\overline{d} – \overline{u} Asymmetry in the Proton in Chiral Effective Theory

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We compute the $d - \bar{u}$ asymmetry in the proton in chiral effective theory, including both nucleon and Δ degrees of freedom, within the relativistic and heavy baryon frameworks. In addition to the distribution at $x > 0$, we compute the corrections to the asymmetry from zero momentum contributions from pion rainbow and bubble diagrams at $x = 0$, which have not been accounted for in previous analyses. We find that the empirical x dependence of $\bar{d} - \bar{u}$ as well as the integrated asymmetry can be well reproduced in terms of a transverse momentum cutoff parameter.

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The observation of the $\bar{d} - \bar{u}$ flavor asymmetry in the light quark sea of the proton [1–[4\]](#page-4-0) has been one of the seminal results in hadronic physics over the past two decades, leading to a major reevaluation of our understanding of the quark structure of the nucleon. In particular, the measurement revealed the importance of 5-quark Fock state components of the nucleon's wave function, and the crucial role played by chiral symmetry breaking. This asymmetry had been anticipated by Thomas [\[5\]](#page-4-1) a decade earlier, and has subsequently been studied using various nonperturbative models [6–[10\]](#page-4-2). However, despite some successes in reproducing the general features of the data, it has proved very difficult to obtain direct connection between the models and QCD.

An important development in establishing a formal link between models of $\bar{d} - \bar{u}$ and QCD came with the realization that the moments of parton distribution functions (PDFs) could be formally expanded in chiral effective field theory in terms of power series in the pion mass squared, m_{π}^2 . The leading nonanalytic (LNA) contributions were found to depend on the (model-independent) long range structure of the pion cloud, with a characteristic $m_{\pi}^2 \log m_{\pi}^2$ dependence [\[11,12\].](#page-4-3) This idea was later applied to the chiral extrapolation of lattice QCD moments of the $u - d$ distribution to reconcile the lattice data at large m_{π}^2 with experiment [\[13\]](#page-4-4).

Initial calculations of pion loop effects were performed in the context of the "Sullivan" process [\[14\],](#page-4-5) using pseudoscalar pion-nucleon coupling, which involves only the pion "rainbow" diagram. Analysis within the chiral effective theory for the pseudovector coupling reveals differences in the off-shell behavior of the loops [\[15\],](#page-4-6) as well as the presence of additional pion bubble terms [\[16,17\]](#page-4-7) at $x = 0$ [\[18\].](#page-4-8) The relationship between the pseudoscalar and pseudovector theories was recently discussed in Refs. [\[18,19\].](#page-4-8)

While the structure of the pion loops constrains the behavior of the PDF moments at small m_{π}^2 , the total moments depend also on the short-distance contributions, parametrized by coefficients of analytic terms in the chiral expansion. In principle, these can be fitted to data, and the PDFs reconstructed from the moments assuming a functional form for the dependence on the parton momentum fraction x [\[20\]](#page-4-9). Recently, however, a method for computing the PDFs directly in x space has been developed, by matching nonlocal operators within heavy baryon (HB) chiral effective theory [\[21\].](#page-4-10) The results allow the total distributions to be computed in the form of convolutions of bare PDFs in the pion with pion light-cone momentum distributions associated with pion rainbow, bubble, and Kroll-Ruderman terms [\[18\]](#page-4-8).

In this Letter, we apply this formalism for the first time to analyze the $\bar{d} - \bar{u}$ distribution in the proton within chiral effective theory. We consider both the covariant and nonrelativistic HB formulations of the low-energy chiral theory, including both nucleon N and Δ contributions to loop integrals. While the on-shell components of the N and Δ rainbow diagrams give rise to distributions that closely resemble earlier model calculations, we find in addition unique signatures of off-shell components and bubble diagrams that are nonzero at $x = 0$. Although difficult to access directly through experiment, these terms modify the value of $\bar{d} - \bar{u}$ integrated over all x, which is the benchmark for the magnitude of the flavor symmetry violation. To illustrate the phenomenological application of our approach, we also compute the shape of the $\bar{d} - \bar{u}$ distribution using a transverse momentum cutoff to regularize the ultraviolet contributions from the pion loops.

The pion light-cone distributions can be calculated in the effective chiral theory from the diagrams shown in Fig. [1](#page-1-0) [\[18,19,21\].](#page-4-8) Other contributions, such as the rainbow diagrams with coupling to the baryon, pion tadpoles, or Kroll-Ruderman diagrams, also give nonzero corrections to PDFs, but do not contribute to $\bar{d} - \bar{u}$ if one assumes a flavor symmetric sea in the bare proton.

Starting from the lowest-order chiral Lagrangian [\[22,23\]](#page-4-11), these have previously been derived for the diagrams in Figs. [1\(a\)](#page-1-0) and [1\(c\)](#page-1-0) involving nucleons and pions [\[18,21\]](#page-4-8). Including also contributions from the Δ intermediate states in Fig. [1\(b\),](#page-1-0) the $\bar{d} - \bar{u}$ difference in the proton can be written as [\[12\]](#page-4-12)

$$
\bar d - \bar u = (f_{\pi^+ n} + f_{\pi^+ \Delta^0} - f_{\pi^- \Delta^{++}} + f_{\pi(\text{bub})}) \otimes \bar q^\pi_v, \quad (1)
$$

where generally the convolution is defined as $f \otimes q =$ $\int_0^1 dy \int_0^1 dz \delta(x - yz) f(y) q(z)$, with $y = k^+/p^+$ the lightcone fraction of the proton's momentum (p) carried by the pion (k). The pion light-cone momentum distributions $f_{\pi N}$ and $f_{\pi \Delta}$ correspond to the pion rainbow diagrams in Figs. [1\(a\)](#page-1-0) and [1\(b\),](#page-1-0) while $f_{\pi(\text{bub})}$ represents the pion bubble diagram in Fig. [1\(c\).](#page-1-0) The contributions from individual charge states in Eq. [\(1\)](#page-1-1) arise from the fluctuations $p \to \pi^+ n$, $\pi^+ \Delta^0$ or $\pi^- \Delta^{++}$. The convolution in Eq. [\(1\)](#page-1-1) is obtained from the crossing symmetry properties of the light-cone distributions [\[12\]](#page-4-12), $f(-y) = f(y)$, and the valence pion PDF, $\bar{q}^{\pi}(x) = -q^{\pi}(-x)$, for which we have assumed charge symmetry, $\bar{q}_{v}^{\pi} \equiv \bar{d}^{\pi^{+}} - d^{\pi^{+}} = \bar{u}^{\pi^{-}} - u^{\pi^{-}}$.

Following [\[18\],](#page-4-8) the $f_{\pi^+ n}$ distribution can be written as

$$
f_{\pi^+ n}(y) = 2 \Big[f_N^{(\text{on})}(y) + f_N^{(\delta)}(y) \Big], \tag{2}
$$

where $f_N^{(\text{on})}$ and $f_N^{(\delta)}$ are the on-shell and δ -function contributions from the pion rainbow diagram, respectively. The on-shell nucleon term for $y > 0$ is [\[5,7,8,18\]](#page-4-1)

$$
f_N^{(\text{on})}(y) = \frac{g_A^2 M^2}{(4\pi f_\pi)^2} \int dk_\perp^2 \frac{y(k_\perp^2 + y^2 M^2)}{(1 - y)^2 D_{\pi N}^2},\qquad(3)
$$

where *M* is the nucleon mass, $g_A = 1.267$ is the axial charge, $f_{\pi} = 93$ MeV is the pion decay constant, and $D_{\pi N} = -[k_{\perp}^2 + y^2 M^2 + (1 - y)m_{\pi}^2]/(1 - y)$ is the pion virtuality $(k^2 - m_\pi^2)$ for an on-shell nucleon intermediate state. In contrast, the off-shell contribution arises from pions with zero light-cone momentum,

FIG. 1. Contributions to the pion light-cone momentum distributions in the proton, from the pion rainbow diagram with (a) a nucleon or (b) Δ intermediate state, and (c) from the pion bubble diagram.

$$
f_N^{(\delta)}(y) = \frac{g_A^2}{4(4\pi f_\pi)^2} \int dk_\perp^2 \log \frac{\Omega_\pi}{\mu^2} \delta(y), \tag{4}
$$

where $\Omega_{\pi} = k_{\perp}^2 + m_{\pi}^2$, and μ is an ultraviolet cutoff on the k[−] integration. Since $f^{(\delta)}$ is nonzero only at y = 0, it contributes to $\bar{d} - \bar{u}$ only at $x = 0$. While it cannot be measured directly, it nevertheless affects the determination of $\overline{D} - \overline{U} \equiv \int_0^1 dx (\overline{d} - \overline{u})$ (as in the Gottfried sum rule [\[24\]](#page-4-13)) when extrapolating to $x = 0$.

The $\pi\Delta$ contribution can be computed from the effective $\pi N\Delta$ interaction $\bar{\psi}_{\Delta}^{\mu} [g_{\mu\nu} - (Z + 1/2)\gamma_{\mu}\gamma_{\nu}] \partial^{\nu}\phi_{\pi}\psi_{N}$, where Z parametrizes the off-shell behavior of the spin- $3/2$ field ψ_{Δ}^{μ} . The resulting $\pi\Delta$ distribution function can be written as a sum of three terms,

$$
f_{\pi^+\Delta^0}(y) = f_{\Delta}^{(\text{on})}(y) + f_{\Delta}^{(\text{end point})}(y) + f_{\Delta}^{(\delta)}(y). \tag{5}
$$

For $Z = -1/2$, the on-shell (Δ -pole) contribution at $y > 0$ is given by

$$
f_{\Delta}^{(\text{on})}(y)
$$

= $C_{\Delta} \int dk_{\perp}^2 \frac{y(\overline{M^2} - m_{\pi}^2)}{1 - y}$

$$
\times \left[\frac{(\overline{M^2} - m_{\pi}^2)(\Delta^2 - m_{\pi}^2)}{D_{\pi\Delta}^2} - \frac{3(\Delta^2 - m_{\pi}^2) + 4MM_{\Delta}}{D_{\pi\Delta}} \right],
$$

(6)

where $D_{\pi\Delta} = -\left[k_{\perp}^2 - y(1-y)M^2 + yM_{\Delta}^2 + (1-y)m_{\pi}^2\right]/(1-y)$ is the pion virtuality for an on-shell Δ intermediate state, and we define $\Delta \equiv M_{\Delta} - M$ and $\overline{M} \equiv M_{\Delta} + M$. The coefficient $C_{\Delta} = g_{\pi N\Delta}^2/[(4\pi)^2 18M_{\Delta}^2]$, and using SU(6) symmetry and the Gell-Mann–Oakes–Renner relation the $\pi N\Delta$ coupling constant is $g_{\pi N\Delta} = (3\sqrt{2}/5)g_A/f_\pi \approx$ 11.8 GeV^{-1} [\[25\].](#page-4-14)

In addition, the pole contribution involves an end-point singularity, which gives a δ -function at $y = 1$,

$$
f_{\Delta}^{(\text{end point})}(y)
$$

= $C_{\Delta} \int dk_{\perp}^2 \delta(1 - y)$

$$
\times \left\{ [\Omega_{\Delta} - 2(\Delta^2 - m_{\pi}^2) - 6MM_{\Delta}] \log \frac{\Omega_{\Delta}}{\mu^2} - \Omega_{\Delta} \right\},
$$

(7)

where $\Omega_{\Delta} = k_{\perp}^2 + M_{\Delta}^2$. The off-shell components of the Δ propagator introduce a δ -function term at $y = 0$,

$$
f_{\Delta}^{(\delta)}(y) = C_{\Delta} \int dk_{\perp}^{2} \delta(y)
$$

$$
\times \left\{ \left[3(\Omega_{\pi} + m_{\pi}^{2}) + \overline{M}^{2} \right] \log \frac{\Omega_{\pi}}{\mu^{2}} - 3\Omega_{\pi} \right\}. \quad (8)
$$

Although this gives a nonzero PDF only at $x = 0$, since it contributes to the integral of \bar{d} − \bar{u} , it will indirectly affect the normalization for $x > 0$. For the $p \to \Delta^{++} \pi^$ dissociation, the distribution function is given by $f_{\pi^-\Lambda^{++}} = 3f_{\pi^+\Lambda^0}$. Note also that for values of the off-shell parameter $Z \neq -1/2$, the additional interaction term ~ $\gamma_{\mu} \gamma_{\nu}$ contributes only to the off-shell component $f_{\Delta}^{(\delta)}$ without modifying our LNA result.

The Δ contribution has been considered in several previous studies, both within pion cloud models [\[7,8,10,25\]](#page-4-15) and in chiral effective theory in the large- N_c limit [\[26\].](#page-4-16) In the phenomenological approaches, one computes the Sullivan process with the Δ intermediate state by taking the Δ -pole contribution, $(p-k)^2 - M_{\Delta}^2 \rightarrow 0$, which gives the distribution usually found in the literature [\[7,8,10,11,25\]](#page-4-15),

$$
f_{\Delta}^{(\text{Sul})}(y) = C_{\Delta} \int dk_{\perp}^{2} y
$$

$$
\times \frac{[k_{\perp}^{2} + (\Delta + yM)^{2}][k_{\perp}^{2} + (\overline{M} - yM)^{2}]^{2}}{(1 - y)^{4}D_{\pi\Delta}^{2}}.
$$
 (9)

Note that the power of k_{\perp} in the numerator here is k_{\perp}^6 , while in the on-shell contribution in Eq. [\(6\)](#page-1-2) it is k_{\perp}^2 . This difference arises because the Sullivan process neglects the end-point contributions, which give rise to the $\delta(1-y)$ term in Eq. [\(7\)](#page-1-3), and also cancel the $\mathcal{O}(k_{\perp}^4)$ and $\mathcal{O}(k_{\perp}^6)$ terms. The correct calculation of the Δ -pole contribution therefore yields $f_{\Delta}^{(on)} + f_{\Delta}^{(end point)}$. The off-shell contribution proportional to $\delta(y)$ is not included in the Sullivan approach, for either the Δ or nucleon intermediate states. For the latter, the Sullivan method yields only the on-shell component [\[18,19\]](#page-4-8), $f_N^{(Sul)}(y) = f_N^{(on)}(y)$.

Finally, for the pion bubble diagram in Fig. [1\(c\)](#page-1-0), the distribution $f_{\pi(\text{bub})}$ has a form similar to the δ -function part of $f_{\pi^+ n}$ [\[18\]](#page-4-8),

$$
f_{\pi(\text{bub})}(y) = -\frac{2}{g_A^2} f_N^{(\delta)}(y). \tag{10}
$$

This term originates with the Weinberg-Tomozawa part of the chiral Lagrangian, and is independent of g_A .

Because the leading chiral behavior of matrix elements is not affected by baryon masses, for simplicity most earlier studies of chiral corrections to PDF moments [\[12,16,17\]](#page-4-12), as well as generalized parton distributions [\[27\]](#page-4-17), were performed in the HB limit. It is instructive therefore to compare the results for the x dependence of PDFs in the HB and relativistic approaches [\[19,28\]](#page-4-18).

In the HB limit ($m_\pi \ll M$, $y \ll 1$), the nonrelativistic analog of the on-shell function in Eq. [\(3\)](#page-1-4) is obtained by the replacement $(1 - y)D_{\pi N} \to \tilde{D}_{\pi N} = -(k_{\perp}^2 + y^2 M^2 + m_{\pi}^2)$. The δ -function and bubble contributions $f_N^{(\delta)}$ and $f_{\pi(\text{bub})}$, however, remain unchanged.

For the Δ intermediate state, in the HB limit both the N and Δ masses are large, $\overline{M} \rightarrow \infty$, while the difference is kept finite, $\Delta/\overline{M} \rightarrow 0$. In this case the on-shell function reduces to

$$
\tilde{f}_{\Delta}^{(\text{on})}(y) = \frac{8g_{\pi N\Delta}^2 M^2}{9(4\pi)^2} \int dk_\perp^2 \frac{y(\Delta^2 - m_\pi^2 - \tilde{D}_{\pi\Delta})}{\tilde{D}_{\pi\Delta}^2}, \quad (11)
$$

where the nonrelativistic analog of $D_{\pi\Delta}$ in Eq. [\(6\)](#page-1-2) is $\widetilde{D}_{\pi\Delta} = -(k_\perp^2 + y^2M^2 + 2yM\Delta + m_\pi^2)$. Similarly, in the HB limit the δ -function contribution in Eq. [\(8\)](#page-1-5) becomes

$$
\tilde{f}_{\Delta}^{(\delta)}(y) = \frac{2g_{\pi N\Delta}^2}{9(4\pi)^2} \int dk_{\perp}^2 \delta(y) \log \frac{\Omega_{\pi}}{\mu^2}.
$$
 (12)

Because in this limit one has $y \ll 1$, there is no analogous nonrelativistic end-point contribution to that in Eq. [\(7\).](#page-1-3)

The consistency of the above results with the chiral symmetry of QCD can be verified by examining the LNA behavior of $\overline{D} - \overline{U}$. Since the valence pion PDF \overline{q}_v^{π} is normalized to unity, $\overline{D} - \overline{U}$ is given entirely by the moments of the pion distribution functions in Eq. [\(1\)](#page-1-1). Expanding these in m_π , the LNA behavior of $\overline{D} - \overline{U}$ is then given by

$$
(\overline{D} - \overline{U})_{\text{LNA}} = \frac{3g_A^2 + 1}{2(4\pi f_\pi)^2} m_\pi^2 \log m_\pi^2 - \frac{g_{\pi N\Delta}^2}{12\pi^2} J_1,\qquad(13)
$$

where $J_1 = \frac{(m_\pi^2 - 2\Delta^2) \log m_\pi^2 + 2\Delta r \log[(\Delta - r)/(\Delta + r)],$ with $r = \sqrt{\Delta^2 - m_\pi^2}$. The nucleon intermediate state contribution in Eq. [\(13\)](#page-2-0) coincides with the result obtained in Ref. [\[12\]](#page-4-12), and the expression for J_1 agrees with that in Ref. [\[16\].](#page-4-7) Note that the end-point component $f_{\Delta}^{(\text{end point})}$ does not contain any nonanalytic structure in m_{π}^2 , and therefore does not contribute to the LNA behavior. Since the chiral properties of the pion distributions are independent of the short-distance part of the pion-nucleon interaction, the LNA contribution computed in the HB limit is identical to that in Eq. [\(13\)](#page-2-0). In the $\Delta \rightarrow 0$ limit, the coefficient of the LNA $m_{\pi}^2 \log m_{\pi}^2$ term is $[(27/50)g_A^2 + 1/2]/(4\pi f_\pi)^2$. Compared to the coefficient of the LNA term in Eq. [\(13\)](#page-2-0) from the nucleon alone, the Δ intermediate state leads to a reduction in the LNA coefficient by more than 50%.

On the other hand, in the Sullivan approach, which only accounts for the on-shell components, the LNA behavior in the $\Delta \rightarrow 0$ limit is [\[11\]](#page-4-3)

$$
(\overline{D} - \overline{U})_{\text{LNA}}^{(\text{Sul})} = \left[\frac{2g_A^2}{(4\pi f_\pi)^2} - \frac{g_{\pi N\Delta}^2}{9\pi^2} \right] m_\pi^2 \log m_\pi^2. \tag{14}
$$

For the nucleon rainbow contribution in Fig. [1\(a\)](#page-1-0), the onshell (Sullivan) approximation is therefore a factor of $4/3$ larger than the exact result in Eq. [\(13\)](#page-2-0). Interestingly, the contribution from the Δ rainbow diagram in Fig. [1\(b\)](#page-1-0) in the Sullivan approach is also $4/3$ larger than the full expression in the $\Delta \rightarrow 0$ limit with the δ -function components. Using SU(6) couplings, the coefficient of the LNA term in the Sullivan approximation is $(18/25)g_A^2/(4\pi f_\pi)^2$.

In Fig. [2](#page-3-0) we show the individual contributions to $\overline{D} - \overline{U}$ from the nucleon, Δ and bubble diagrams as a function of μ and the k_1 cutoff parameter Λ . For μ ranging between 0.1 and 1 GeV, Λ is fixed by matching the calculation to the value of the $d-\bar{u}$ integral extracted from the E866 Drell-Yan data over the measured x range, $\int_{0.015}^{0.35} dx (\bar{d} - \bar{u}) =$ $0.0803(11)$ [\[4\].](#page-4-19) The resulting variation in Λ is relatively mild, ranging from $\Lambda \approx 0.18$ to 0.23 GeV, and the total \overline{D} – \overline{U} is similar to the empirical results from the Drell-Yan [\[4\]](#page-4-19) and deep-inelastic scattering [\[1,2\]](#page-4-0) data. One should caution, however, that the experimental values are obtained by extrapolating the data to $x = 0$ and $x = 1$ under the assumption that there is no contribution at $x = 0$. The δ-function contributions $f_N^{(\delta)}$, $f_{\Delta}^{(\delta)}$, and $f_{\pi(\text{bub})}$ will give nonzero corrections to the extrapolated moment.

Numerically, the most important contribution is from the nucleon on-shell distribution $f_N^{(on)}$, but this is canceled to some extent by the negative $f_N^{(\delta)}$ distribution. The on-shell Δ contribution $f_{\Delta}^{(on)}$ is negative and \approx 1/2 the magnitude of the on-shell N component. On the other hand, the Δ

FIG. 2 (color online). Contributions to $\overline{D} - \overline{U}$ from the dia-grams in Fig. [1](#page-1-0) as a function of the cutoff parameters μ and Λ . The nucleon on-shell and δ-function, and $Δ$ on-shell, end-point, and δ-function contributions are shown individually, together with the pion bubble contribution and the total. The calculations are compared with data from the NMC [\[1\]](#page-4-0), HERMES [\[2\],](#page-4-21) and E866 [\[4\]](#page-4-19) experiments, and from a recent lattice QCD calculation [\[29\].](#page-4-22)

contributions at $y = 0$ and $y = 1$ are both positive, with the sum of the two largely canceling the negative on-shell Δ term. The net result is a significantly smaller Δ component than that found in many previous analyses. The pion bubble contribution is positive for most values of μ and slightly enhances the on-shell nucleon term. From Eq. [\(10\)](#page-2-1) one expects the $f_N^{(\delta)}$ contribution to cancel strongly with $f_{\pi(\text{bub})}$, and partially with the positive $f_{\Delta}^{(\delta)}$. Thus we see strong cancellations between all the singular $x = 0$ pieces, leaving the total pionic contributions to $\overline{D} - \overline{U}$ determined largely by the nucleon on-shell part [\[5\].](#page-4-1) This explains for the first time the relative success of the phenomenological descriptions of the data through the Sullivan process in terms of on-shell nucleon contribution alone. In practice, the δ-function pieces at $x = 0$ can contribute up to $\approx 16\%$ (for $\mu = 1$ GeV) of the $\bar{d} - \bar{u}$ difference integrated over the measured region of x , which is a relatively small contribution to the Gottfried sum violation.

For the nonrelativistic calculation, with the same values of the cutoff parameters, there is a small reduction in the N contribution, reflecting the $D_{\pi N} \to \tilde{D}_{\pi N}$ modification in the on-shell component. On the other hand, the absence of the end-point term in the HB calculation means that the nonrelativistic on-shell contribution is significantly more negative, and cancels much more of the total nucleon contribution.

Within the framework of the present calculation, we can also estimate the x dependence of $\bar{d} - \bar{u}$ from the convolution in Eq. [\(1\)](#page-1-1), with the light-cone distributions computed in terms of the same parameters as in Fig. [2](#page-3-0). For the valence PDFs in the pion we use the global para-metrization from Ref. [\[30\].](#page-4-20) The resulting $\bar{d} - \bar{u}$ asymmetry is illustrated in Fig. [3](#page-3-1) at a scale $Q^2 = 54 \text{ GeV}^2$, with the

FIG. 3 (color online). Flavor asymmetry $\bar{d} - \bar{u}$ from the N and Δ intermediate states, and the total, for cutoffs $\mu = 0.3$ and $\Lambda = 0.2$ GeV, compared with the asymmetry extracted at leading order from the E866 Drell-Yan data [\[4\]](#page-4-19) at $Q^2 = 54 \text{ GeV}^2$. The band indicates the uncertainty on the total distribution from the cutoff parameters (for μ between 0.1 and 1 GeV) and from the empirical $\overline{D} - \overline{U}$ normalization.

nucleon and Δ contributions at $x > 0$ computed for $\mu =$ 0.3 GeV and $\Lambda = 0.2$ GeV (the δ-function contributions $f_N^{(\delta)}, f_\Delta^{(\delta)}$, and $f_{\pi(\text{bub})}$ exist only at $x = 0$). The asymmetry is dominated by the nucleon on-shell component, with the Δ on-shell contribution providing some cancellation at small x, but becoming negligible for $x \gtrsim 0.1$. The shaded band illustrates the uncertainty in the calculation, with the envelope representing the extremal values of the cutoffs $\mu = 0.1$ and 1 GeV, and the experimental uncertainty on the E866 data [\[4\].](#page-4-19) Without attempting to fine-tune the parameters, the overall agreement between the calculation and experiment is very good. As with all previous pion loop calculations, the apparent trend of the E866 data towards negative $\bar{d} - \bar{u}$ values for $x \gtrsim 0.3$ is not reproduced in this analysis. The new SeaQuest experiment [\[31\]](#page-4-23) at Fermilab is expected to provide new information on the shape of $d - \bar{u}$ for $x \leq 0.45$.

The analysis described here can be applied to other nonperturbative quantities in the proton, such as the flavor asymmetry of the polarized sea, $\Delta \bar{u} - \Delta d$, or the strangeantistrange asymmetry $s - \bar{s}$ using an SU(3) generalization of the effective chiral theory. Beyond this, the methodology can be further extended to study the systematics of chiral loop corrections to partonic observables such as transverse momentum dependent distributions and generalized parton distributions.

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