# Linear Mode Stability of the Kerr-Newman Black Hole and Its Quasinormal Modes

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We provide strong evidence that, up to 99.999% of extremality, Kerr-Newman black holes (KNBHs) are linear mode stable within Einstein-Maxwell theory. We derive and solve, numerically, a coupled system of two partial differential equations for two gauge invariant fields that describe the most general linear perturbations of a KNBH. We determine the quasinormal mode (QNM) spectrum of the KNBH as a function of its three parameters and find no unstable modes. In addition, we find that the lowest radial overtone QNMs that are connected continuously to the gravitational $\ell = m = 2$ Schwarzschild QNM dominate the spectrum for all values of the parameter space ($m$ is the azimuthal number of the wave function and $\ell$ measures the number of nodes along the polar direction). Furthermore, the (lowest radial overtone) QNMs with $\ell = m$ approach $\mathrm{Re}\omega = m\Omega_H^{\mathrm{ext}}$ and $\mathrm{Im}\omega = 0$ at extremality; this is a universal property for any field of arbitrary spin $|s| \leq 2$ propagating on a KNBH background ($\omega$ is the wave frequency and $\Omega_H^{\mathrm{ext}}$ the black hole angular velocity at extremality). We compare our results with available perturbative results in the small charge or small rotation regimes and find good agreement.

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*Introduction.*—The uniqueness theorems [1,2] state that the Kerr-Newman black hole (KNBH) [3,4] is the unique, most general family of stationary asymptotically flat black holes (BHs) of Einstein-Maxwell theory. It is characterized by three parameters: mass $M$, angular momentum $J \equiv Ma$, and charge $Q$. The Kerr, Reissner-Nordström (RN) and Schwarzschild (SCHW) BHs constitute limiting cases: $Q = 0$, $a = 0$, and $Q = a = 0$, respectively.

Given their uniqueness, the most relevant question to consider is the linear mode stability of these BHs. It is known that the Kerr, RN, and SCHW BHs are linear mode stable. Indeed, the perturbation study of the linearized Einstein(-Maxwell) equation gives the quasinormal mode (QNM) spectrum of frequencies (that describes the damped oscillations of the BH back to equilibrium) resulting in no unstable modes [5–21] (see review [22]). Remarkably, for these BHs, the QNM spectrum turns out to be encoded in a single separable equation—known as the (odd mode) Regge-Wheeler and (even mode) Zerilli equations [5–7] (for RN and SCHW) and the Teukolsky equation [13] (for Kerr, RN, and SCHW)—that effectively yields a pair of ordinary differential equations (ODEs). Even though the nonlinear stability of Kerr remains an open problem (see, however, [23–25] for recent progress), it is also believed to be stable beyond the linear level.

Unfortunately, it does not seem possible to cast a general perturbation of a KNBH as a single partial differential equation (PDE). Therefore, obtaining the QNM spectrum of KNBHs requires solving coupled PDEs. Naïvely, one expects to find a system of nine coupled PDEs. However, working in the so-called phantom gauge Chandrasekhar reduced the problem to the study of two coupled PDEs [15] (see also [26]). Despite this significant progress, finding the QNM spectrum and addressing the problem of the linear mode stability of the KNBH has remained a major open problem of Einstein-Maxwell theory since the 1980s, when Chandrasekhar's seminal work [15] was published.

Recently, there have been some notable efforts to address this problem. References. [26–28] have found the QNM spectrum in a perturbative small rotation and charge, respectively, expansion around the RN and Kerr BHs. These works find no sign of linear instability; however, such an instability is more likely to be found in extreme regimes where both $Q$ and $a$ are large. Another remarkable effort to infer the (non-)linear stability of KNBHs has been made in [29], where the full time evolution of some KNBH with a given initial perturbation is considered, finding no sign of a nonlinear instability. However, since nonlinear simulations are computationally costly, the search in moduli space is modest.

In this Letter, we derive two coupled PDEs that reduce to the Chandrasekhar coupled PDE system upon gauge fixing and compute the QNM spectrum of the KNBH to a high degree of accuracy. Up to 99.999% of extremality, we find no sign of a linear mode instability for any of the gravitoelectromagnetic modes that are described by $\ell = 1, 2, 3, 4$ and $|m| \leq \ell$. We use two distinct numerical methods that have been developed to solve efficiently similar systems of (several coupled) ODEs and PDEs that appear in QNM, superradiant and ultraspinning instability studies [30–39]. One of these methods formulates the problem as a quadratic eigenvalue problem in the frequency and employs a pseudospectral grid collocation. The other method searches directly for specific QNMs using a Newton-Raphson root-finding algorithm. We refer the reader to [30–39] for details. The pseudospectral exponential convergence of our method, and the use of quadruple precision, guarantees that the results are accurate up to, at least, the tenth decimal place.

*Notation.*—We use the standard nomenclature of the Newman-Penrose formalism to denote components of the curvatures, electromagnetic field strength, and connections [40]. $X^{(0)}$ denotes background quantities, while $X^{(1)}$ denotes a perturbed quantity at the linear order.

*Formulation of the problem.*—We write the KNBH solution in standard Boyer-Lindquist coordinates $\{t, r, \theta, \phi\}$ (time, radial, polar, azimuthal coordinates) [4]. Its event horizon, with angular velocity $\Omega_H$ and temperature $T_H$, is generated by the Killing vector $K = \partial_t + \Omega_H \partial\phi$. The location of the horizon $r_+$ is given by the largest root of the function $\Delta$. These quantities are given in terms of the parameters $\{M, a, Q\}$ as follows:

$$\begin{aligned}\Delta &= r^2 - 2Mr + a^2 + Q^2,\\ r_+ &= M + \sqrt{M^2 - a^2 - Q^2},\\ \Omega_H &= \frac{a}{r_+^2 + a^2},\\ T_H &= \frac{1}{4\pi r_+}\frac{r_+^2 - a^2 - Q^2}{r_+^2 + a^2}.\end{aligned} \tag{1}$$

The KNBH has a regular extremal configuration when its temperature vanishes and its angular velocity reaches a maximum. For fixed $M$ and $Q$, this occurs for the extremal rotation parameter $a = a_{\text{ext}} = \sqrt{M^2 - Q^2}$. Thus, at extremality (ext) we have

$$T_H^{\text{ext}} = 0 \Leftrightarrow \Omega_H^{\text{ext}} = \frac{\sqrt{M^2 - Q^2}}{2M^2 - Q^2} = \frac{a_{\text{ext}}}{M^2 + a_{\text{ext}}^2}. \tag{2}$$

We consider the most general perturbation of a KNBH (except for trivial modes that shift the parameters of the solution). Using the fact that $\partial_t$ and $\partial_\phi$ are Killing vector fields of the Kerr-Newman (KN) background, we Fourier decompose its perturbations as $e^{-i\omega t}e^{im\phi}$. This introduces the frequency $\omega$ and azimuthal quantum number $m$ of the perturbation. By formulating the perturbation problem in the Newman-Penrose (NP) formalism, we obtain a set of two coupled partial differential equations that describe the perturbations of a KNBH

$$\begin{aligned}&\left(\mathcal{O}_{-2} + \Phi_{11}^{(0)}\mathcal{P}_{-2}\right)\varphi_{-2} + \Phi_{11}^{(0)}\mathcal{Q}_{-2}\varphi_{-1} = 0,\\ &\left(\mathcal{O}_{-1} + \Phi_{11}^{(0)}\mathcal{P}_{-1}\right)\varphi_{-1} + \Phi_{11}^{(0)}\mathcal{Q}_{-1}\varphi_{-2} = 0,\end{aligned} \tag{3}$$

where differential operators $\{\mathcal{O}, \mathcal{P}, \mathcal{Q}\}$ are given in the Supplemental Material [41], $\varphi_{-2} = \Psi_4^{(1)}$ and $\varphi_{-1} = 2\Phi_1^{(0)}\Psi_3^{(1)} - 3\Psi_2^{(0)}\Phi_2^{(1)}$ (the $\Psi$'s and $\Phi$'s are standard NP scalars defined in the Supplemental Material [41]).

Substituting the background values of the NP quantities, the above equations reduce to

$$\begin{aligned}&(\mathcal{F}_{-2} + Q^2\mathcal{G}_{-2})\psi_{-2} + Q^2\mathcal{H}_{-2}\psi_{-1} = 0,\\ &(\mathcal{F}_{-1} + Q^2\mathcal{G}_{-1})\psi_{-1} + Q^2\mathcal{H}_{-1}\psi_{-2} = 0,\end{aligned} \tag{4}$$

where second order differential operators $\{\mathcal{F}, \mathcal{G}, \mathcal{H}\}$ are given in the Supplemental Material [41] and

$$\begin{aligned}\psi_{-2} &= (\bar{r}^*)^4\Psi_4^{(1)},\\ \psi_{-1} &= \frac{(\bar{r}^*)^3}{2\sqrt{2}\Phi_1^{(0)}}\left(2\Phi_1^{(0)}\Psi_3^{(1)} - 3\Psi_2^{(0)}\Phi_2^{(1)}\right),\end{aligned} \tag{5}$$

with $\bar{r} = r + ia\cos\theta$. We emphasise that $\psi_{-2}$ and $\psi_{-1}$ (as well as $\varphi_{-2}$ and $\varphi_{-1}$) are gauge invariant perturbed quantities; i.e., they are invariant under both linear diffeomorphisms and tetrad rotations. Furthermore, these are the NP scalars that are relevant for the study of perturbations that are outgoing at future null infinity and regular at the future horizon [42]. Fixing a gauge in which $\Phi_0^{(1)} = \Phi_1^{(1)} = 0$, we obtain the Chandrasekhar coupled PDE system [15] (see also the derivation in [26]). Finally, note that in the limit $Q \to 0$ and/or $a \to 0$ these equations decouple yielding the Teukolsky equation.

In order to solve these equations, we need to impose appropriate boundary conditions. The $t - \phi$ symmetry of the KNBH guarantees that we can consider only modes with $m \geq 0$, say, as long as we consider both positive and negative $\mathrm{Re}(\omega)$; when $a = 0$, this enhances to a $t \to -t$ symmetry and the QNM frequencies form pairs of $\{\omega, -\omega^*\}$.

At spatial infinity, a Frobenius analysis of (4), and the requirement that we have only outgoing waves, fixes the decay to be ($s = -2, -1$)

$$\psi_s|_\infty \simeq e^{i\omega r}r^{-(2s+1)+i\omega\frac{r_+^2+a^2+Q^2}{r_+}}\left(\alpha_s(\theta) + \frac{\beta_s(\theta)}{r} + \cdots\right),$$

where $\beta_s(\theta)$ is a function of $\alpha_s(\theta)$ and its derivatives along $\theta$, whose exact form is fixed by expanding (4) around spatial infinity.

At the horizon, a Frobenius analysis, and requiring only regular modes in ingoing Eddington-Finkelstein coordinates, yields the near-horizon expansion

$$\psi_s|_H \simeq (r - r_+)^{-s-\frac{i(\omega - m\Omega_H)}{4\pi T_H}}[a_s(\theta) + b_s(\theta)(r - r_+) + \cdots],$$

where $b_s(\theta)$ is related to $a_s(\theta)$ and its tangential derivatives along $\theta$.

At the North (South) pole $x \equiv \cos\theta = 1(-1)$, regularity dictates that the fields must behave as ($\varepsilon = 1$ for $m \geq 2$, while $\varepsilon = -1$ for $m = 0, 1$ modes)

$$\psi_s|_{N,(S)} \simeq (1 \mp x)^{\varepsilon^{(1\pm 1/2)}(s+m/2)}[A_s^{\pm}(r) + B_s^{\pm}(r)(1 \mp x) + \cdots],$$

where $B_s^+(r)[B_s^-(r)]$ is a function of $A_s^+(r)[A_s^-(r)]$ and its derivatives along $r$, whose exact form is fixed by expanding (4) around the North (South) pole.

We consider only modes with the lowest radial overtone ($n = 0$) because these are the ones that have smaller $|\mathrm{Im}\omega|$, and thus, they are the ones that dominate a time evolution and are more likely to become unstable near extremality. Note, also, that we can scale out one of the three parameters of the solution. Thus, we work with the adimensional parameters $\{a/M, Q/M\}$ (or $\{a/r_+, Q/r_+\}$) and $\omega M$.

*Results and discussion.*—Our primary aim is to find whether KNBHs can be linear mode unstable. For that, we study the QNM spectrum and check if there are modes with $\mathrm{Im}\omega > 0$. Note that, for $Q$, $a \to 0$, we ought to recover the SCHW QNMs. In this limit, there are two families of QNMs, namely the Regge-Wheeler (odd or axial) modes and the Zerilli (even or polar) modes. These families are isospectral; i.e., they have exactly the same spectrum [15]. Thus, we only need to distinguish the gravitational modes (described in Table V of page 262 [15]—hereafter, Table of [15]—by the eigenfunction $Z_2$) from the electromagnetic modes (described in Table of [15] by the eigenfunction $Z_1$). (These QNMs are also computed more accurately in more recent studies; see [22]). Each of these is specified by the harmonic number $\ell = 1, 2, 3, \cdots$ ($Z_2$ modes with $\ell = 1$ are pure gauge modes). When the BH has charge and rotation, we have to scan a two parameter space in $\{Q/M, a/M\}$. The above two families become coupled gravitoelectromagnetic QNMs and the Schwarzschild eigenvalue $\ell$ does not appear explicitly in the KN PDEs (4). However, we can still count the number of nodes along the polar direction of the eigenfunctions of (4) and this gives $\ell$.

We perform a complete scan in $\{Q/M, a/M\}$ for modes with $\ell = 1, 2, 3$, $|m| \leq \ell$ (both in the $Z_1$ and $Z_2$ sectors). For each family, we focus on QNMs with the lowest radial overtone and smallest $|\mathrm{Im}\omega|$ because these are the least damped and could eventually become unstable for large $J$ and $Q$. Modes with $\ell = 4$ are also studied, but there, we focus on modes that approach $\mathrm{Im}\omega = 0$ at extremality. As one of our main results, we do not find any unstable mode with $\mathrm{Im}\omega > 0$, even when we probe regions in parameter space for which $a/a_{\mathrm{ext}} = 0.99999$. We see this as good numerical evidence that the KNBH is linearly mode stable.

To illustrate our search, in Fig. 1, we take KNBHs with $Q = a$ and we display all the QNMs that are continuously connected to the gravitational $Z_2$ SCHW QNM with

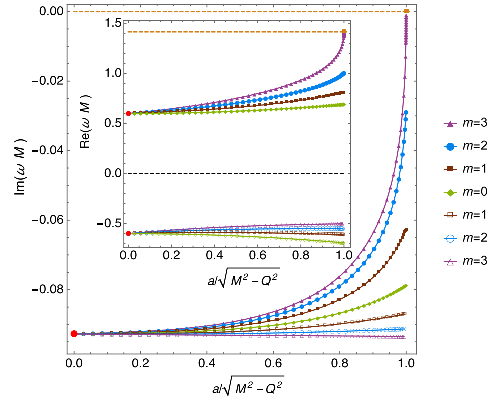


FIG. 1 (color online). All lowest radial overtone QNMs of $Q = a$ KNBHs that start at the $\ell = 3$ SCHW gravitational QNM (red disc). Note that the family of axisymmetric QNMs ($m = 0$) form pairs of $\{\omega, -\omega^*\}$.

$\ell = 3$, namely $\omega M = 0.59944329 - 0.09270305i$ (see Table of [15]). The different QNMs are distinguished by their azimuthal number $m = 0, 1, 2, 3$ and by whether they have positive or negative $\mathrm{Re}\omega$ (modes with $m = 0$ have a pair of QNMs $\{\omega, -\omega^*\}$; see discussion above). All these modes become degenerate in the Schwarzschild limit (red disk in Fig. 1). We plot the imaginary (main plot) and real (inset plot) parts of the frequency $\omega M$ as a function of $a/a_{\mathrm{ext}} = a/\sqrt{M^2 - Q^2}$. We see that the most likely mode to be unstable is the $\ell = 3$ mode with $\mathrm{Re}\omega > 0$ (magenta triangles). However, we follow this mode up to $a/a_{\mathrm{ext}} = 0.99999$ and find that, although the $\mathrm{Im}\omega$ quickly approaches zero as $a \to a_{\mathrm{ext}}$, it never crosses $\mathrm{Im}\omega = 0$.

It is also relevant to ask what are the dominant QNMs, i.e., the modes with the smallest $|\mathrm{Im}\omega|$. We find that the QNM family that, in the $Q$, $a \to 0$ limit, approaches the $Z_2$ SCHW QNM with $\ell = m = 2$ with $\omega M = 0.37367168 - 0.08896232i$ (Table of [15]) is the one that always (i.e., for a given $Q$ and $a$) has the smallest $|\mathrm{Im}\omega|$. Therefore, these QNMs must be the dominant modes in a time evolution of the KNBH. Since this mode is the most relevant in a time evolution process, hereafter, we will use it to illustrate our discussions (other modes will be presented elsewhere).

The plots of Fig. 2 (real part of $\omega M$) and Fig. 3 (imaginary part of $\omega M$) give details of the $Z_2$, $\ell = m = 2$ mode. We represent the QNMs of the $Q = a$ KNBH by blue disks, but we also present the QNMs for KNBHs with fixed charge $Q/r_+$ (see plot legends) as the rotation grows from zero to $a_{\mathrm{ext}}/M$. Figure 2 shows that, as extremality is approached, we always have $\mathrm{Re}\omega \to 2\Omega_H^{\mathrm{ext}}$. On the other hand, Fig. 3 shows that $\mathrm{Im}\omega \to 0^-$ as extremality is approached. Again, we emphasize that the last point of each of these curves is, at least, at $a = 0.99999a_{\mathrm{ext}}$. Figure 1, Fig. 2, and Fig. 3 illustrate a general property of the KN QNMs with $\ell = m$ (both in the $Z_1$ and $Z_2$ sectors): as extremality is approached one has $\mathrm{Re}\omega \to m\Omega_H^{\mathrm{ext}}$ and $\mathrm{Im}\omega \to 0^-$. Collecting previous results [14,20,43,44],

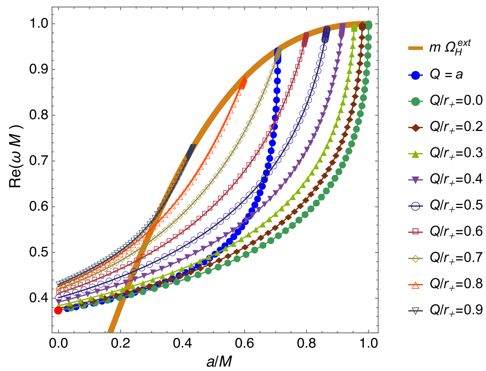


FIG. 2 (color online). Real part of the QNM frequencies with $\ell = m = 2$ of KNBHs with fixed $Q/r_+ = 0.0, 0.2, \cdots, 0.9$ that start at the SCHW gravitational QNM with $\ell = 2$ (red disc). We also show the $\ell = m = 2$ QNMs of $Q = a$ KNBHs.

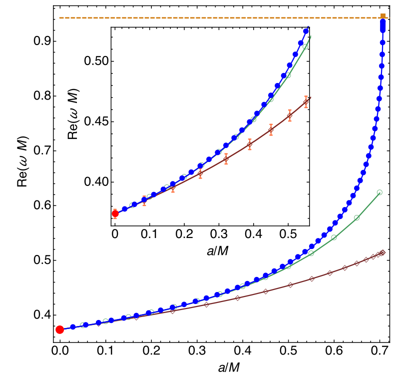


FIG. 4 (color online). Comparison (for Re$\omega$) between the exact $\ell = m = 2$ $Z_2$ QNMs of KN with $Q = a$ (blue disks) with the small $a$ approximations of [27] (red diamonds with their 1% error bar) and with the small $Q$ approximations of [26] (green circles).

particularly in [45], we can now state that this is a universal property for any perturbation spin $s$. Note that [46] proved that, at the onset of an instability, i.e., when Im$\omega = 0$, the superradiant inequality Re$(\omega - m\Omega_H) \leq 0$ must necessarily be obeyed. We find that, as extremality is approached, the linearized modes of KN saturate this superradiant condition (but do not become unstable). As we discussed above, in a three-dimensional $\{Q/M, a/M, \mathrm{Im}(\omega M)\}$ plot, all the $\ell = m \neq 2$ QNMs are below the $Z_2$, $\ell = m = 2$.

Previously, there were some attempts to find the QNMs of the KNBH using a perturbative analysis, notably for small $a$ around the RN QNMs [27,28] and for small $Q$ around the Kerr QNMs [26]. We use these perturbative results to further check our results for small $a$ or $Q$, thus, establishing the regimes of validity of the aforementioned approximations. To compare the two perturbative analyses in a single graphic, it is convenient to look at QNMs with $Q = a$, see Fig. 4 (real part) and Fig. 5 (imaginary part). We see that the approximations of [27,28] ([26]) are within 1% of the exact results up to ~25% (~70%) of extremality. The different accuracy of the two results is most probably due to the fact that the weakly charged result is accurate up to $\mathcal{O}(Q^2/M^2)$ [26], while the slowly rotating results are accurate only up to $\mathcal{O}(a/M)$ [27,28].

Reference [29] considered the full time evolution of some KNBHs with a given initial perturbation and found no sign of a nonlinear instability, which is consistent with our full parameter scan of the QNMs up to $a/a_{\mathrm{ext}} = 0.99999$. Reference [29] also finds numerical evidence that some $\ell = m = 2$ QNMs of a KNBH (with $a/Q > 1$) should have the scaling $\omega = \omega(a/\sqrt{M^2 - Q^2})$. We can test this claim with higher accuracy and we find that it does not hold (although it must be emphasized that our linear results are well within the numerical accuracy of [29]; they differ by, at most, 1%; the error in our results is $\pm 10^{-10}$ which is some order of magnitudes smaller than this difference). Coming back to the fact that these modes approach $m\Omega_H^{\mathrm{ext}} + 0i$ at extremality, this had to be the case since $\Omega_H^{\mathrm{ext}} \neq \Omega_H^{\mathrm{ext}}(a/\sqrt{M^2 - Q^2})$. To summarize, although the proposed scaling fails to hold just by a small relative amount (less than 1%), our numerical and analytical considerations show that it is only approximate, but not exact.

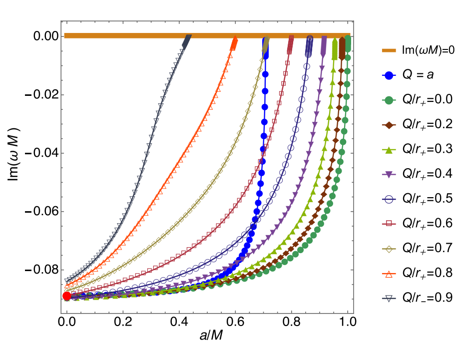


FIG. 3 (color online). Similar to Fig. 2 but now for Im$(\omega M)$.

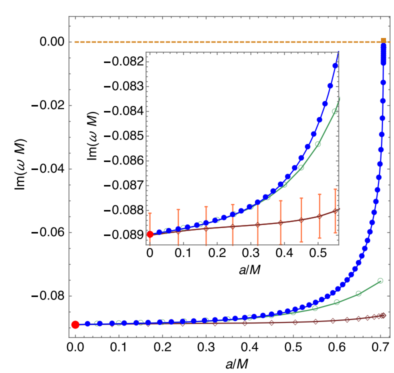


FIG. 5 (color online). Similar comparison to that in Fig. 4 but now for Im$\omega$.

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[41] See Supplemental Material at http://link.aps.org/supplemental/10.1103/PhysRevLett.114.151101 for a brief self-contained description of the Newman-Penrose formalism. This formalism is then used to derive the set of Eqs. (3) and (4) for the two gauge invariant fields (5) that describe the perturbations of the KNBH. We also give the explicit expressions for the differential operators that are introduced in Eqs. (3) and (4).

[42] There is a set of two coupled PDEs—related to (4) by a Geroch-Held-Penrose [12] transformation—for the quantities $\psi_2$ and $\psi_1$ that are the positive spin counterparts of (4); however, these would be relevant if we were interested in perturbations that were outgoing at past null infinity.

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