Critical Properties of the Superfluid—Bose-Glass Transition in Two Dimensions

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We investigate the superfluid (SF) to Bose-glass (BG) quantum phase transition using extensive quantum Monte Carlo simulations of two-dimensional hard-core bosons in a random box potential. $T = 0$ critical properties are studied by thorough finite-size scaling of condensate and SF densities, both vanishing at the same critical disorder $W_c = 4.80(5)$. Our results give the following estimates for the critical exponents: $z = 1.85(15)$, $\nu = 1.20(12)$, $\eta = -0.40(15)$. Furthermore, the probability distribution of the SF response $P(\ln\rho_{\mathrm{SF}})$ displays striking differences across the transition: while it narrows with increasing system sizes L in the SF phase, it broadens in the BG regime, indicating an absence of self-averaging, and at the critical point $P(\ln\rho_{\mathrm{SF}} + z\ln L)$ is scale invariant. Finally, high-precision measurements of the local density rule out a percolation picture for the SF-BG transition.

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Introduction.—The interplay between disorder and interactions in condensed matter systems, while intensively studied during the last decades, remains today puzzling in many respects for both experimental and theoretical investigations [1]. First raised by experiments in the late 1980s on superfluid ^{4}He in porous media [2,3], the theoretical question of interacting bosons in the presence of disorder has been addressed at the same time by several pioneer works [4–8]. It was then rapidly understood that for two-dimensional (2D) bosons with repulsive interaction, superfluidity is robust to weak disorder.

A breakthrough came with the thorough study of the critical properties of the quantum ($T = 0$) phase transition between superfluid (SF) and localized Bose-glass (BG) regimes by Fisher *et al.* [8]. In particular, a generalization of the Josephson scaling relations [9] was given, thus predicting new critical exponents (see first line of Table I). Following this work a great endeavor has been made, using exact numerical techniques such as quantum Monte Carlo (QMC) simulations [10–26] or the density matrix renormalization group [27–29], in order to explore in detail the phase diagram of the disordered Bose-Hubbard model. Nevertheless, a general consensus regarding the precise values of the critical exponents at the SF-BG transition is still lacking, despite huge analytical [6,8,30–36] and numerical [12–20,24–26] efforts.

At the same time a wealth of new experiments have been developed, using different techniques and setups: (i) ultracold bosonic atoms in a random potential [37–40]; (ii) strongly disordered superconducting films where preformed Cooper pairs can localize [41–44]; (iii) impurity doped quantum magnets at high field [45–49]. They all have shed new light on the problem of boson localization but raised important theoretical questions, regarding, e.g., the precise nature of the critical point [32–35], the inhomogeneous character of the SF and BG phases [41,42,50,51].

In this Letter, we address two important issues of the Bose-glass problem using the most advanced available exact numerical technique, namely, the stochastic series expansion (SSE) QMC method. The quantum critical behavior at the onset of boson localization and the delicate estimate of the critical exponents are first discussed. Then the inhomogeneous nature of the SF and BG phases is addressed through the study of the probability distribution of the SF response which shows strikingly different properties when increasing lattice sizes. Shrinking in the SF phase, it clearly broadens in the BG regime, thus indicating the absence of self-averaging [52]. We also demonstrate that all sites remain compressible, ruling out a percolation picture. Our conclusions are supported by careful ground-state (GS) simulations through the so-called β-doubling scheme, disorder averaging over a very large number of realizations, detailed error bar evaluation, and systematic finite-size scaling analysis.

Model and quantum Monte Carlo approach.—We consider hard-core bosons at half-filling on a two-dimensional square lattice, described by

$$\mathcal{H} = -t\sum_{\langle ij\rangle}(b_i^\dagger b_j + b_j^\dagger b_i) - \sum_i \mu_i b_i^\dagger b_i, \qquad (1)$$

where hopping between nearest neighbors is fixed to $t = 1/2$, and the random chemical potential μ_i is drawn from a uniform distribution $[-W, W]$; i.e., half-filling is statistically achieved, on average [53]. This model, also relevant to describe many aspects of strongly disordered superconductors [4,5,41,42,50,54], exhibits a quantum ($T = 0$) phase transition between a Bose condensed SF and a localized BG regime at sufficiently strong disorder [13,15,19].

The intrinsic difficulties to simulate with QMC methods the low temperature properties of such a strongly

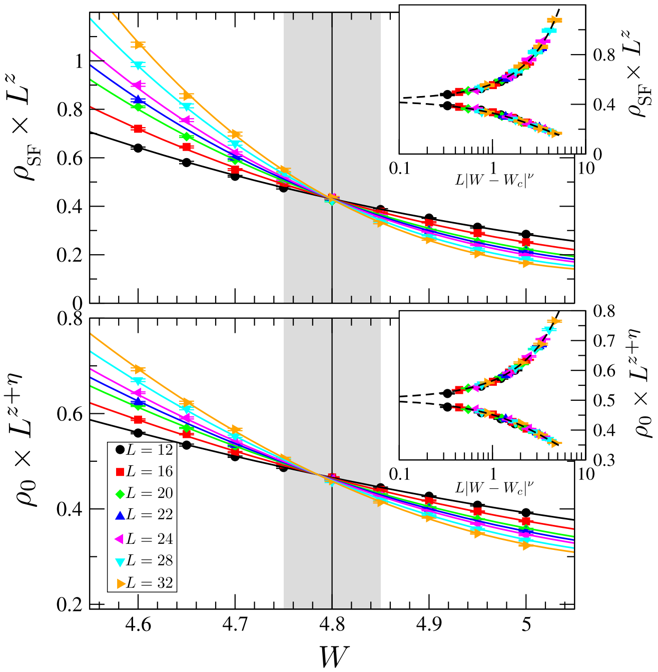

FIG. 1 (color online). Scaling analysis of the SF ρ_{SF} (top) and BEC ρ_0 (bottom) densities. Solid lines show best fits to the universal scaling functions Eqs. (2) and (3) for the full data set with $z \simeq 1.85$, $W_c^{SF} \simeq 4.8$, $W_c^0 \simeq 4.79$, $z + \eta \simeq 1.42$, $\nu_{SF} \simeq 1.1$, $\nu_0 \simeq 1.2$, and $G_{SF|0}$ 3rd order polynomials. The distance from the critical point $W_c = 4.80(5)$ (grey area), when rescaled by $L^{-1/\nu}$ with $\nu = 1.2$, yields a perfect data collapse (insets).

disordered quantum system are twofold: (i) accessing ground-state properties means very long equilibration and simulation times; (ii) statistical uncertainties of the measured physical observables originate from both MC sampling with N_{MC} steps and random sample to sample fluctuations with $\mathcal{N}_s$ samples. Therefore, the simulation time grows very fast as $L^2 \times \beta \times N_{MC} \times \mathcal{N}_s$ which limits the largest system size L reachable. The strategy we adopt to tackle this problem, using the SSE algorithm [55], is as follows (simulation details are discussed in the Supplemental Material [56]). First we use the β-doubling scheme to speed up equilibration towards very low temperature [56,58,59], after what we perform for each sample a number of measurement steps N_{MC}^s (sample dependent) large enough that the SF density is efficiently measured [56]. This procedure is then repeated for a very large number of disorder realizations $\mathcal{N}_s = \mathcal{O}(10^4)$. We have noticed that GS convergence is in practice extremely hard to achieve rigorously for all samples, as some samples may exhibit finite-size gaps smaller than the infrared cutoff of the β-doubling expansion, which is fixed on average. Nevertheless, we have checked that intrinsic MC errors induced by such a slow GS convergence remain smaller than statistical errors. The results, at $\beta t = 2^h$ with $h = 7$ for $L = 12$ up to $h = 9$ for the largest sizes, can therefore be safely interpreted as $T = 0$ ones [56].

Finite-size scaling.—Motivated by the fact that previous works disagree on the values of the critical parameters (see [13,15,19] and Table I), we now discuss our determination of these parameters by the finite-size scaling approach for disorder averaged QMC estimates of the SF and Bose condensed densities.

The ordered regime is characterized by a finite SF density ρ_{SF}, efficiently estimated using the winding number fluctuations in the QMC algorithm [60]. In the vicinity of the 2D quantum critical point, the finite-size scaling of the SF density is

$$\rho_{SF}(L) = L^{-z}\mathcal{G}_{SF}[L^{1/\nu}(W - W_c)], \qquad (2)$$

where z is the dynamical critical exponent, ν the correlation length exponent, W_c the critical disorder, and $\mathcal{G}_{SF}$ a universal function.

Beyond the SF response, one can also probe Bose-Einstein condensation (BEC), occurring in 2D at $T = 0$ where U(1) symmetry can be broken. The BEC density $\rho_0 = \sum_{ij} G_{ij}/N^2$, obtained from the equal time Green's function [61] $G_{ij} = \langle b_i^\dagger b_j \rangle$, plays the role of the order parameter, with a critical scaling

$$\rho_0(L) = L^{-z-\eta}\mathcal{G}_0[L^{1/\nu}(W - W_c)]. \qquad (3)$$

Our QMC data are very nicely described by the above scaling forms, as shown in Fig. 1 for both SF and BEC densities. Strikingly, BEC and SF densities vanish at the same disorder strength $W_c = 4.80(5)$. The values of the critical exponents are given in Table I. This determination results from fits of our data set by Taylor expanding the scaling functions $\mathcal{G}_{SF}$ and $\mathcal{G}_0$ around W_c up to an order large enough that the goodness of fit is acceptable (3rd order in Fig. 1, see Ref. [56]). We have performed a careful error analysis using the bootstrap approach in order to estimate statistical errors of the fit parameters, as well as potential systematic errors by fitting over various ranges of disorder strengths and sizes [56]. This results in conservative uncertainties for the estimates of the critical parameters, as visible in Table I.

TABLE I. Various estimates of critical exponents and disorder strength W_c for the 2D SF-BG transition of model Eq. (1), *n.a.* not available.

z	ν	η	W_c	Reference
2	≥ 1	≤ 0		Fisher *et al.* [8]
0.5(1)	2.2(2)	n.a.	2.5	Makivić *et al.* [13]
2.0(4)	0.90(13)	n.a.	4.95(20)	Zhang *et al.* [15]
1.40(2)	1.10(4)	−0.22(6)	4.42(2)	Priyadarshee *et al.* [19]
1.85(15)	1.20(12)	−0.40(15)	4.80(5)	This work

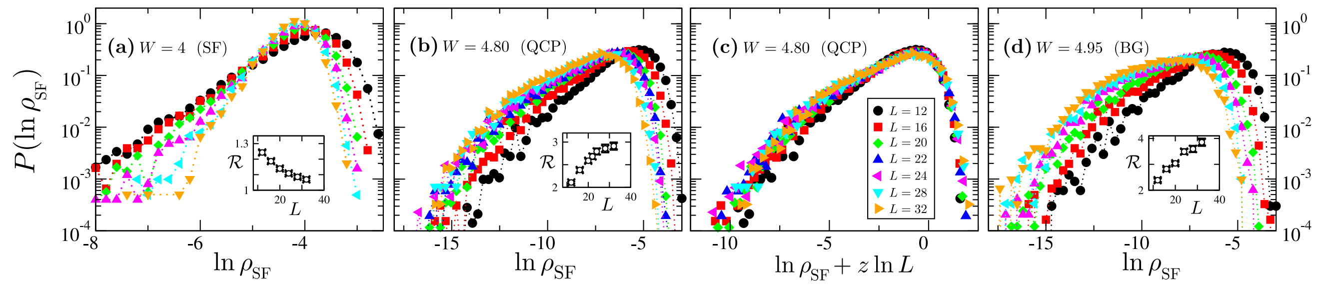

FIG. 2 (color online). Histogram of QMC estimates for $\ln \rho_{\rm SF}$ performed over $\mathcal{N}_s \sim 10^4$ disordered samples for each size L. (a) In the SF regime $W = 4$, distributions get narrower with increasing L whereas in the BG phase (d) for $W = 4.95$ they broaden. At criticality $W_c = 4.8$ [(b),(c)] the broadening stops above $L = 20$ and $P(\ln \rho_{\rm SF} + z \ln L)$ displays a good collapse using $z = 1.85$. The insets show the ratio $\mathcal{R} = \rho_{\rm SF}^{\rm avg}/\rho_{\rm SF}^{\rm typ}$ vs system size L.

We observe a good agreement with the predicted bounds from Fisher *et al.* [8] for $\nu = 1.20(12) \geq 1$ and $\eta = -0.40(15) \leq 0$. Regarding the more debated question of the dynamical exponent [32], while still compatible with $z = 2$ within error bars our best estimate gives a smaller number $z = 1.85(15)$, in agreement with a recent careful estimate for quantum rotors [25]. Comparing with other studies in Table I, our results, obtained with much larger system sizes, agree within error bars with Ref. [15], whereas results in Refs. [13,19] are probably biased due to finite temperature effects and too small disorder averaging.

Distributions and absence of self-averaging in the BG.— In order to go beyond the analysis of the critical properties based on disorder averaged observables, we now turn to the much less studied issue of distributions. The question of a possible broadening of the responses, linked to the issue of self-averaging, has not been studied for 2D bosons, although it may be crucial as discussed for disordered Ising models [62–64] and strongly disordered superconductors [41,42,50]. Here we focus on the probability distribution $P(\ln \rho_{\rm SF})$, obtained by building histograms of QMC estimates for $\ln \rho_{\rm SF}$ over $\mathcal{N}_s$ independent samples, with $\mathcal{N}_s \approx 2 \times 10^4$ for $L \leq 22$ and $\mathcal{N}_s \approx 10^4$ for $L \geq 24$, shown in Fig. 2 for three values of the disorder strength.

In the SF regime [panel (a), $W = 4 < W_c$] the distribution narrows upon increasing the size L, thus demonstrating that the SF response is self-averaging in the ordered phase. Conversely, as visible in panel (d) for the BG regime at $W = 4.95 > W_c$, $P(\ln \rho_{\rm SF})$ broadens when L increases, and moves towards large negative values, as expected in the thermodynamic limit where the SF stiffness vanishes. We therefore expect a difference between average and typical SF densities in the BG: as shown in the insets of Fig. 2, the ratio $\mathcal{R} = \rho_{\rm SF}^{\rm avg}/\rho_{\rm SF}^{\rm typ}$ clearly increases with L in the BG regime [Fig. 2(d)] whereas it goes to 1 in the SF phase [Fig. 2(a)]. At the critical point $W_c = 4.8$ [panels (b) and (c) of Fig. 2], the histograms first broaden for small sizes and then, above $L = 20$ the curves appear self-similar, simply shifted relative to each other. This absence of broadening at large scales is also visible in the inset of Fig. 2(b), where the ratio $\mathcal{R}$ tends to saturate to a constant value. The shift of the distributions can be corrected for by adding $z \ln L$ to $\ln \rho_{\rm SF}$ using our best estimate $z = 1.85$. Indeed, as shown in Fig. 2(c) $P[\ln(L^z \rho_{\rm SF})]$ yields a collapse onto a scale invariant distribution, particularly good above $L = 20$.

The fact that all distributions at W_c are identical up to a shift suggests that, while typical and average SF densities scale differently in the BG regime, their critical scalings are described by the same exponents. Indeed, the typical SF density, defined as $\rho_{\rm SF}^{\rm typ} = \exp(\overline{\ln \rho_{\rm SF}})$ [where $\overline{(\cdots)}$ stands for disorder averaging], can be analyzed using a scaling hypothesis similar to the average Eq. (2), but including additional irrelevant corrections [65]

$$\rho_{\rm SF}^{\rm typ}(L) = L^{-z}(\mathcal{G}_{\rm SF}^{\rm typ}[L^{1/\nu}(W - W_c)] + cL^{-y}). \qquad (4)$$

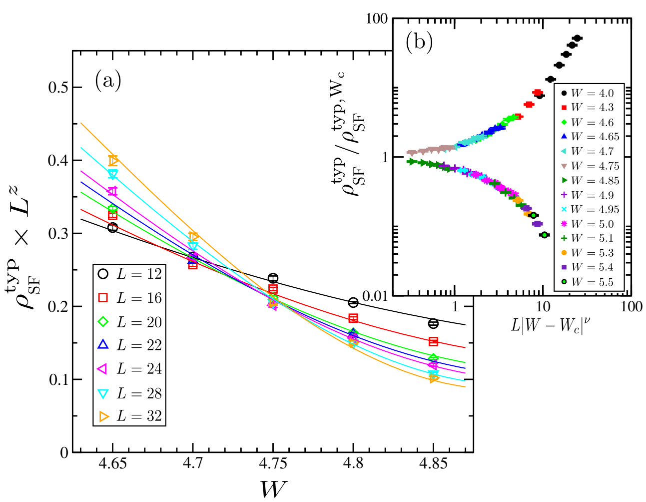

FIG. 3 (color online). Typical SF density (a) plotted as $\rho_{\rm SF}^{\rm typ} \times L^z$ vs W where the crossing at $W_c = 4.8$ has a visible drift, captured by $\mathcal{G}_{\rm SF}^{\rm typ}[L^{1/\nu}(W - W_c)] + cL^{-y}$ with fixed $\nu = 1.2$ and $z = 1.85$, and an estimated irrelevant exponent $y = 0.97(4)$. In panel (b), $\rho_{\rm SF}^{\rm typ}/\rho_{\rm SF}^{{\rm typ},W_c}$ plotted against $L|W - W_c|^\nu$ exhibits an almost perfect collapse of the data for $4 \leq W \leq 5.5$ and $12 \leq L \leq 32$ with no additional parameters.

Because of the presence of irrelevant corrections, a fit of our data set by Eq. (4) with a polynomial $\mathcal{G}_{\mathrm{SF}}$ is unstable unless we fix the critical parameters W_c, z, and ν to our best estimates (Table I). The crossing of $\rho_{\mathrm{SF}}^{\mathrm{typ}} \times L^z$ vs W plotted in Fig. 3(a) displays a non-negligible drift, well captured by irrelevant corrections in Eq. (4) with $y = 0.97(4)$. A nice way to achieve a scaling plot for the typical SF density is then to divide $\rho_{\mathrm{SF}}^{\mathrm{typ}}$ by its value at W_c, this in order to cancel out the irrelevant corrections $\sim L^{-y}$. Next, a rescaling of the length L by the correlation length $\xi = |W - W_c|^{-\nu}$ with $\nu = 1.2$ and $W_c = 4.8$, gives an almost perfect collapse, without any additional adjustable parameters, as shown in Fig. 3(b) for $4.0 \leq W \leq 5.5$ and all available system sizes $12 \leq L \leq 32$. This demonstrates that the quantum critical behaviors of average and typical SF densities are similar, in particular their critical exponents $z_{\mathrm{avg}} = z_{\mathrm{typ}} = 1.85(15)$ and $\nu_{\mathrm{typ}} = \nu_{\mathrm{avg}} = 1.20(12)$.

Coming back to the distributions, the drift observed for the typical stiffness in Fig. 3(a) is related to the transient (irrelevant) broadening of $P(\ln \rho_{\mathrm{SF}})$ observed at small sizes in Fig. 2(b). In order to take such a crossover into account and get rid of irrelevant corrections, we study the broadening of $P(\ln \rho_{\mathrm{SF}})$ using the corrected standard deviation (StD) $\tilde{\sigma}_{\ln \rho_{\mathrm{SF}}} = \sigma_{\ln \rho_{\mathrm{SF}}} - \sigma^c$, where σ^c is the StD at criticality. This is plotted in Fig. 4 vs $L|W - W_c|^{\nu} = L/\xi$, where a very good collapse of the data is achieved without any adjusted parameters. In the SF regime, $\tilde{\sigma}$ converges towards $-\sigma^c$ as $1/\sqrt{N}$ (dashed curve), a consequence of self-averaging. More interestingly, the BG phase features an opposite qualitative behavior with $\tilde{\sigma}$ growing with system size, as $(L/\xi)^{\omega}$ (full line). A careful study of such very broad distributions hits the limits of our numerics, leading to quite large statistical errors, despite the very large number of samples $\mathcal{N}_s = \mathcal{O}(10^4)$, but nevertheless allows us to estimate the exponent $\omega = 0.5(2)$. We interpret this result as follows: The prediction [54] that the stiffness is dominated by quasi-1D paths suggests that one may understand the global SF response ρ_{SF} as a purely local quantity in the BG insulator. Moreover, an analogy [42,50] between the BG and the disordered phase of the random transverse-field Ising model [62], as supported by recent 1D results [66], suggests that the BG is governed by directed-polymer physics in dimension $1+1$ [67]. This predicts an exponent $\omega = 1/3$ [68] for local quantities which is compatible with our estimate.

Local density and absence of percolation.—Finally, we want to discuss some microscopic properties of the insulating BG state. For this we focus on the local bosonic density $\rho_i = \langle b_i^{\dagger} b_i \rangle$, shown in Fig. 5 in the BG regime ($W = 5$) for 16×16, at low enough temperature $\beta t = 1024$ such that the total number of bosons does not fluctuate (see Ref. [56]). Clearly, the average behavior is always compressible, which contrasts with the clean case where the system is incompressible whenever $|\mu| > 2$ [69]. Furthermore, the fraction of incompressible sites with $\rho_i = 0$ or 1 decreases with the number of MC steps and seems to vanish in the exact limit (inset). This shows that percolation through compressible sites is present even in the BG phase, at least from such a single particle view, and is therefore not related to the SF-BG transition, in contrast with some recent discussions [51,70]. Moreover, our

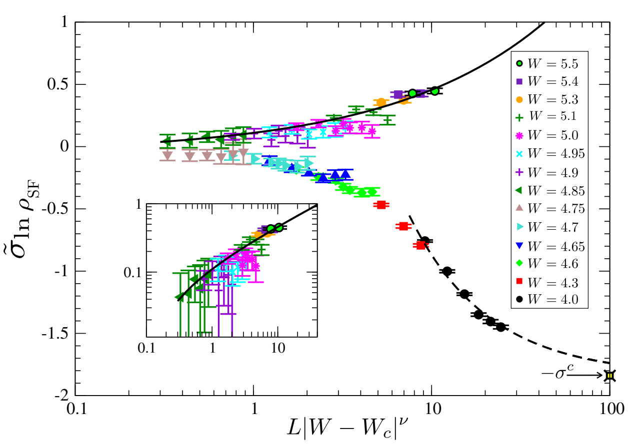

FIG. 4 (color online). Corrected standard deviation of the logarithm of the SF response $\tilde{\sigma}_{\ln \rho_{\mathrm{SF}}} = \sigma_{\ln \rho_{\mathrm{SF}}} - \sigma^c$ vs system size in units of the typical length scale $\xi = |W - W_c|^{-\nu}$. In the SF phase $W < 4.8$, $\tilde{\sigma}$ tends to $-\sigma^c$ as $1/L$ (dashed line), whereas in the BG regime $W > 4.8$, $\tilde{\sigma}$ grows as L^{ω} (full black line) with $\omega = 0.5(2)$. Inset: zoom on the BG regime.

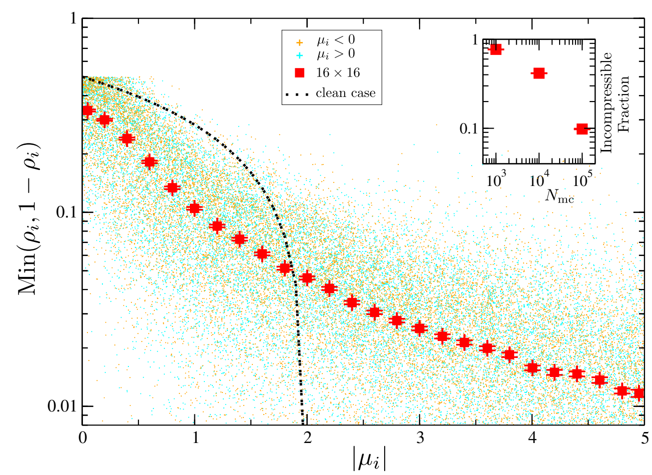

FIG. 5 (color online). Local particle (hole) densities ρ_i $(1 - \rho_i)$ plotted vs local chemical potentials $|\mu_i|$. Small blue (orange) points show QMC results of $\mathcal{N}_s = 150$ samples of size 16×16 measured at $\beta t = 1024$ with $N_{\mathrm{MC}} = 10^5$ steps. The red data points show averages over windows of the chemical potential of size $\bar{\mu} - 0.1 \leq \mu_i \leq \bar{\mu} + 0.1$ and the clean ($W = 0$) result is shown by the dashed line, yielding exactly zero for $|\mu| > 2$ for $\mathrm{Min}(\rho_i, 1 - \rho_i)$ [69]. The inset quantifies the incompressible fraction, i.e., the fraction of sites with $\rho_i = 0$ or $\rho_i = 1$ as a function of MC steps showing that in the exact limit of infinite Markov chains the incompressible fraction tends to zero.

estimates of the critical exponents appear clearly incompatible with those of standard percolation in 2D [71].

Conclusions.—Large-scale QMC simulations of the SF-BG transition supplemented by finite-size scaling show that SF and BEC densities disappear at the same critical disorder strength $W_c \approx 4.8$, with critical exponents $z \approx 1.85$, $\nu \approx 1.2$, and $\eta \approx -0.4$. The SF density distribution becomes infinitely broad upon increasing system size in the BG insulator, a characteristic signature of the absence of self-averaging, supporting the fact that the SF density is a purely local quantity at strong disorder. Our results also rule out a classical percolation scenario of incompressible sites in the BG.

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Note added.—Recently, we became aware of a parallel work [72], which reaches comparable conclusions.

[1] T. Giamarchi, C.R. Phys. **14**, 637 (2013).
[2] D. Finotello, K. A. Gillis, A. Wong, and M. H. W. Chan, Phys. Rev. Lett. **61**, 1954 (1988).
[3] J. D. Reppy, J. Low Temp. Phys. **87**, 205 (1992).
[4] M. Ma and P. A. Lee, Phys. Rev. B **32**, 5658 (1985).
[5] M. Ma, B. I. Halperin, and P. A. Lee, Phys. Rev. B **34**, 3136 (1986).
[6] T. Giamarchi and H. J. Schulz, Europhys. Lett. **3**, 1287 (1987).
[7] D. S. Fisher and M. P. A. Fisher, Phys. Rev. Lett. **61**, 1847 (1988).
[8] M. P. A. Fisher, P. B. Weichman, G. Grinstein, and D. S. Fisher, Phys. Rev. B **40**, 546 (1989).
[9] B. D. Josephson, Phys. Lett. **21**, 608 (1966).
[10] W. Krauth, N. Trivedi, and D. Ceperley, Phys. Rev. Lett. **67**, 2307 (1991).
[11] R. T. Scalettar, G. G. Batrouni, and G. T. Zimanyi, Phys. Rev. Lett. **66**, 3144 (1991).
[12] E. S. Sørensen, M. Wallin, S. M. Girvin, and A. P. Young, Phys. Rev. Lett. **69**, 828 (1992).
[13] M. Makivić, N. Trivedi, and S. Ullah, Phys. Rev. Lett. **71**, 2307 (1993).
[14] P. B. Weichman, Phys. Rev. Lett. **74**, 1038 (1995).
[15] S. Zhang, N. Kawashima, J. Carlson, and J. E. Gubernatis, Phys. Rev. Lett. **74**, 1500 (1995).
[16] J. Kisker and H. Rieger, Phys. Rev. B **55**, R11981 (1997).
[17] F. Alet and E. S. Sørensen, Phys. Rev. E **67**, 015701 (2003).
[18] N. Prokof'ev and B. Svistunov, Phys. Rev. Lett. **92**, 015703 (2004).
[19] A. Priyadarshee, S. Chandrasekharan, J.-W. Lee, and H. U. Baranger, Phys. Rev. Lett. **97**, 115703 (2006).
[20] P. Hitchcock and E. S. Sørensen, Phys. Rev. B **73**, 174523 (2006).
[21] V. Gurarie, L. Pollet, N. V. Prokof'ev, B. V. Svistunov, and M. Troyer, Phys. Rev. B **80**, 214519 (2009).
[22] J. Carrasquilla, F. Becca, A. Trombettoni, and M. Fabrizio, Phys. Rev. B **81**, 195129 (2010).
[23] F. Lin, E. S. Sørensen, and D. M. Ceperley, Phys. Rev. B **84**, 094507 (2011).
[24] S. G. Söyler, M. Kiselev, N. V. Prokof'ev, and B. V. Svistunov, Phys. Rev. Lett. **107**, 185301 (2011).
[25] H. Meier and M. Wallin, Phys. Rev. Lett. **108**, 055701 (2012).
[26] Z. Yao, K. P. C. da Costa, M. Kiselev, and N. Prokof'ev, Phys. Rev. Lett. **112**, 225301 (2014).
[27] R. V. Pai, R. Pandit, H. R. Krishnamurthy, and S. Ramasesha, Phys. Rev. Lett. **76**, 2937 (1996).
[28] S. Rapsch, U. Schollwöck, and W. Zwerger, Europhys. Lett. **46**, 559 (1999).
[29] G. Roux, T. Barthel, I. P. McCulloch, C. Kollath, U. Schollwöck, and T. Giamarchi, Phys. Rev. A **78**, 023628 (2008).
[30] K. G. Singh and D. S. Rokhsar, Phys. Rev. B **46**, 3002 (1992).
[31] I. F. Herbut, Phys. Rev. Lett. **79**, 3502 (1997).
[32] P. B. Weichman and R. Mukhopadhyay, Phys. Rev. Lett. **98**, 245701 (2007).
[33] E. Altman, Y. Kafri, A. Polkovnikov, and G. Refael, Phys. Rev. B **81**, 174528 (2010).
[34] Z. Ristivojevic, A. Petković, P. Le Doussal, and T. Giamarchi, Phys. Rev. Lett. **109**, 026402 (2012).
[35] S. Iyer, D. Pekker, and G. Refael, Phys. Rev. B **85**, 094202 (2012).
[36] J. P. Álvarez Zúñiga and N. Laflorencie, Phys. Rev. Lett. **111**, 160403 (2013).
[37] M. White, M. Pasienski, D. McKay, S. Q. Zhou, D. Ceperley, and B. DeMarco, Phys. Rev. Lett. **102**, 055301 (2009).
[38] B. Deissler, M. Zaccanti, G. Roati, C. D'Errico, M. Fattori, M. Modugno, G. Modugno, and M. Inguscio, Nat. Phys. **6**, 354 (2010).
[39] S. Krinner, D. Stadler, J. Meineke, J.-P. Brantut, and T. Esslinger, Phys. Rev. Lett. **110**, 100601 (2013).
[40] C. D'Errico, E. Lucioni, L. Tanzi, L. Gori, G. Roux, I. P. McCulloch, T. Giamarchi, M. Inguscio, and G. Modugno, Phys. Rev. Lett. **113**, 095301 (2014).
[41] B. Sacépé, T. Dubouchet, C. Chapelier, M. Sanquer, M. Ovadia, D. Shahar, M. Feigel'man, and L. Ioffe, Nat. Phys. **7**, 239 (2011).
[42] G. Lemarié, A. Kamlapure, D. Bucheli, L. Benfatto, J. Lorenzana, G. Seibold, S. C. Ganguli, P. Raychaudhuri, and C. Castellani, Phys. Rev. B **87**, 184509 (2013).
[43] E. F. C. Driessen, P. C. J. J. Coumou, R. R. Tromp, P. J. de Visser, and T. M. Klapwijk, Phys. Rev. Lett. **109**, 107003 (2012).
[44] M. Mondal, A. Kamlapure, S. C. Ganguli, J. Jesudasan, V. Bagwe, L. Benfatto, and P. Raychaudhuri, Sci. Rep. **3**, 1357 (2013).
[45] T. Hong, A. Zheludev, H. Manaka, and L.-P. Regnault, Phys. Rev. B **81**, 060410 (2010).
[46] D. Hüvonen, S. Zhao, M. Månsson, T. Yankova, E. Ressouche, C. Niedermayer, M. Laver, S. N. Gvasaliya, and A. Zheludev, Phys. Rev. B **85**, 100410 (2012).
[47] R. Yu, L. Yin, N. S. Sullivan, J. S. Xia, C. Huan, A. Paduan-Filho, N. F. Oliveira Jr, S. Haas, A. Steppke,

C. F. Miclea, F. Weickert, R. Movshovich, E.-D. Mun, B. L. Scott, V. S. Zapf, and T. Roscilde, Nature (London) **489**, 379 (2012).
[48] M. Vojta, Phys. Rev. Lett. **111**, 097202 (2013).
[49] A. Zheludev and T. Roscilde, C.R. Phys. **14**, 740 (2013).
[50] M. V. Feigel'man, L. B. Ioffe, and M. Mézard, Phys. Rev. B **82**, 184534 (2010).
[51] S. Krinner, D. Stadler, J. Meineke, J.-P. Brantut, and T. Esslinger, arXiv:1311.5174.
[52] A. Hegg, F. Krüger, and P. W. Phillips, Phys. Rev. B **88**, 134206 (2013).
[53] P. B. Weichman and R. Mukhopadhyay, Phys. Rev. B **77**, 214516 (2008).
[54] G. Seibold, L. Benfatto, C. Castellani, and J. Lorenzana, Phys. Rev. Lett. **108**, 207004 (2012).
[55] O. F. Syljuåsen and A. W. Sandvik, Phys. Rev. E **66**, 046701 (2002).
[56] See Supplemental Material at http://link.aps.org/supplemental/10.1103/PhysRevLett.114.155301 for simulation details and data analysis methods, which includes Ref. [57].
[57] P. Young, arXiv:1210.3781.
[58] A. W. Sandvik, Phys. Rev. B **66**, 024418 (2002).
[59] N. Laflorencie, S. Wessel, A. Läuchli, and H. Rieger, Phys. Rev. B **73**, 060403 (2006).
[60] E. L. Pollock and D. M. Ceperley, Phys. Rev. B **36**, 8343 (1987).
[61] A. Dorneich and M. Troyer, Phys. Rev. E **64**, 066701 (2001).
[62] D. S. Fisher, Phys. Rev. B **51**, 6411 (1995).
[63] A. P. Young and H. Rieger, Phys. Rev. B **53**, 8486 (1996).
[64] S. Wiseman and E. Domany, Phys. Rev. Lett. **81**, 22 (1998).
[65] A. Rodriguez, L. J. Vasquez, K. Slevin, and R. A. Römer, Phys. Rev. B **84**, 134209 (2011).
[66] H. Javan Mard, J. A. Hoyos, E. Miranda, and V. Dobrosavljević, Phys. Rev. B **90**, 125141 (2014).
[67] C. Monthus and T. Garel, J. Phys. A **45**, 095002 (2012).
[68] D. A. Huse, C. L. Henley, and D. S. Fisher, Phys. Rev. Lett. **55**, 2924 (1985).
[69] T. Coletta, N. Laflorencie, and F. Mila, Phys. Rev. B **85**, 104421 (2012).
[70] A. E. Niederle and H. Rieger, New J. Phys. **15**, 075029 (2013).
[71] A. Aharony and D. Stauffer, *Introduction to Percolation Theory* (Taylor & Francis, London, 2003).
[72] R. Ng and E. S. Sørensen, arXiv:1501.05981.