emphasize that this effect is a unique feature of SOC; in contrast, the SL mode remains undamped (at $q=0$) even if the Zeeman energy becomes comparable to the Fermi energy.

In a particular model of the screened Coulomb potential, the effect of damping appears to be rather weak. For isotropic SOC, Eq. (11) yields $C = r_s^2 \ln r_s^{-1}/12\pi$. Using parameters for an InGaAs/InAlAs quantum well, we then find $\gamma \approx 2\times 10^{-3}\bar{\Delta}/E_F$ for an electron number density of 1.6×10^{12} cm^2. On the other hand, chiral-spin waves are also damped by disorder via the Dyakonov-Perel' mechanism [8]. For a mobility of 2×10^5 cm^2/V $\cdot$ s, damping due to disorder is stronger than that by *eei* by a factor of 10. One should not forget, however, that Eq. (11) is valid only for $r_s \ll 1$ and the actual numbers may differ from quoted above as r_s increases. Although damping from disorder appears to be the dominant effect in solid-state systems, damping due to interaction should be dominant in (fermionic) cold-atom systems with synthetic SOC [3,4], which have virtually no disorder. In this case, the interaction is short-ranged but, as we have already mentioned, this should only affect the prefactor in Eq. (11).

In conclusion, we showed that *eei* in the presence of SOC not only gives rise to a new type of collective modes but also leads to their damping. This damping occurs even at $q = 0$ and its rate scales as the square of the spin-orbit splitting. This effect occurs because none of the three components of magnetization is a conserved in the presence of SOC. This prediction should be important for the experimental studies of such modes via absorption of electromagnetic waves, as discussed in Refs. [8,9,11].

We would like to thank M. Imran and V. Zyuzin for useful discussions. S. M. acknowledges support as a Dirac Post-Doctoral Fellow at the National High Magnetic Field Laboratory, which is supported by the National Science Foundation via Cooperative agreement No. DMR-1157490, the State of Florida, and the U.S. Department of Energy. D. L. M. acknowledges support from the National Science Foundation via Grant No. NSF DMR-1308972.

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