

Multiple Transitions in Rotating Turbulent Rayleigh-Bénard Convection

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Sometimes it is thought that sharp transitions between potentially different turbulent states should be washed out by the prevailing intense fluctuations and short coherence lengths and times. Contrary to this expectation, we found a *sequence* of such transitions in turbulent rotating Rayleigh-Bénard convection as the rotation rate was increased. This phenomenon was observed in cylindrical samples with aspect ratios (diameter/height) $\Gamma = 1.00$ and 0.50 . It became most prominent at very large Rayleigh numbers up to 2×10^{12} , where the fluctuations are extremely vigorous, and was manifested most clearly for $\Gamma = 1.00$. It was found in the heat transport as well as in the temperature gradient near the sample center. We conjecture that the transitions are between different large-scale structures which involve changes of symmetry and thus cannot be gradual [L. Landau, *Zh. Eksp. Teor. Fiz.* 7, 19 (1937); L. D. Landau, *Phys. Z. Sowjetunion* 11, 26 (1937); L. D. Landau, in *Collected Papers of L. D. Landau*, (Oxford University Press, Oxford, 1965), pp. 193–216].

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Turbulent flows involve large fluctuations and loss of coherence in space and time. Thus it was argued at times that Kolmogorov's theory of turbulence [1] implies that turbulent flows become featureless when the Reynolds number is large enough (for a discussion of this issue see, for instance, Ref. [2]). Apparently in contradiction to this expectation, several experiments recently showed a sharp transition between two different turbulent states [2–6]. However, all of these investigations were carried out on systems with geometrical constraints in all physical directions, and it is not clear whether the sharp transitions are caused by boundary conditions or whether they would survive in the theoretically envisioned [1] unconstrained system. Indeed for one, namely, turbulent rotating Rayleigh-Bénard convection (RBC), measurements were made as a function of the lateral extent (aspect ratio), and it was found that the observed transition moves toward zero rotation rate as the lateral system size approaches infinity [7,8].

In this Letter we report that there is a *sequence* of sharp transitions in rotating turbulent RBC that occurs with increasing rotation rate. They have been observed most clearly at very large Rayleigh numbers (Ra) up to 2×10^{12} , where fluctuations are extremely vigorous and coherence lengths are much smaller than the sample size. While the first transition may be attributed to the finite system size [7], it is unlikely that this explanation can be applied to the subsequent transitions.

Turbulent RBC occurs in a fluid confined between parallel horizontal plates heated from below [9–13]. Most of the temperature drop across the sample is sustained by two thin thermal boundary layers, one below the top plate and the other above the bottom plate [14–17]. The temperature throughout the bulk of the sample, even though

vigorously fluctuating, remains nearly constant in the time average.

Rotating a sample of turbulent RBC about its vertical axis at an angular frequency Ω introduces a host of interesting new phenomena (see, for instance, Ref. [18]). The associated Coriolis force affects the velocity and temperature fields, and the heat transport as expressed in terms of the dimensionless Nusselt number Nu . Slow rotation at constant Ra leads to an *increase* of Nu [19] when Ra is not too large and the Prandtl number Pr is not too small. This phenomenon is due to Ekman pumping that occurs when thermal plumes that emerge from the thermal boundary layers are turned into vortices by the Coriolis force; these vortices pump hot or cold fluid from the bottom or top boundary layer into the bulk of the sample [5,8,19–26].

A remarkable experimental discovery was that Ekman pumping does not occur immediately when rotation starts. Rather, at a critical dimensionless rotation rate $1/Ro_c$ (Ro is the Rossby number, proportional to $1/\Omega$) a sharp transition occurs where Ekman pumping sets in [5]. The value for $1/Ro_c$ is proportional to $1/\Gamma$, where $\Gamma \equiv D/L$ with D the diameter and L the height of a cylindrical sample, showing that the laterally unbounded system has no transition [7,8].

In view of the above discussion, it came as a surprise to us that the transition at $1/Ro_c$ is not the only one in this system. For a fluid with $Pr = 12.3$ and larger Ra values than had been used before to study $1/Ro_c$, we found two additional sharp transition, to be labeled $1/Ro_{c,2}$ and $1/Ro_{c,3}$, which for $Ra \approx 10^{11}$ and $\Gamma = 1.00$, for instance (where $1/Ro_c \approx 0.28$), occurred roughly at $2/Ro_c$ and $5/Ro_c$. These transitions were found in the data for Nu , as well as in measurements of the vertical temperature

gradient near the sample center. The detailed origin of these transitions between turbulent states is not known to us, but we presume that it involves a sudden change of large-scale structures of the turbulent flow. Although of-hand it seems surprising that a sharp bifurcation should exist in this highly fluctuating turbulent state with Ra as large as 2×10^{12} , changes of large-scale structures involve symmetry changes that cannot occur gradually [27–29].

Experiments on thermal convection often are done in cylindrical vessels of diameter D and height L confined by a warm plate of temperature T_b from below and a colder plate at temperature T_t from above. The Rayleigh number $Ra = \alpha g \Delta T L^3 / \nu \kappa$ and the Prandtl number $Pr = \nu / \kappa$ characterize this system. Here α , κ , and ν denote the isobaric thermal expansion coefficient, the thermal diffusivity, and the kinematic viscosity of the fluid. These properties are evaluated at the mean temperature $T_m = (T_b + T_t)/2$. The gravitational acceleration is g , and $\Delta T \equiv T_b - T_t$. The angular rotation frequency Ω is expressed by the Rossby number $Ro = \sqrt{\alpha g \Delta T} L / 2\Omega$. We shall use $1/Ro$ because it is proportional to Ω . The Nusselt number is given by $Nu = QL / A \lambda \Delta T$. We shall give the relative Nusselt number $Nu_r = Nu(1/Ro) / Nu(0)$.

The apparatus was described before [30–32]. We used cylindrical samples with $L = D$ ($\Gamma = 1.00$) and $L = 2.00D$ ($\Gamma = 0.50$), where $D = 19.0$ cm. The fluid was a fluorocarbon C_6F_{14} (3M Fluorinert™ FC72) with $Pr = 12.3$ and $T_m = 25^\circ C$. The sidewalls were made of Lexan. The top and bottom plates of the cell were made of copper. The rotating table and the corresponding outer frame were as described in detail in an earlier publication [30]. We held Ra and Pr constant while measuring Nu for several different Ω . We also determined time-averaged temperatures near the center of the sample, as described in Ref. [32].

Figure 1(a) displays results for Nu_r as a function of $1/Ro$. They are for $\Gamma = 1.00$, with $Ra = 2.07 \times 10^{11}$ (solid black circles) and 1.00×10^{11} (red open circles), and extend by a decade to larger Ra than had previously been explored for $3 \lesssim Pr \lesssim 6$ [30]. Initially, Nu_r decreased with increasing $1/Ro$. This decrease occurs only at large Ra , and to our knowledge it has not been explained. Nu_r reached a minimum at $1/Ro_c \approx 0.280$. For $1/Ro > 1/Ro_c$, Nu_r initially increased linearly with $1/Ro$ due to Ekman pumping. Within the resolution of the data, Nu_r is independent of Ra over this range of $1/Ro$.

A remarkable and unexpected new feature of the data is that $Nu_r(1/Ro)$ showed a second sharp transition at $1/Ro_{c,2} \approx 0.492$, where the slope of Nu_r decreased abruptly while no discontinuity of Nu_r was resolved. For $1/Ro > 1/Ro_{c,2}$ Nu_r depended on Ra , being smaller for larger Ra . Yet another transition, with a discontinuous change in slope, is indicated by the data near $1/Ro_{c,3} \approx 1.55$ for $Ra = 2.07 \times 10^{11}$ and near $1/Ro_{c,3} \approx 1.33$ for $Ra = 1.00 \times 10^{11}$.

Figure 1(b) shows data for the dimensionless vertical temperature gradient $\partial\Theta/\partial(z/L)$ near the sample center.

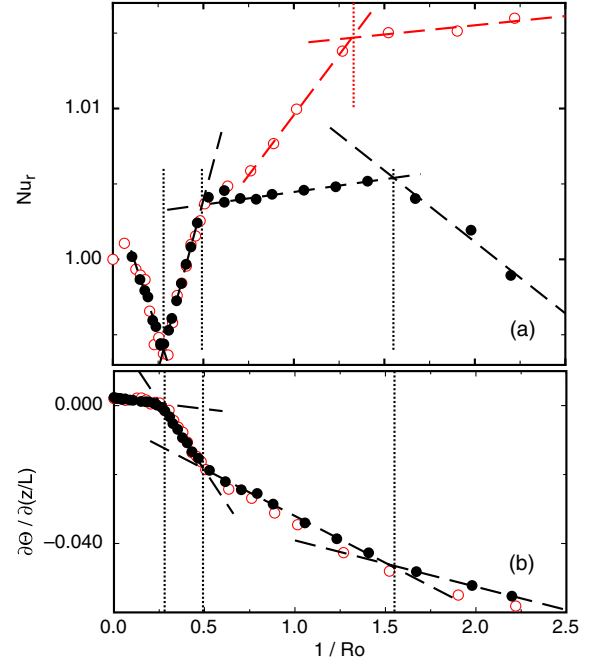


FIG. 1 (color online). (a) Nu_r , and (b) the temperature gradient at the sample center, both as a function of $1/Ro$. Here and in subsequent figures the dashed lines are straight-line fits to various segments of data and serve as guides to the eye. Black vertical dotted lines, from left to right: the location of $1/Ro_c = 0.280$, $1/Ro_{c,2} = 0.492$, and $1/Ro_{c,3} \approx 1.55$, respectively. Solid black circles: $Ra = 2.07 \times 10^{11}$ [$Nu(0) = 344$]. Open red circles: $Ra = 1.00 \times 10^{11}$ [$Nu(0) = 273$].

Here $\Theta \equiv (\langle T \rangle - T_m) / \Delta T$, $\langle \dots \rangle$ indicates a time average and z is the vertical coordinate. This local parameter also shows the three transitions, although the one at $1/Ro_{c,3}$ is not as well defined particularly at the smaller Ra .

Our observation of a sequence of transitions suggested a reexamination of earlier measurements. Some of those data are shown in Fig. 2. There, one sees that those results are

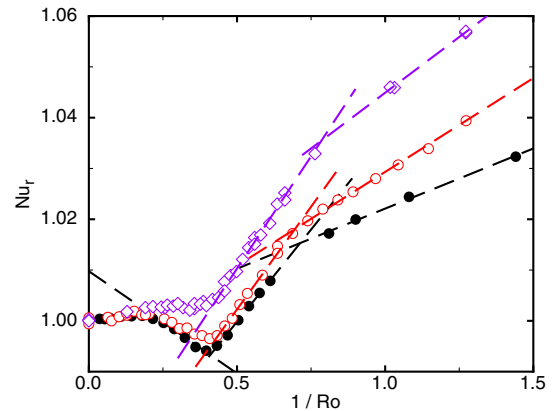


FIG. 2 (color online). The reduced Nusselt numbers Nu_r as a function of $1/Ro$ for water ($Pr = 4.38$), $\Gamma = 1.00$, $Ra = 2.3 \times 10^9$ (open purple diamonds), $Ra = 9.0 \times 10^9$ (open red circles), and $Ra = 1.8 \times 10^{10}$ (solid circles). Based on data from Ref. [30].

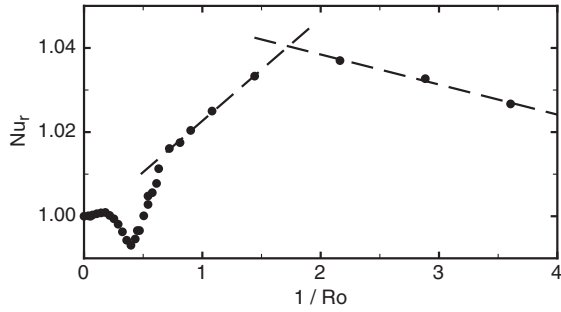


FIG. 3. An example of the reduced Nusselt number Nu_r as a function of $1/Ro$ for water over a wider range of $1/Ro$ than in Fig. 2. For this case, $Pr = 4.38$ and $Ra = 1.8 \times 10^{10}$. Based on data from Ref. [30].

also consistent with both the transition at $1/Ro_c$ and a second transition at $1/Ro_{c,2}$. Figure 3 shows that evidence also for the third transition at $1/Ro_{c,3}$ can be found.

In Fig. 4 we show all transition points that we were able to identify for $\Gamma = 1.00$, both in the new measurements for $Pr = 12.3$ and in the older ones for $3.62 \leq Pr \leq 6.26$, as a function of Ra . One sees that the phenomenon of multiple transitions is not limited to large Ra , although there it becomes more obvious in the data, as can be seen in Fig. 1 for $Ra = 2.07 \times 10^{11}$.

Turning now to $\Gamma = 0.50$, we show in Fig. 5 new measurements of Nu_r as a function of $1/Ro$ for $Pr = 12.3$. Qualitatively, the results are consistent with those of Ref. [33], where, for instance, a reduction of Nu_r to about 0.966 was found for $Ra = 4.3 \times 10^{15}$, $Pr = 5.9$, and $1/Ro = 2$, but the data of Ref. [33] did not identify any transitions. In Fig. 5 one finds $1/Ro_c = 0.612$, which, consistent with previous work [7], is slightly larger than twice the value $1/Ro_c = 0.280$ found for $\Gamma = 1.00$ (see Fig. 1). For $Ra = 8.0 \times 10^{11}$, a second transition at $1/Ro_{c,2} = 2.10$ is found, but for $Ra = 1.66 \times 10^{12}$ there is

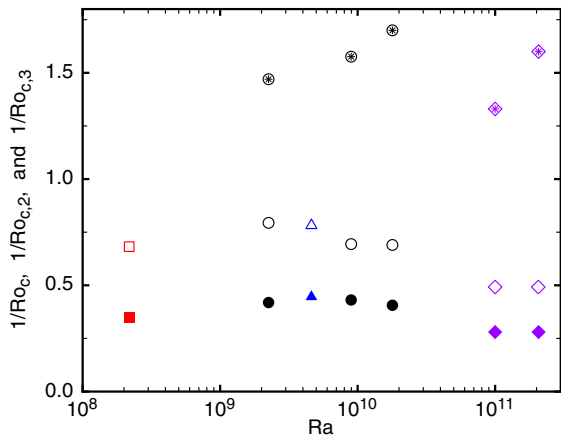


FIG. 4 (color online). Results for $1/Ro_c$ (solid symbols), $1/Ro_{c,2}$ (open symbols), and $1/Ro_{c,3}$ (open symbols with stars) as a function of Ra . Blue triangles: $Pr = 3.62$. Black circles: $Pr = 4.38$. Red squares: $Pr = 6.26$. Purple diamonds: $Pr = 12.3$.

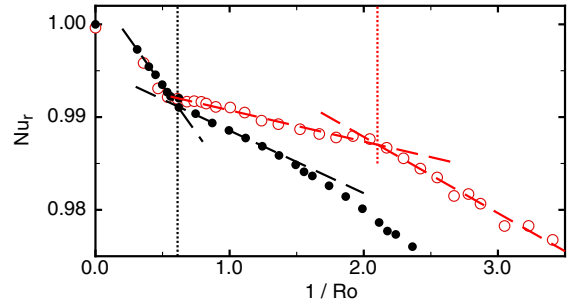


FIG. 5 (color online). The reduced Nusselt number Nu_r as a function of $1/Ro$ for $\Gamma = 0.50$, $Pr = 12.3$, $Ra = 1.66 \times 10^{12}$ [solid black circles, $Nu(0) = 680$], and $Ra = 8.0 \times 10^{11}$ [open red circles, $Nu(0) = 540$]. The vertical dotted lines indicate the locations of $1/Ro_c = 0.612$ and $1/Ro_{c,2} = 2.10$.

no evidence for a second sharp transition over the $1/Ro$ range covered by those data.

Also for $\Gamma = 0.50$ it is worth reexamining data published before. Thus, in Fig. 6 we show results from Ref. [8] for $Pr = 4.38$ and $Ra = 2.3 \times 10^9$, which extend to rather large values of $1/Ro$ and which were thought to vary smoothly with $1/Ro$ for $1/Ro > 1/Ro_c \approx 0.85$. While $1/Ro_c$ is not resolved in this figure, evidence for three transitions at $1/Ro_{c,2} = 2.38$, $1/Ro_{c,3} = 4.53$, and $1/Ro_{c,4} = 16.5$ can be found. We note that the result for $1/Ro_{c,2}$ is consistent with the result $1/Ro_{c,2} = 2.10$ for $Pr = 12.3$ displayed in Fig. 5.

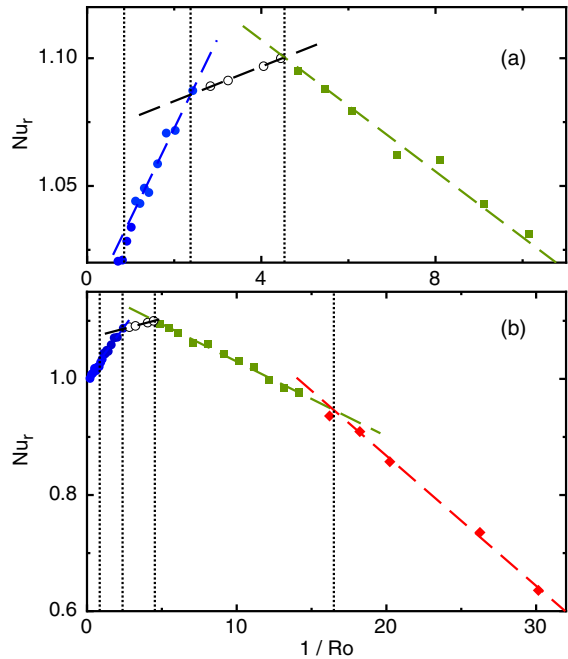


FIG. 6 (color online). The reduced Nusselt number Nu_r as a function of $1/Ro$ on two different scales for $\Gamma = 0.50$, $Pr = 4.38$, and $Ra = 2.3 \times 10^9$. The vertical dotted lines, from left to right, indicate the location of $1/Ro_c = 0.85$, $1/Ro_{c,2} = 2.38$, $1/Ro_{c,3} = 4.53$, and $1/Ro_{c,4} = 16.5$. The data are from Ref. [8].

In this Letter we presented new measurements of the heat transport by and the temperature gradient in the center of turbulent convection in the presence of rotation for samples of aspect ratios 1.00 and 0.50. We reached values of Ra up to 2×10^{12} at a Prandtl number $Pr = 12.3$.

It was known from previous measurements that $Nu_r(1/Ro)$ undergoes a sharp transition at a value of $1/Ro = 1/Ro_c$ that depends on Pr (but apparently not on Ra) and in all cases studied was well below 1, but it was thought that for $1/Ro > 1/Ro_c$, $Nu_r(1/Ro)$ varied smoothly with $1/Ro$ without any further singularities [18]. The new data show that actually there is a sequence of transitions, and that to a good approximation $Nu_r(1/Ro)$ between successive transitions is well represented by straight lines. We confirm this finding by a close examination of previous measurements that had been interpreted as representing a smoothly varying $Nu_r(1/Ro)$, but which upon closer examination also reveal sequences of several transitions.

While the observations presented in this Letter are contrary to the common belief that intense turbulent fluctuations would round out any transitions between different states that potentially could occur as Ra or $1/Ro$ is increased, they are consistent with the idea that the transitions are between different large-scale structures of different symmetries. As enunciated by Landau [27–29], symmetry-breaking transitions cannot be gradual, and instead lead to sharp, albeit possibly continuous, transitions. This is in no way altered by the presence of fluctuations, for if it were, then there also would be no continuous phase transitions (i.e., critical points) in equilibrium systems.

The task ahead, of course, is to identify the structures involved in these transitions. For $1/Ro_c$ it was well established that the new structure just beyond it consists of Ekman vortices that form near the plates, and the “order parameter” [27,28] of the new phase was determined from direct numerical simulations (see Fig. 4 of Ref. [7]). Determining experimentally the changes of large-scale structures at $1/Ro_{c,2}$ and $1/Ro_{c,3}$ will be challenging. It cannot be done in our apparatus where the extensive thermal shielding needed for the high-resolution (0.1% or better) heat-transport measurements reported in Figs. 1 and 5 prevents optical access. Perhaps it may be done more readily by direct numerical simulations.

Possibly relevant to the symmetry of the structures is the formation of a Stewartson layer [34] as $1/Ro$ increases. Here it is interesting to note that the Stewartson layer thickness reported in Refs. [34,35] for $\Gamma = 1.00$ also suggests the existence of more than one transition as $1/Ro$ is increased. Structures at larger $1/Ro$ were reported, for instance, based on the numerical work of Ref. [36]. It is not clear how or whether these structures correlate with the experimentally observed transitions. But it is clear that the system cannot evolve smoothly between structures that are

vertically coherent over the sample height and are subject to a symmetry transformation about the horizontal mid-plane, on the one hand, and structures that are vertically uncorrelated over the sample height, on the other hand (see, e.g., Fig. 1 of Ref. [36]).

Finally, one may ask whether our observation of a sequence of transitions is specific to finite systems with lateral boundaries. Although we do not know the answer for sure, we note that the structures of different symmetry reported in Ref. [36] are characteristic of the infinite system, and thus the unavoidably sharp transition between them will not depend on lateral boundaries. More globally, we call attention to the observation of cloud streets (see, for instance, Ref. [37]) in the laterally unbounded sky, which undergo symmetry-breaking transitions [38], for instance, due to the cross-roll instability, as “control parameters” (i.e., atmospheric conditions) change. These symmetry-breaking pattern changes also cannot be smooth and must involve sharp transitions according to Landau’s arguments.

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