Experimental Estimation of Average Fidelity of a Clifford Gate on a 7-Qubit Quantum Processor

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One of the major experimental achievements in the past decades is the ability to control quantum systems to high levels of precision. To quantify the level of control we need to characterize the dynamical evolution. Full characterization via quantum process tomography is impractical and often unnecessary. For most practical purposes, it is enough to estimate more general quantities such as the average fidelity. Here we use a unitary 2-design and twirling protocol for efficiently estimating the average fidelity of Clifford gates, to certify a 7-qubit entangling gate in a nuclear magnetic resonance quantum processor. Compared with more than 10^8 experiments required by full process tomography, we conducted 1656 experiments to satisfy a statistical confidence level of 99%. The average fidelity of this Clifford gate in experiment is 55.1%, and rises to at least 87.5% if the signal's decay due to decoherence is taken into account. The entire protocol of certifying Clifford gates is efficient and scalable, and can easily be extended to any general quantum information processor with minor modifications.

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Introduction.—Benchmarking protocols for characterizing the level of coherent control are fundamental in evaluating potential quantum information processing (QIP) devices. They provide an objective comparison of quantum control capabilities between diverse QIP devices, and also indicate the prospects of a given platform with respect to fault-tolerant quantum computation [1]. Quantum process tomography (QPT) [2,3], the standard method for completely characterizing a quantum channel, requires a number of measurements that scale exponentially with the number of qubits n ($\approx 2^{4n}$), making it impractical even in relatively small systems. To date, QPT has been applied to at most 3-qubit systems in experiment [4–11]. Fortunately, full characterization is not necessary for many practical purposes and more accessible properties of the gates are sufficient. To benchmark a gate it is enough to estimate the distance between the implemented channel and the ideal gate. A standard figure of merit for benchmarking the level of control is the average fidelity, and several methods have been proposed to evaluate it in an efficient manner. These include randomized benchmarking [12–14], twirling [15–17], and Monte Carlo estimations [18,19], each with its own restrictions and drawbacks. Here, in order to benchmark our coherent controls on a 7-qubit nuclear magnetic resonance (NMR) system, we adopted the twirling protocol [17] to estimate the average fidelity of an important Clifford gate in QIP. The gate of interest generates maximal coherence from single coherence with the aid of local rotations, and is of critical importance to many QIP tasks such as the creation of a cat state in multiqubit systems. The estimation method is scalable and independent of the number of qubits, and is straightforward to implement in other quantum information processing architectures.

For the twirling protocol we conducted only 1656 experiments compared with about 2.7×10^8 experiments required for fully characterizing the 7-qubit gate via QPT. The average fidelity of the certified gate is 55.1% before accounting for decoherence and rises above 87.5% by factoring out the decoherence effect. Moreover, the NMR spectra based on the application of this Clifford gate are in excellent agreement with the simulation results.

Theory.—Let $\mathcal{U}$ be a superoperator representation of the Clifford gate U that we want to implement and $\tilde{\mathcal{U}} = \Lambda \circ \mathcal{U}$ be the superoperator representation of the real evolution in the laboratory experiment. We call Λ the noise superoperator and our task is to estimate its average fidelity with respect to the identity. The method described below is based on twirling [20] and the construction of a unitary 2-design [16].

Given a fiducial pure state $|\psi\rangle$, the average fidelity (with respect to the identity) is the quantum fidelity

$\langle\psi|\Lambda(|\psi\rangle\langle\psi|)|\psi\rangle$ averaged over all pure states $V|\psi\rangle$, where V is an arbitrary unitary transformation. Averaging over the entire Hilbert space can be done using the Haar measure $d\mu(\mathcal{V})$ [12], so

$$\bar{F}(\Lambda) = \int d\mu(\mathcal{V}_U)\langle\psi|\mathcal{V}_U^\dagger\circ\Lambda\circ\mathcal{V}_U(|\psi\rangle\langle\psi|)|\psi\rangle. \quad (1)$$

Here, $\mathcal{V}$ is the superoperator representation of a unitary V and $\mathcal{V}_U = \mathcal{U}\circ\mathcal{V}$. In this notation it is easy to see that the average fidelity depends only on Λ.

Using a unitary 2-design based on the Clifford group, it is possible to simplify Eq. (1) to

$$\bar{F}(\Lambda) = \frac{1}{|\mathcal{C}_n|}\sum_{\mathcal{C}_i\in\mathcal{C}_n}\langle\psi|\mathcal{C}_i^\dagger\circ\Lambda\circ\mathcal{C}_i(|\psi\rangle\langle\psi|)|\psi\rangle, \quad (2)$$

where $\mathcal{C}_n$ is the n-qubit Clifford group $\mathcal{C}_n$. The average fidelity is therefore equivalent to the fidelity of the average channel

$$\bar{\Lambda}_{\mathcal{C}_n} = \frac{1}{|\mathcal{C}_n|}\sum_{\mathcal{C}_i\in\mathcal{C}_1}\mathcal{C}_i^\dagger\circ\Lambda\circ\mathcal{C}_i. \quad (3)$$

This is a depolarizing channel $\bar{\Lambda}_{\mathcal{C}_n}(\rho) = P_0\rho + [1 - P_0]\mathcal{I}$ with P_0 the probability for no error. The average fidelity of Eq. (2) is therefore a function of the parameter P_0.

To estimate P_0 in a scalable way we can make use of an identification involving the $C_1^{\otimes n}\Pi$-twirled channel. This is the channel Λ twirled over the composition of the n-fold tensor product of the 1-qubit Clifford group $C_1^{\otimes n}$ and the permutation group Π

$$\bar{\Lambda}_{\mathcal{C}_1^{\otimes n}\Pi} = \frac{1}{|\mathcal{C}_1^{\otimes n}\Pi|}\sum_{\mathcal{C}_i\in\mathcal{C}_1^{\otimes n}\Pi}\mathcal{C}_i^\dagger\circ\Lambda\circ\mathcal{C}_i. \quad (4)$$

It has a Pauli form

$$\bar{\Lambda}_{\mathcal{C}_1^{\otimes n}\Pi}(\rho) = \sum_{w=0}^{n}\Pr(w)\left(\frac{1}{3^w\binom{n}{w}}\sum_{i=1}^{3^w\binom{n}{w}}\mathcal{P}_{i,w}\rho\mathcal{P}_{i,w}\right), \quad (5)$$

where $\Pr(w)$ is the probability that a Pauli error of weight w occurs. The identification $P_0 = \Pr(0)$ [21] gives

$$\bar{F}(\Lambda) = \frac{2^n\Pr(0) + 1}{2^n + 1}. \quad (6)$$

The task of finding the average fidelity of the noisy channel Λ is now reduced to finding Pr(0), i.e., the probability that the twirled channel $\bar{\Lambda}_{\mathcal{C}_1^{\otimes n}\Pi}$ does not cause an error.

To obtain Pr(0), we can start from the input state $|0\rangle^{\otimes n}$, apply the $\mathcal{C}_1^{\otimes n}$ twirled channel, and measure the output state in the n-bit string basis [15]. Equivalently, for an ensemble system we can replace $|0\rangle^{\otimes n}$ by n distinct input states $\rho_w = Z^{\otimes w}I^{\otimes n-w}$ where Z represents the Pauli matrix σ_z, followed by a permutation operation Π_n, and measure accordingly as shown in Fig. 1(a).

From an experimental perspective this is still a difficult task. Ideally we want to make as few assumptions as possible about the ability to perform arbitrary Clifford operations since in practice we can only implement $\tilde{\mathcal{U}}_c = \Lambda\circ\mathcal{U}_c$. Moussa *et al.* [17] modified the original twirling protocol in the following way. By inserting the identity $\mathcal{U}_c\circ\mathcal{U}_c^\dagger$ appropriately, the circuit depicted in the upper panel of Fig. 1(a) can be transformed to the lower one. The input state $\rho_i = \mathcal{C}_i\circ\Pi_n(\rho_w)$ is the input Pauli operator and the measurement $M_{\rho_i,\mathcal{U}_c} = \mathcal{U}_c(\rho_i)$ (that can be calculated efficiently [24]), is also a Pauli operator.

By implementing the circuit in the lower panel of Fig. 1(a), the probability of no error is [25]

$$\Pr(0) = \frac{1}{4^n}\left(1 + \frac{1}{2^n}\sum_{i=1}^{4^n-1}\mathrm{Tr}(\tilde{\mathcal{U}}_c(\rho_i)M_{\rho_i,\mathcal{U}_c})\right). \quad (7)$$

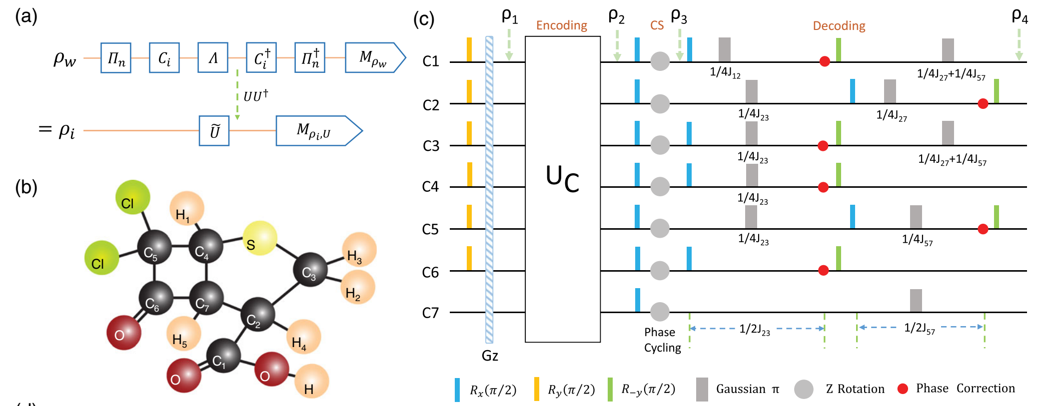
Then substituting Eq. (7) in Eq. (6) will yield the average fidelity of the faulty Clifford gate $\tilde{\mathcal{U}}_c$.

Note that the above twirling protocol is limited to the certification of Clifford gates. For a general unitary gate, it is often impractical to realize the measurement operator $M_{\rho_i,\mathcal{U}} = \mathcal{U}\rho_i\mathcal{U}^\dagger$, whereas for a Clifford gate it can be decomposed efficiently [24]. It is possible to develop fault-tolerant quantum computing where Clifford gates and magic state preparation are the basic building blocks [26,27]. In these architectures they are the only gates that need to be benchmarked [28]. For example, the encoding operation of the 3-qubit quantum error correction code is a Clifford gate comprising two controlled-NOT (CNOT) gates and a single qubit Hadamard gate, and has been certified in a 3-qubit solid-state NMR system [17].

In spite of the simplification of the aforementioned way to estimate the average fidelity of Clifford gates, the complexity remains exponential as $4^n - 1$ distinct Pauli states need to be prepared. Actually, measuring all of the expectation values is unnecessary if one only desires to approximate the average with a given confidence level and confidence interval [15]. Hoeffding's inequality [29] states that if $x_1, \ldots, x_m$ are independent realizations of a random variable x, confined to the interval $[a, b]$ and with statistical mean $\mathbb{E}(x) = \mu$, then for any $\delta > 0$ we have

$$\mathrm{Prob}(|\bar{x} - \mu| > \delta) \leq 2e^{-2\delta^2 m/(b-a)^2}, \quad (8)$$

where $\bar{x} = (1/m)\sum_{i=1}^m x_i$ is the estimator of the exact mean μ, and $\mathrm{Prob}(\epsilon)$ denotes the probability of event $\epsilon: |\bar{x} - \mu| > \delta$ which we want to minimize. Explicitly, Hoeffding's inequality provides an upper bound on the probability that the estimated mean is off by a value greater than δ.



(d)

	Number of Expt.		Calibration		Average Fidelity		
Weight	k	k_w	t (ms)	F_i	F_e	E_d	$F_{d,c}$
w=1	21	3	2	0.977±0.024	0.782±0.012	18.9%	0.965
w=2	189	22	26	0.915±0.029	0.701±0.047	23.4%	0.915
w=3	945	101	34	0.895±0.039	0.671±0.042	28.3%	0.936
w=4	2835	272	49	0.866±0.025	0.582±0.033	33.2%	0.872
w=5	5103	505	53	0.838±0.041	0.552±0.033	37.6%	0.885
w=6	5103	524	55	0.861±0.030	0.518±0.026	39.5%	0.856
w=7	2187	229	60	0.865±0.031	0.489±0.029	43.1%	0.860
Total	16383	1656	N/A	0.858	0.547	37.4%	0.874

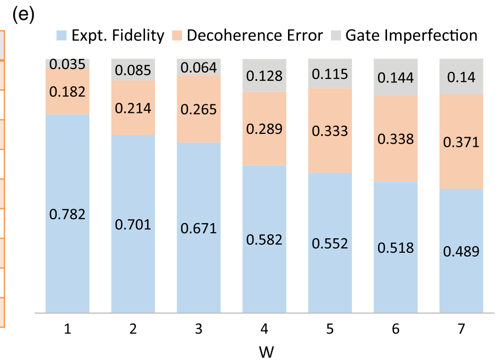


FIG. 1 (color online). (a) Twirling protocols for quantum memories (top) and Clifford gates (bottom). Top: $\rho_w = Z^{\otimes w} I^{\otimes n-w}$ represents n distinct Pauli states, $\mathcal{C}_i$ is a 1-qubit Clifford operation in $\mathcal{C}_1^{\otimes n}$, and Π_n is a permutation operation. Bottom: $\rho_i = \mathcal{C}_i \Pi_n \rho_w \Pi_n^\dagger \mathcal{C}_i^\dagger$ spreads over the entire Pauli group $\mathcal{P}_n$, $\tilde{\mathcal{U}}_c = \mathcal{U}_c \circ \Lambda$ is the noisy Clifford gate, and $M_{\rho_i, \mathcal{U}_c} = \mathcal{U}_c \rho_i \mathcal{U}_c^\dagger$. (b) Molecular structure of Dichlorocyclobutanone, where C_1 to C_7 form a 7-qubit system. (c) Pulse sequence for the creation of labeled PPS via the method in Ref. [22]. It consists of three parts: encoding, coherence selection (CS), and decoding. $\mathcal{U}_c$, realized by a 80 ms GRAPE pulse (compare with $0.5s < T_2 < 2s$ for all seven spins [23]), is the Clifford gate to be certified. The instantaneous states are (unnormalized) $\rho_1 = I^{\otimes 6} \otimes Z$, $\rho_2 = Z^{\otimes 7}$, $\rho_3 = |0\rangle\langle 0|^{\otimes 7} + |1\rangle\langle 1|^{\otimes 7}$, and $\rho_4 = |0\rangle\langle 0|^{\otimes 6} \otimes Z_7$, respectively. (d) Experimental result for the certification of $\mathcal{U}_c$. $k = 3^w \binom{7}{w}$ is the number of Pauli operators for weight w, while k_w is the number of experiments via the sampling; t is the typical time for the input Pauli state preparation, and F_i is the calibration to capture the errors in preparation and measurement; F_e is the experimental result of the probability of no error, and E_d is the numerically simulated signal attenuation (in terms of percentage) due to decoherence. $F_{d,c}$ is the same quantity as F_e but without decoherence effect E_d. (e) Relationship among the experimental remaining signals (blue), decoherence errors (orange), and gate imperfections (gray) for different w.

The confidence level and confidence interval are $1 - \text{Prob}(\epsilon)$ and $[-\delta, \delta]$, respectively.

When μ is the average fidelity we have $a = 0$ and $b = 1$. Hence, for a given $\text{Prob}(\epsilon)$ and δ, the number of experiments calculated by taking the log of Eq. (8) is

$$m \leq \frac{\ln(2/\text{Prob}(\epsilon))}{2\delta^2}. \quad (9)$$

Note that the number of experiments is independent of the number of qubits n, once the desired $\text{Prob}(\epsilon)$ and δ have been given. This result reveals that the estimation of the average fidelity of Clifford gates via the twirling protocol is efficient and scalable. For instance, given a 99% confidence level, i.e., $\text{Prob}(\epsilon) = 1\%$ and $\delta = 0.04$, the total number of experiments is 1656, independent of n.

Experiment.—In the experiment we chose $\mathcal{U}_c$ to be the Cifford gate used to generate maximal (7-qubit) coherence from single (1-qubit) coherence, up to single-qubit gates. It evolves $ZI^{\otimes n-1}$ to $Z^{\otimes n}$ and is the basic encoding process for the pseudo-pure state (PPS) preparation method of Ref. [22] shown in Fig. 1(c). It also plays a role in the creation of cat states. The gate can be decomposed into a sequence of elementary Clifford gates of the type

$$e^{-i(\pi/4)X_i} e^{-i(\pi/4)Z_i Z_j} e^{-i(\pi/4)Y_i}, \quad (10)$$

that increase the order of coherence by evolving Z_i to $Z_i Z_j$. Implementing $\mathcal{U}_c$ in experiment is nontrivial as it requires $2(n-1)$ single qubit operations and $(n-1)$ 2-qubit operations.

Our 7-qubit NMR processor is the per-^{13}C-labeled dichlorocyclobutanone derivative [30] shown in Fig. 1(b) dissolved in d6-acetone. The carbon nuclei labeled C_1 to C_7 denote the seven qubits. Details of the molecular structure can be found in the Supplemental Material [23]. ^{1}H nuclei were decoupled by the Waltz-16 sequence throughout all experiments. The internal Hamiltonian of this system can be described as

$$\mathcal{H}_{\text{int}} = \sum_{j=1}^{7} \pi \nu_j Z_j + \sum_{j<k,=1}^{7} \frac{\pi}{2} J_{jk} Z_j Z_k, \tag{11}$$

where ν_j is the resonance frequency of the jth spin and J_{jk} is the scalar coupling strength between spins j and k. All experiments were conducted on a Bruker DRX 700 MHz spectrometer at room temperature.

The entire procedure to estimate the average fidelity of $\mathcal{U}_c$ can be divided into four parts, as follows:

(i) Sampling. To achieve a confidence level 99% and precision $\delta = 0.04$, we computed that the required number of experiments is 1656 via Eq. (9). Then we randomly sampled 1656 distinct Pauli states out of the entire 7-qubit Pauli group, which has in total $4^7 - 1 = 16\,383$ elements. We distributed all 1656 input Pauli states to seven subgroups according to their Pauli weights $w = 1$ to $w = 7$. The primary reason for this distribution is that a quantum gate such as $\mathcal{U}_c$ here is usually more prone to error when applied to higher weight Pauli states. Additionally, the preparations of input Pauli states with different weights w are distinct.

The sampling result is shown in Fig. 1(d), where the number of sampled experiments k_w for weight w is around one tenth of the total number $k = 3^w \binom{n}{w}$.

(ii) Preparation and calibration. For the creation of every input Pauli state, we employed an efficient sequence compiling program [31] to produce the corresponding pulse sequence. All pulses in the preparation sequences are selective and generated by Gaussian shapes. We then compared the state preparation results with the thermal equilibrium state as a calibration of the certification procedure, aiming to capture the errors in preparation and measurements. The typical duration t for preparing a weight w Pauli state and the related calibration results F_i are both listed in Fig. 1(d).

(iii) Evolution. The target operation $\mathcal{U}_c$ was optimized by a Gradient Ascent Pulse Engineering (GRAPE) pulse [32]. Traditional shaped pulses in NMR are usually optimized for rotations of single qubits, and are not effective at suppressing undesired coupling evolution during the pulse in a multiqubit setting. We utilize the GRAPE algorithm to construct $\mathcal{U}_c$, and find that this ensures $\mathcal{U}_c$ is a Clifford gate to a very good approximation. The GRAPE pulse of $\mathcal{U}_c$ was obtained with the pulse width chosen as 80 ms and a simulated fidelity of 0.99. A special calibration method was used in the experiment to ensure that the pulse acting on the spins was a very close approximation to the simulated (theoretical) pulse [5].

(iv) Measurement. After applying the GRAPE pulse of $\mathcal{U}_c$ to each input Pauli state in the experiment, we measured the corresponding output Pauli state by local readout pulses, and recorded the ratio of the remaining signal to

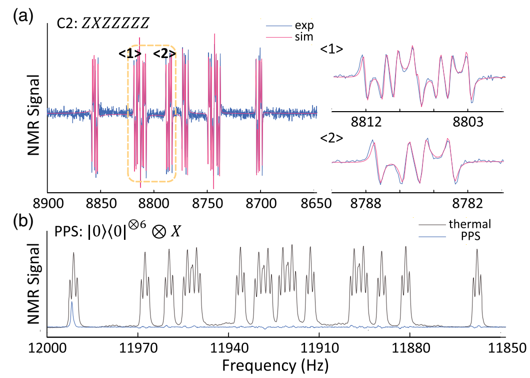


FIG. 2 (color online). (a) NMR spectrum of $Z^{\otimes 7}$ under the observation of C_2. The simulated (red) spectrum is rescaled for line shape comparison with the experimental (blue) one. (b) PPS spectrum (blue) based on the network in Fig. 1(c), where $\mathcal{U}_c$ was employed as the encoding process. The spectrum of the thermal equilibrium state (black) is also shown.

that of the reference input state. Next, we averaged the results with respect to different weights w, as shown by F_e in Fig. 1(d). It is expected that the ratio will decrease as w increases, since higher coherences are less robust to the decoherence occurring during $\mathcal{U}_c$.

The probability of no error [F_e in Fig. 1(d)] is $\Pr(0) \approx 54.7\%$. The average fidelity of $\mathcal{U}_c$ via Eq. (6) is then $\bar{F}(\Lambda) \approx 55.1\%$. It is possible to decompose $\mathcal{U}_c$ into twelve 1-qubit gates and six 2-qubit gates [33]. To quantify the decoherence contribution during $\mathcal{U}_c$, we followed the approach of phase damping [34] to simulate the dynamical process step by step. This gives a conservative estimate of decoherence due to T_2 processes. The average signal attenuation due to decoherence is shown by E_d in Fig. 1(d). Under the assumption that the decoherence error can be factorized, the probability of no error $\Pr(0)$ after theoretically removing the decoherence is 87.4%, which means the average fidelity is 87.5%. In the equivalent implementation using elementary gates, the average error per gate would then be $< 1\%$. The remaining errors are mainly attributed to imperfection in the design and implementation of the GRAPE pulse. Figure 1(e) shows the relationship between the raw experimental results, decoherence effects, and gate imperfections for each w.

Figure 2(a) shows the spectrum of $Z^{\otimes 7}$ after $\mathcal{U}_c(Z_7)$ under the observation of C_2. Comparing the simulated and experimental spectra gives a qualitative indication of the level of coherent control achieved in this 7-qubit system. For another comparison we followed $\mathcal{U}_c$ by a set of operations to extract the PPS as in Fig. 1(c). The PPS spectrum by observing the labeled spin C_7 is shown in Fig. 2(b).

Conclusion.—We estimated the average fidelity of a nontrivial 7-qubit Clifford gate using a twirling protocol together with a random sampling method. This is the largest gate characterization reported in an experiment to date and is currently the only average fidelity benchmark for

> 3-qubit gates. The experimental spectra demonstrate reliable coherent control of this 7-qubit system while our benchmarking protocol gives an average gate fidelity of 55.1% before accounting for decoherence, and 87.5% after theoretically removing the contribution of decoherence. An important feature of the protocol is the relatively small number of experiments required: 1656 ($< 2^{11}$) compared to $2.7 \times 10^8 (\approx 2^{28})$ for process tomography.

We believe the level of control demonstrated here is state of the art for a quantum system above 4 qubits. Lack of average fidelity measurements for 7-qubit gates in other systems prevents direct comparison. A rough comparison can be made using the decomposition into a circuit of 18 one- and two-qubit gates. An overall average fidelity of 55.1% requires each gate to have an average fidelity of at least 96% (99% after accounting for decoherence) on a 7-qubit quantum processor. The best average fidelity reported in NMR is 86% (99% after accounting for decoherence) for a 3-qubit gate and 99.5% for a 2-qubit gate, both on a 3-qubit sample [14,17]. A comparable system is a 5 Xmon qubit superconducting circuit with benchmarked average fidelities of 99.0%–99.4% for nearest neighbor 2-qubit gates [35]. It is likely that state-of-the-art ion traps may achieve similar average fidelities [36]. With further developments in experimental quantum information processing we expect that these and other systems can be compared directly with the 7-qubit NMR system using the protocol outlined in this Letter.

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[1] J. Preskill, Proc. R. Soc. A **454**, 385 (1998).
[2] I. L. Chuang and M. A. Nielsen, J. Mod. Opt. **44**, 2455 (1997).
[3] J. F. Poyatos, J. I. Cirac, and P. Zoller, Phys. Rev. Lett. **78**, 390 (1997).
[4] A. M. Childs, I. L. Chuang, and D. W. Leung, Phys. Rev. A **64**, 012314 (2001).
[5] Y. S. Weinstein, T. F. Havel, J. Emerson, N. Boulant, M. Saraceno, S. Lloyd, and D. G. Cory, J. Chem. Phys. **121**, 6117 (2004).
[6] J. L. O'Brien, G. J. Pryde, A. Gilchrist, D. F. V. James, N. K. Langford, T. C. Ralph, and A. G. White, Phys. Rev. Lett. **93**, 080502 (2004).
[7] M. Riebe, K. Kim, P. Schindler, T. Monz, P. O. Schmidt, T. K. Körber, W. Hänsel, H. Häffner, C. F. Roos, and R. Blatt, Phys. Rev. Lett. **97**, 220407 (2006).
[8] J. M. Chow, J. M. Gambetta, L. Tornberg, J. Koch, L. S. Bishop, A. A. Houck, B. R. Johnson, L. Frunzio, S. M. Girvin, and R. J. Schoelkopf, Phys. Rev. Lett. **102**, 090502 (2009).
[9] R. C. Bialczak *et al.*, Nat. Phys. **6**, 409 (2010).
[10] D. Kim *et al.*, Nature (London) **511**, 70 (2014).
[11] G. Feng, G. Xu, and G. Long, Phys. Rev. Lett. **110**, 190501 (2013).
[12] J. Emerson, R. Alicki, and K. Zyczkowski, J. Opt. B **7**, S347 (2005).
[13] E. Knill, D. Leibfried, R. Reichle, J. Britton, R. B. Blakestad, J. D. Jost, C. Langer, R. Ozeri, S. Seidelin, and D. J. Wineland, Phys. Rev. A **77**, 012307 (2008).
[14] C. A. Ryan, M. Laforest, and R. Laflamme, New J. Phys. **11**, 013034 (2009).
[15] J. Emerson, M. P. da Silva, O. Moussa, C. A. Ryan, M. Laforest, J. Baugh, D. G. Cory, and R. Laflamme, Science **317**, 1893 (2007).
[16] C. Dankert, R. Cleve, J. Emerson, and E. Livine, Phys. Rev. A **80**, 012304 (2009).
[17] O. Moussa, M. P. da Silva, C. A. Ryan, and R. Laflamme, Phys. Rev. Lett. **109**, 070504 (2012).
[18] S. T. Flammia and Y.-K. Liu, Phys. Rev. Lett. **106**, 230501 (2011).
[19] M. P. da Silva, O. Landon-Cardinal, and D. Poulin, Phys. Rev. Lett. **107**, 210404 (2011).
[20] C. H. Bennett, D. P. DiVincenzo, J. A. Smolin, and W. K. Wootters, Phys. Rev. A **54**, 3824 (1996).
[21] M. P. da Silva, Ph.D. thesis, University of Waterloo, 2008.
[22] E. Knill, R. Laflamme, R. Martinez, and C.-H. Tseng, Nature (London) **404**, 368 (2000).
[23] See Supplemental Material at http://link.aps.org/supplemental/10.1103/PhysRevLett.114.140505 for a detailed description of both theory and experiment.
[24] S. Aaronson and D. Gottesman, Phys. Rev. A **70**, 052328 (2004).
[25] D. Trottier, Master thesis, University of Waterloo, 2013.
[26] S. Bravyi and A. Kitaev, Phys. Rev. A **71**, 022316 (2005).
[27] A. M. Souza, J. Zhang, C. A. Ryan, and R. Laflamme, Nat. Commun. **2**, 169 (2011).
[28] Note that benchmarking of measurements and preparation is unavoidable.
[29] S. Venkatesh, *The Theory of Probability: Explorations and Applications* (Cambridge University Press, Cambridge, England, 2012).
[30] J. W. Johnson, D. P. Evanoff, M. E. Savard, G. Lange, T. R. Ramadhar, A. Assoud, N. J. Taylor, and G. I. Dmitrienko, J. Org. Chem. **73**, 6970 (2008).
[31] C. A. Ryan, C. Negrevergne, M. Laforest, E. Knill, and R. Laflamme, Phys. Rev. A **78**, 012328 (2008).
[32] N. Khaneja, T. Reiss, C. Kehlet, T. Schulte-Herbrüggen, and S. J. Glaser, J. Magn. Reson. **172**, 296 (2005).
[33] The pulse was optimized using GRAPE, so the operation $\mathcal{U}_c$ is not implemented by a sequence of 1- and 2-qubit elementary gates. However, the initial guess for the GRAPE pulse was generated from an elementary gate sequence.
[34] L. M. K. Vandersypen, M. Steffen, G. Breyta, C. S. Yannoni, M. H. Sherwood, and I. L. Chuang, Nature (London) **414**, 883 (2001).
[35] R. Barends *et al.*, Nature (London) **508**, 500 (2014).
[36] J. Benhelm, G. Kirchmair, C. F. Roos, and R. Blatt, Nat. Phys. **4**, 463 (2008).