Gauge Field Optics with Anisotropic Media

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By considering gauge transformations on the macroscopic Maxwell's equations, a two-dimensional gauge field, with its pseudomagnetic field in the real space, is identified as tilted anisotropy in the constitutive parameters. We show that the optical spin Hall effect with broadband response and one-way edge states become possible simply by using anisotropic media. The proposed gauge field also allows us to obtain unidirectional propagation for a particular pseudospin based on the Aharonov-Bohm effect. Our approach will be useful in spoof magneto-optics with arbitrary magnetic fields mimicked by metamaterials with subwavelength unit cells. It also serves as a generic way to design polarization-dependent devices.

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Artificial magnetism from nonmagnetic constituents has been showing great promise in light manipulation, particularly in the optical regime where the magnetic effect is weak. One possible way to realize artificial magnetism is to consider the "Lorentz force" of a photon in the geometrical optics limit. While the spin-orbit coupling of light can induce a pseudomagnetic field in the momentum space in giving rise to the optical spin Hall effect [1–4], recent research works show that it is possible to realize a pseudomagnetic field in the real space instead. It complements the refractive index gradient, the pseudoelectric field in the Lorentz force, and provides an additional degree of freedom for designing optical devices through the nonconservative nature of the magnetic force. Most of the current approaches are based on a photonic lattice of coupled resonators or waveguides. Then an effective gauge field is generated by either making the coupling direction dependent through optical path difference [5–7], dynamic modulation [8–12], or making the coupling inhomogeneous through a strain [13]. While these "bottom-up" approaches have experimentally confirmed the existence of the pseudomagnetic field in the real space, one is curious to ask whether there is a material realization of such pseudofields; in particular, we have flexible metamaterials to realize any required prescriptions of material parameters in a spatial profile [14–26]. Such an approach allows us to scale down the required structures to the subwavelength regime, and to have broadband response if necessary.

In this work, we consider gauge transformations on the macroscopic Maxwell's equations. While it is common to associate the refractive index to the scalar potential for a photon, such a "top-down" approach allows us to also directly associate the gauge field, or the pseudovector potential, to the constitutive parameters. We will show that the gauge field corresponds to a particular type of anisotropy, i.e., tilted anisotropy, in the permittivity and permeability tensors. It splits the originally degenerated local dispersion surfaces of two opposite pseudospins. Then the gauge field can be implemented by flexible metamaterials with subwavelength unit cells. This abstraction, in a similar spirit to transformation optics (TO), provides an additional way to bend light and enables a versatile and macroscopic approach in designing various optical devices, including the TO ones, such as invisibility cloaks and optical illusion devices [14–26].

For simplicity, we consider two-dimensional in-plane wave propagations on the x-y plane with both material parameters and fields invariant in the z direction. The fields have two polarizations and can be described by a 2×1 column vector (E_z, iH_z), indicating the z components of the electric and magnetic fields, while the in-plane transverse fields are denoted by $(\boldsymbol{E}_T, i\boldsymbol{H}_T)$. We begin our discussion by considering a specific class of field transformations (FT) in "rotating" polarization [27]:

$$\begin{pmatrix} E_z \\ iH_z \end{pmatrix} = \begin{pmatrix} \cos k_0\phi & -\sin k_0\phi \\ \sin k_0\phi & \cos k_0\phi \end{pmatrix} \begin{pmatrix} E_z^{(0)} \\ iH_z^{(0)} \end{pmatrix},$$
$$\begin{pmatrix} \boldsymbol{E}_T \\ i\boldsymbol{H}_T \end{pmatrix} = \begin{pmatrix} \cos k_0\phi & \sin k_0\phi \\ -\sin k_0\phi & \cos k_0\phi \end{pmatrix} \begin{pmatrix} \boldsymbol{E}_T^{(0)} \\ i\boldsymbol{H}_T^{(0)} \end{pmatrix}, \qquad (1)$$

where the fields and the tensors discussed later with (without) superscript "(0)" denote the relevant objects before (after) transformation, $k_0 = 2\pi/\lambda_0$ is the wave number in vacuum, and the function ϕ indicates the degree of polarization rotation at each location. While applications in TO (and also in Ref. [27]) explore the difference between the two configurations before and after transformation (e.g., in changing size, polarization signature of an object in perception), here we purposely dismiss the polarization as an internal degree of freedom. The FT in Eq. (1) is regarded as a (unitary) symmetry operation to keep $|E_z|^2 + |H_z|^2$ and also the in-plane Poynting vector invariant. This is similar to the discussion of gauge symmetry of the Schrödinger equation in quantum mechanics that multiplying the wave function ψ by local phase shift $\exp(i\phi)$ keeps $|\psi|^2$ and also the probability current density invariant so that standard procedures in identifying gauge fields can be adopted in the following. In our case, a constant FT is a so-called global symmetry of the vacuum Maxwell's equations; i.e., the FT with constant ϕ leaves the vacuum Maxwell's equations invariant. This global symmetry can actually be extended to the macroscopic Maxwell's equations with any media being coordinate transformed from vacuum within the TO framework, with equal permittivity and permeability tensor

$$\bar{\bar{\epsilon}}^{(0)} = \bar{\bar{\mu}}^{(0)} = \begin{pmatrix} \epsilon_{xx} & \epsilon_{xy} & 0 \\ \epsilon_{xy} & \epsilon_{yy} & 0 \\ 0 & 0 & \epsilon_{zz} \end{pmatrix}.$$

We still call this medium the "free space" with global symmetry. Next, we promote the global symmetry to a local one by considering a spatially varying ϕ. According to Ref. [27], a FT with spatially varying ϕ introduces an addition to the constitutive tensors by

$$\Delta\bar{\bar{\epsilon}} = -\Delta\bar{\bar{\mu}} = \begin{pmatrix} 0 & 0 & \partial_y\phi \\ 0 & 0 & -\partial_x\phi \\ \partial_y\phi & -\partial_x\phi & 0 \end{pmatrix}, \qquad (2)$$

as a choice of induced material transformation so that the Maxwell's equations are kept form invariant. (Note that global symmetry can also be verified by $\Delta\bar{\bar{\epsilon}} = \Delta\bar{\bar{\mu}} = 0$ with constant ϕ.) Therefore, the transformed medium goes outside the original free space and has a form of constitutive tensors

$$\bar{\bar{\epsilon}} = \begin{pmatrix} \epsilon_{xx} & \epsilon_{xy} & A_y \\ \epsilon_{xy} & \epsilon_{yy} & -A_x \\ A_y & -A_x & \epsilon_{zz} \end{pmatrix},$$

$$\bar{\bar{\mu}} = \begin{pmatrix} \epsilon_{xx} & \epsilon_{xy} & -A_y \\ \epsilon_{xy} & \epsilon_{yy} & A_x \\ -A_y & A_x & \epsilon_{zz} \end{pmatrix}, \qquad (3)$$

where $\mathbf{A} = A_x\hat{x} + A_y\hat{y} = \nabla\phi$ is recognized as the pure gauge field; i.e., it is still equivalent to the free space. For completeness, we apply the FT with spatially varying ϕ again in Eq. (3), but now with arbitrary $\mathbf{A}$ such that the associated field strength $\boldsymbol{B}_{\text{eff}} = \nabla \times \mathbf{A}$ is generally nonzero. In this case, the transformation in Eq. (2) is actually still valid [27]. The transformed medium is again in the form of Eq. (3) as a local symmetry while $\mathbf{A}$ is transformed according to

$$\mathbf{A} \to \mathbf{A} + \nabla\phi, \qquad (4)$$

which is therefore identified as the gauge transformation with arbitrary $\mathbf{A}$, now being called the gauge field. In our case, the Maxwell's equations (in Heaviside-Lorentz units) are decoupled into pseudo-spin-up (-down) $\psi_{+/-} = E_z \pm H_z$ (a similar definition of pseudospin is found in Ref. [28]):

$$(\nabla \pm ik_0\mathbf{A}) \cdot \frac{1}{m} \cdot (\nabla \pm ik_0\mathbf{A})\psi + k_0^2\epsilon_{zz}\psi = 0, \qquad (5)$$

where

$$m = \begin{pmatrix} \epsilon_{yy} & -\epsilon_{xy} \\ -\epsilon_{xy} & \epsilon_{xx} \end{pmatrix},$$

which plays a similar role to the mass in the Schrödinger equation. Therefore, the wave equation of the transformed medium stays the same with respect to the one without gauge field through the notion of a 2D covariant derivative $\nabla \to \nabla \pm ik_0\mathbf{A}$, as expected in a gauge theory. An immediate implication is that the local dispersion surfaces split by $\boldsymbol{k} \to \boldsymbol{k} \pm k_0\mathbf{A}$, a common signature of a real-space gauge field [12]. This gauge field provides an alternative way to manipulate light by shifting the centers rather than varying the sizes or shapes of the local dispersion surfaces. On the other hand, if we consider the ray trajectory at a fixed frequency in the geometrical optics limit, an additional magnetic force that is perpendicular to the ray direction and proportional to B_{eff} arises. For example (also for all examples in this work), we set $\epsilon_{xx} = \epsilon_{yy} = \epsilon_{zz} = n$, $\epsilon_{xy} = 0$, and simply call n the index of the medium. Then the ray equation (see Supplemental Material for the development [29]) can be written as

$$\frac{d}{ds}\left(n\frac{d\boldsymbol{r}}{ds}\right) = \nabla n \mp \hat{s} \times \boldsymbol{B}_{\text{eff}}, \qquad (6)$$

where s measures the arc length of the ray, $\boldsymbol{r}$ and $\hat{s}$ denote the position vector and the propagating direction of a point on the ray. The equation, with the right-hand side being interpreted as the Lorentz force for a photon, is an extended version of the ray equation describing a photon propagating in a gradient index medium of isotropic indices [30]. The additional term arises from $\boldsymbol{B}_{\text{eff}}$, which can now be interpreted as a pseudomagnetic field (pointing along the z direction) while $\mathbf{A}$ can be interpreted as the gauge field. Such a pseudomagnetic force, having the same magnitude but opposite signs for the two pseudospins, provides spin-dependent bending of the photon trajectory.

The above treatment is inspired by previous efforts in gauging the electromagnetic duality from vacuum while now with material parameters as potentials [31]. Moreover, if we gauge the electromagnetic duality within macroscopic Maxwell's equations, nontrivial gyrotropic media are required (see Supplemental Material [29]). However, our choice of FT, also a kind of duality operation, is specially designed for 2D and has an advantage that the gauge field can be materialized by purely reciprocal and anisotropic media. This media, different from the TO media without gauge fields, is called tilted anisotropic media here. They can be constructed by reciprocal anisotropic metamaterials, e.g., split rings with one principal axis tilted away from the z axis by 45 deg (see Supplemental Material [29] for details and also for a reduced parameter approximation). Our approach can thus be potentially pushed to optical frequencies and to have broadband response.

First, we show that the materialization of the pseudomagnetic field using anisotropic media allows the demonstration of optical spin Hall effect (cf. the similar effect induced by a pseudomagnetic field in momentum space [1–4]) when a photon travels in a region of constant $\boldsymbol{B}_{\mathrm{eff}} = B_0\hat{z}$ (with $B_0 = 0.1$) for $r = \sqrt{x^2+y^2} \le r_0$ (with $r_0 = 6.5$, the black circle in Fig. 1) while there is no pseudomagnetic field outside the region. The whole medium has a constant index $n = 1$, and we fix the nonunique $\mathbf{A}$ (from the prescribed $\boldsymbol{B}_{\mathrm{eff}}$) by choosing the Coulomb gauge with

$$\mathbf{A} = \begin{cases} 1/2B_0 r\hat{\phi} & (r \le r_0) \\ 1/2B_0 r_0^2/r\hat{\phi} & (r > r_0), \end{cases} \tag{7}$$

where $\hat{\phi}$ is the unit vector in the angular direction. We have performed full-wave simulations (using COMSOL MULTIPHYSICS) for the propagation of a Gaussian beam (of wavelength 0.4 and beam width 1.6) within such a medium. Within the region of nonzero $\boldsymbol{B}_{\mathrm{eff}}$, the beam with spin-up [Fig. 1(a)] (spin-down [Fig. 1(b)]) is bent in the anticlockwise (clockwise) direction. It undergoes a circular motion with radius of n/B_0 (=10, the red curve). Within an area of zero magnetic field (when the beam exits), the modes exhibit no bending forces and will keep propagation with negligible deflection. Such bending effect [Eq. (6)] is broadband in nature (see Supplemental Material [29]). We note that it is also possible to induce polarization splitting by applying TO individually to the TE or TM polarizations, e.g., in designing beam splitters and multifunctional devices [32,33] without the additional bending effect from spin-orbit interaction. In our approach, we associate gauge field or vector potential directly to material parameters. It is also possible to construct vector potential using spatial derivative of bianisotropic or gyromagnetic material parameters [28,34], but it will rely on the semiclassical correction to geometrical optics with a less prominent effect (see Supplemental Material [29]).

The magnetic force (here from spatially shifting local dispersion surfaces) allows wave guiding with a very different mechanism. For example, with an interface between regions of opposite pseudomagnetic fields, the photon can be guided on the interface, as an edge state [9], with the two opposite bending forces illustrated in the ray picture [see Fig. 2(a)]. It supports a one-way transport mode, which can travel around sharp corners without backscattering. We have simulated such a phenomenon in Fig. 2(c). The whole domain (with constant $n = 1$) has three different regions with constant pseudomagnetic fields of $B_1 = 2$ (the inner region), $B_2 = -2$ (the ring), and $B_3 = 0$ (outside). The required materialization of the gauge field is again fixed (numerically) by the Coulomb gauge [29]. A point source located at a position near the interface (the solid dot) emits a spin-up photon (with a free space wavelength of 2), which travels to the right and bends around the corners without reflection. We have also obtained the dispersion diagram, shown in Fig. 2(b), of this edge state on the flat surface (see Supplemental Material for more details [29]), where $\gamma = |B_1| = |B_2|$ is the magnitude of the pseudomagnetic field on the two sides. Black (red) curves show the spin-up (spin-down) modes. They are mirror copies ($k_x \to -k_x$) of each other, resulting from the time-reversal symmetry respected by our system. The flat bands [largely negative (positive) k_x

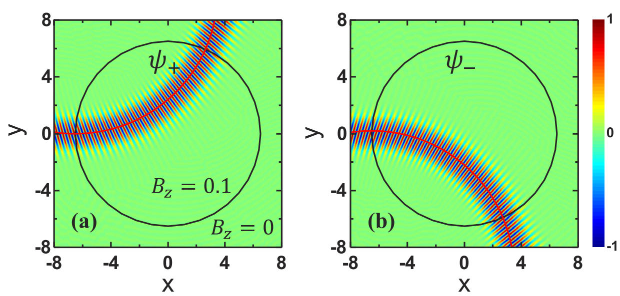

FIG. 1 (color online). The mode ψ_+/ψ_- bends in the (a) anticlockwise and (b) clockwise direction in the pseudomagnetic field region $r \le 6.5$ (the black circle) where $B_z = 0.1$.

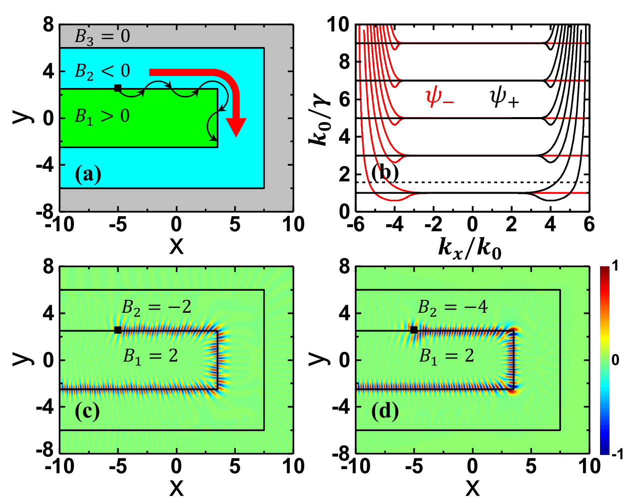

FIG. 2 (color online). (a) Edge state with ray picture (black arrows). (b) Dispersion diagram of the edge state for the two spins ψ_+ (black lines) and ψ_- (red lines) when $\gamma = |B_1| = |B_2|$. Simulated one-way transport of the edge state (ψ_+) for (c) $B_1 = 2$, $B_2 = -2$ and (d) $B_1 = 2$, $B_2 = -4$.

for spin-up (spin-down)] with k_0/γ around odd numbers are the odd and even mode combinations of the cyclotron modes on the two sides approaching the so-called Landau levels with little (evanescent wave) coupling. They evolve and with more prominent splitting (e.g., at larger values of k_x for the spin-up) to form the waveguide modes. The upper (lower) mode has odd (even) symmetry about the interface. The horizontal dashed line in Fig. 2(b) indicates the frequency for the simulation in Fig. 2(c) with free-space wavelength $\lambda_0 = 2$. For spin-up excitation, it has two modes with $k_x \approx 4.3k_0$ and $k_x \approx 5.3k_0$. They beat together, forming the wiggling pattern, and propagate only to the right, as shown in Fig. 2(c). The one-way transport can work for different frequencies as well. As the excitation frequency changes, we can still find edge modes in the dispersion diagram for $k_0/\gamma > 0.59$. Furthermore, the one-way transport is also possible when $|B_1| \neq |B_2|$. Figure 2(d) shows the asymmetric case when B_2 is changed to -4. The two sets of flatbands (the Landau levels) on the two sides split (see Supplemental Material [29]), the working frequency (the horizontal dashed line) in this case only cuts the band of the lower guiding mode, so that the one-way transport is observed without beating [see Fig. 2(d)]. We note that the phenomenon for the spin-down mode is completely opposite (one-way transport to the left) for the same medium.

Apart from modifying the photon trajectory, there is an additional geometric phase ϕ_g along the propagation in the case of a nonzero pseudomagnetic field. For a round-trip, ϕ_g is given by

$$\phi_g = \mp \oint_{\partial\Sigma} k_0 \mathbf{A} \cdot d\boldsymbol{r} = \mp k_0 \iint_{\Sigma} \boldsymbol{B}_{\text{eff}} \cdot d\Sigma, \quad (8)$$

where $\partial\Sigma$ denotes the ray trajectory and Σ denotes the enclosed area (with second equality obtained by Stokes theorem). The upper (lower) sign corresponds to the mode $\psi_+(\psi_-)$. In other words, the propagation phase now becomes path dependent, as the so-called photonic Aharonov-Bohm effect in this tilted anisotropic media (another approach is based on dynamic modulation, see Ref. [8]). The significance of Eq. (8) is that the geometric phase is only related to the enclosed magnetic flux even though there is no pseudomagnetic field along the ray trajectory (see Supplemental Material [29] for the discussion and additional results on Aharonov-Bohm effect). In another perspective, the phase difference is created due to the additional anisotropy $\mathbf{A}$ we put into the medium. If this gauge field has a nonconservative nature, we can have a path-dependent phase. Based on this, we can actually design devices with nontrivial profile of $\mathbf{A}$ (instead of generating $\mathbf{A}$ from specified pseudomagnetic fields) directly.

As the final example, we employ the path-dependent geometric phase to obtain unidirectional propagation for the spin-up mode by coupling a ring resonator with nonzero $\mathbf{A}$ to a straight waveguide [Fig. 3(a)]. The ring resonator has an inner (outer) radius of 4 (5), touching a straight waveguide of unit height. The free-space wavelength here is fixed at 5.5 while the index of the waveguide and the ring is designed to resonate at $n = 2.4$ so that a guiding mode initially traveling along the straight waveguide is completely reflected and scattered (same for both forward incidence and backward incidence), with a transmission dip (gray color) when we vary n in Fig. 3(b). Then, we add a constant $A_\phi = 0.08$ (as tilted anisotropy) pointing in the anticlockwise direction wherever inside the ring. It adds (subtracts) a geometric phase for the spin-up modes propagating in the clockwise (anticlockwise) direction. This splits the resonating condition of the ring resonator, or equivalently the forward (left to right) and backward transmission curve, as shown as the black and red curves in Fig. 3(b). Then, when we shift to work at $n = 2.45$ (the position of the forward transmission dip), the backward and forward spin-up guiding modes have nearly unit and zero transmission ($T_b \cong 0.95$, $T_f \cong 0.01$) with a large contrast [Figs. 3(c) and 3(d)]. We note that the system here has decoupled spin-up (spin-down) mode propagation. We have assumed the device is working with spin-up wave so that the unidirectional functionality is defined with respect to the fundamental spin-up mode of the waveguide in both forward and backward directions. Scattering from other elements are assumed to preserve spin. It is in contrast to the usual discussion of isolators that the backward propagation mode is the time-reversed copy of the forward propagation mode. Our system is still reciprocal, and the time-reversed copy of the forward spin-up propagation mode actually goes to the backward spin-down propagation

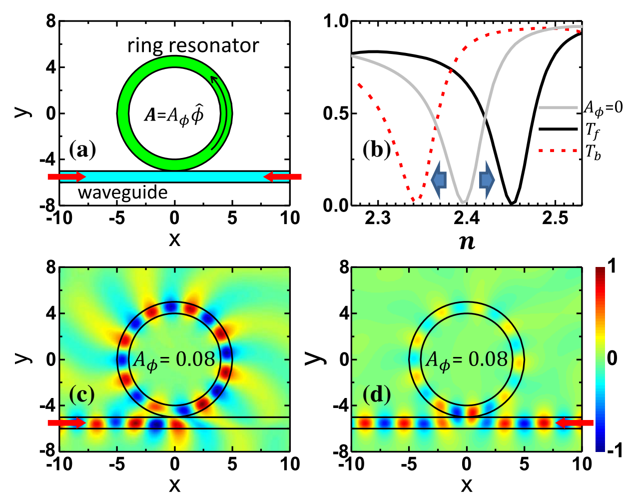

FIG. 3 (color online). Unidirectional propagation for spin-up mode (ψ_+). (a) Schematic of design. (b) Power transmittance T_f (T_b) for forward (backward) incidence versus the waveguide index n when $A_\phi = 0.08$. The gray line shows the symmetric transmittance when $A_\phi = 0$. Field patterns ($\psi_+ = E_z + H_z$) for (c) forward incidence and (d) backward incidence when $A_\phi = 0.08$ and $n = 2.45$.

mode. The gauge transformation approach proposed here can be further combined with TO to provide an additional way to bend light, e.g., to design a cloak as a possible example [35,36].

In conclusion, we have proposed a scheme to realize the gauge field in the real space for photon propagation using reciprocal anisotropic media. Such a materialization using anisotropic media allows us to design optical devices enabled by the additional bending power from pseudo-magnetic fields or the path-dependent geometric phase introduced by a nontrivial gauge field. As illustrations, we have demonstrated optical spin Hall effect, one-way transport of edge states and unidirectional propagation using tilted anisotropic media.

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[29] The profile of **A** in Fig. 2 is obtained by numerically solving $\nabla \times \mathbf{A} = B_z \hat{z}$ with $\mathbf{A} = \partial_y \Phi \hat{x} - \partial_x \Phi \hat{y}$ and zero Φ at 4.5 (8) units from upper (right) boundary of device to have $|\mathbf{A}|$ decaying away from device; see Supplemental Material at http://link.aps.org/supplemental/10.1103/PhysRevLett.114.103902 for other analytic developments and additional results.
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