

Conditional Ramsey Spectroscopy with Synchronized Atoms

Minghui Xu and M. J. Holland

*JILA, National Institute of Standards and Technology and Department of Physics,
University of Colorado, Boulder, Colorado 80309-0440, USA*

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We investigate Ramsey spectroscopy performed on a synchronized ensemble of two-level atoms. The synchronization is induced by the collective coupling of the atoms to a heavily damped mode of an optical cavity. We show that, in principle, with this synchronized system it is possible to observe Ramsey fringes indefinitely, even in the presence of spontaneous emission and other sources of individual-atom dephasing. This could have important consequences for atomic clocks and a wide range of precision metrology applications.

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The precision currently achievable by atomic clocks is remarkable; for example, the instability of state-of-the-art optical lattice clocks lies in the realm of 10^{-18} [1,2]. The pursuit of even more stability is motivated by the potential benefit to a wide range of fields in the physical and natural sciences, facilitating progress in diverse areas such as redefinition of the system of physical units in terms of time [3], clock-based geodesy [4], gravitational wave detection [5], and tests of fundamental physics and cosmology [6,7]. Atomic clock developments have also enabled spin-off applications, including precision measurements [8], quantum state control [9], and investigations of quantum many-body physics [10,11].

Atomic clocks typically operate using the method of Ramsey spectroscopy (RS) [12]. As shown in Fig. 1, RS consists of three steps: (i) the initial preparation of a coherent superposition between two quantum states, (ii) accumulation of a phase difference between the atoms and a local oscillator reference over an interrogation time T , and (iii) mapping of the phase difference to a population readout. Conventional RS is based on independent-atom physics, with the role of a large number of atoms entering only through improving the signal by statistical averaging. The performance of RS is limited by the atomic coherence time, which causes decay of the fringe visibility as a function of T . Because of this decay, an optimal strategy is typically used that involves setting T to be of the order of the coherence time, and filling up the total measurement interval τ by repeated RS cycles [13]. This gives an uncertainty in the frequency difference between the atoms and local oscillator that scales as $1/(\sqrt{N}\tau)$, with the \sqrt{N} coming from the quantum projection noise at each readout. This scaling $\tau^{-1/2}$ is much worse than the fundamental Fourier limit τ^{-1} .

There are two paths to improving on the standard limit for RS, apart from simply increasing N . First, the projection noise can be reduced by preparing spin-squeezed states [14,15]. Pursuing this direction, there have been numerous

efforts to produce spin squeezing in various physical situations [16–23]. It is worth pointing out that entangled states are often fragile and sensitive to decoherence processes, which may limit their potential for providing significant improvements to the sensitivity [24,25]. Second, one can increase the coherence time of atoms. One approach has been to increase the dephasing time of magnetically and optically trapped atomic ensembles by spin self-rephasing induced by the exchange interaction between two identical particles [26,27]. In recent lattice clock experiments [2], the atomic dephasing time T_2 has been pushed to ~ 1 s. Even if further technical improvements are made, there is a fundamental upper limit to the atomic coherence time provided by the lifetime, T_1 , of the long-lived excited clock state (~ 160 s for ^{87}Sr) [28].

In this Letter, we propose an approach to RS that is more robust against decoherence. Our idea is to use atoms that resonantly exchange photons with a heavily damped single

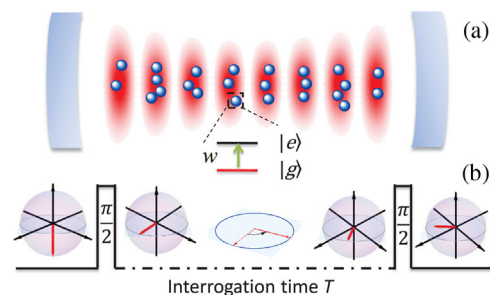


FIG. 1 (color online). (a) Conditional Ramsey spectroscopy where synchronized atoms are coupled collectively to a cavity and pumped individually with incoherent rate w during the interrogation time. (b) Ramsey sequence showing initial preparation in state $|g\rangle$ (pseudospins pointing down to the south pole of the Bloch sphere), the rotation to the equator by illuminating the atoms with a coherent near-resonant $\pi/2$ laser pulse, precession around the equator, and second $\pi/2$ x -axis rotation, after which the z -axis projection carries information about the accumulated phase.

mode of an optical cavity during the interrogation time of the RS sequence [see Fig. 1(a)]. Because of the cavity damping, it is necessary to continuously replenish the energy by incoherently repumping the atoms. One may have thought that this would simply give rise to additional decoherence channels, on top of the usual T_1 and T_2 processes, and cause the RS fringe visibility to decay more rapidly. This is not the case, since the cavity-mediated dissipative coupling between atoms acts to synchronize their phases. We show that the coherence time of the synchronized ensemble does not depend on individual-atom dephasing, as represented by T_1 and T_2 . The synchronized atoms instead undergo only a collective quantum phase diffusion. However, the collective phase can be continuously monitored by observing the cavity output field. Consequently, this system provides a kind of conditional RS, conditioned on the cavity output, where fringes of high visibility may be observed indefinitely.

The atom-cavity system during the interrogation time is described by the Hamiltonian

$$\hat{H} = \frac{\hbar\Delta\nu}{2} \sum_{j=1}^N \hat{\sigma}_j^z + \frac{\hbar g}{2} \sum_{j=1}^N (\hat{a}^\dagger \hat{\sigma}_j^- + \hat{a} \hat{\sigma}_j^+), \quad (1)$$

where $\Delta\nu$ is the frequency difference between the atoms and local oscillator and g is the coupling strength between a single atom and the cavity mode. We introduce the bosonic annihilation and creation operators, \hat{a} and \hat{a}^\dagger , for cavity photons, and the j th atom Pauli operators, $\hat{\sigma}_j^z$ and $\hat{\sigma}_j^- = (\hat{\sigma}_j^+)^{\dagger}$, for the pseudospins representing the two-level system. For simplicity, g is assumed to be identical for all atoms. In principle, this could be achieved by trapping the atoms at the antinodes of the cavity mode by an optical lattice. A less ideal spatial configuration only leads to a reduced effective atom number, which has no impact on the basic conclusions of this Letter [29].

In the presence of decoherence, the evolution is described by the usual Born-Markov quantum master equation for the reduced atom-cavity density matrix ρ ,

$$\begin{aligned} \frac{d\rho}{dt} = & \frac{1}{i\hbar} [\hat{H}, \rho] + \kappa \mathcal{L}[\hat{a}] \rho \\ & + \sum_{j=1}^N \left(w \mathcal{L}[\hat{\sigma}_j^+] + \frac{1}{T_1} \mathcal{L}[\hat{\sigma}_j^-] + \frac{1}{4T_2} \mathcal{L}[\hat{\sigma}_j^z] \right) \rho \end{aligned} \quad (2)$$

where $\mathcal{L}[\hat{O}] \rho = (2\hat{O}\rho\hat{O}^\dagger - \hat{O}^\dagger\hat{O}\rho - \rho\hat{O}^\dagger\hat{O})/2$ denotes the Lindblad superoperator. The cavity decays with rate κ and the incoherent repumping is at rate w . Conventional RS is recovered by setting $g = 0$ and $w = 0$, with the result that the RS fringe visibility then decays exponentially with the single-atom decoherence rate $\Gamma_S = (T_1^{-1} + T_2^{-1})/2$ [see Fig. 2(a)].

We solve for the dynamics in an extreme regime of bad-cavity quantum electrodynamics [31–35], where the cavity decay rate is much greater than any time scale associated

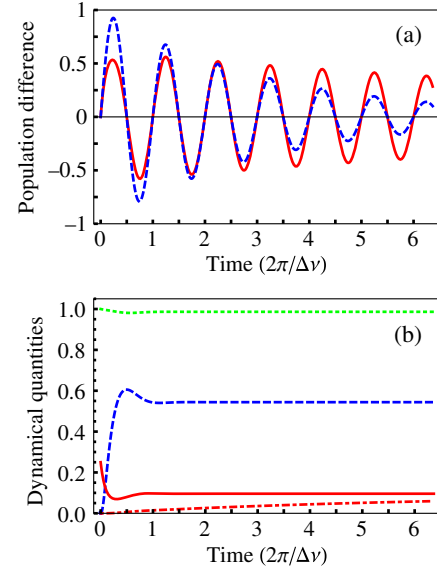


FIG. 2 (color online). Calculations of Eq. (3) with $N = 250$, $\Gamma_C = 0.2/T_1$, $T_2 = T_1$, and $w = N\Gamma_C/2$. (a) Ramsey fringes with synchronized atoms (red solid line) versus T . Conventional Ramsey fringes (blue dashed line) for the same T_1 and T_2 . (b) During the interrogation time, the atomic inversion $\langle \hat{\sigma}_j^z \rangle$ (blue dashed line), spin-spin correlation $\langle \hat{\sigma}_j^+ \hat{\sigma}_k^- \rangle$ (red solid line), $\langle \hat{\sigma}_j^+ \hat{\sigma}_k^- \rangle - \langle \hat{\sigma}_j^+ \rangle \langle \hat{\sigma}_k^- \rangle$ (red dotted-dashed line), and $\langle \hat{\sigma}_j^+ \hat{\sigma}_k^z \rangle / (\langle \hat{\sigma}_j^+ \rangle \langle \hat{\sigma}_k^z \rangle)$ (green dotted line).

with the atomic dynamics [36]. This allows an approximation to be made where the cavity field is entirely eliminated from the equations of motion [37]. The role of the cavity field then is to simply provide a source for a dissipative collective coupling for the atoms. The effective evolution is given by a quantum master equation containing only atoms;

$$\begin{aligned} \frac{d\rho}{dt} = & -\frac{i}{2} \Delta\nu \sum_{j=1}^N [\hat{\sigma}_j^z, \rho] + \Gamma_C \mathcal{L}[\hat{J}^-] \rho \\ & + \sum_{j=1}^N \left(w \mathcal{L}[\hat{\sigma}_j^+] + \frac{1}{T_1} \mathcal{L}[\hat{\sigma}_j^-] + \frac{1}{4T_2} \mathcal{L}[\hat{\sigma}_j^z] \right) \rho, \end{aligned} \quad (3)$$

where $\hat{J}^- = \sum_{j=1}^N \hat{\sigma}_j^-$ is the collective decay operator and $\Gamma_C = C/T_1$ is the collective decay rate, written in terms of the cooperativity parameter of the cavity C [38]. The collective decay rate can be taken to be small, i.e., $\Gamma_C \ll \Gamma_S$, because C is a dimensionless geometric cavity parameter that for real systems is typically much less than 1. For a current generation ^{87}Sr optical clock experiment [2], $\Gamma_S \sim 1 \text{ s}^{-1}$, while Γ_C can be as small as 10^{-3} s^{-1} for $C \approx 0.16$.

It is extremely difficult to find numerical solutions to Eq. (3) for an appreciable number of atoms without further approximation due to the exponential scaling, 4^N , of the dimensionality of the Liouvillian space. Fortunately, an

underlying $SU(4)$ symmetry of the Liouvillian superoperators in Eq. (3) was developed recently, which reduces the complexity of the problem to N^3 [39]. This enables us to obtain numerical solutions up to a few hundred pseudospins.

Figure 2(a) shows numerical calculations of RS fringes with synchronized atoms. The solution of the quantum master equation represents the ensemble average of many experimental trials. A remarkable feature is that the fringe visibility decays much slower than that of conventional RS under the same T_1 and T_2 decoherences, demonstrating the robustness to individual-atom decoherence. When compared to conventional RS with independent atoms, the principal difference here is that strong spin-spin correlations between atoms $\langle \hat{\sigma}_j^+ \hat{\sigma}_k^- \rangle$ ($j \neq k$) develop due to the dissipative coupling, as shown in Fig. 2(b). This feature is a characteristic of phase locking [32,40]. After a brief initial transient evolution, the fringe fits well to an exponentially decaying *sine* function, i.e., $Ae^{-\lambda t} \sin \Delta\nu t$, where λ is the decay rate of the fringe visibility and A is an amplitude (we derive this behavior later.)

Intuitively, one may expect that in order to effectively phase lock the atoms, it should be necessary for the dissipative coupling that provides rephasing to dominate over the “random walk” due to quantum noises that destroy phase correlations. Because of the all-to-all nature of the interaction of atoms through the cavity mode, the dissipative coupling strength scales with N and is given by $N\Gamma_C/2$ [41]. We show the effect of this in the inset of Fig. 3. For small atom number, the individual quantum noises dominate over the rephasing, and the fringe envelope decays more rapidly than in conventional RS, i.e., $\lambda > \Gamma_S$. As N increases, the dissipative coupling increases, and we reach the regime $\lambda < \Gamma_S$. For a large atom number, we find λ approaches Γ_C . The Γ_C limit arises from quantum fluctuations associated with the collective pseudospin decay through the cavity.

There are three time scales one should consider. At short times, quantum correlations develop as the atoms phase

lock. This can be seen in the initial transient part of the evolution of the observables shown in Fig. 2(b), and is characterized by the time scale w^{-1} . This phase-locking time should be less than the atomic coherence time Γ_S^{-1} in order to observe high-visibility fringes. There is also a long time scale provided by the collective decay time Γ_C^{-1} . It is important to operate in the parameter regime in which $w \gg \Gamma_S \gg \Gamma_C$.

A valid question to consider is: Why does the large incoherent repumping rate w not destroy the synchronization? Somewhat paradoxically, repumping is crucial for building up phase correlations among atoms. In Fig. 3, we show the effect of w on the decay rates of the Ramsey fringe visibility λ . When the repumping rate is too small or too large we find $\lambda > \Gamma_S$, so that the system performs worse than conventional RS. This can be understood since an effective synchronization model for Eq. (3) can be derived under the mean field approximation [29]. By parameterizing $\langle \hat{\sigma}_j^+ \rangle$ as $\alpha_j e^{-i\phi_j}$ for each atom j , we obtain the equation of motion for the phases ϕ_j ,

$$\frac{d\phi_j}{dt} = -\Delta\nu + \frac{\Gamma_C}{2} \frac{\langle \hat{\sigma}_j^z \rangle}{\alpha_j} \sum_m \alpha_m \sin(\phi_m - \phi_j). \quad (4)$$

This has the form of a Kuramoto model [42,43] that describes the synchronization of phase oscillators. The model shows that population inversion of the pseudospins is a necessary condition for phase synchronization. The repumping strength must be large enough that there is more probability for the atoms to be in the excited state than in the ground state. However, if the repumping rate is too large, the associated quantum noise destroys the phase correlations before they can develop. As has also been seen in the case of the superradiant laser [32,34], the most coherent system is realized at an intermediate pump strength.

An accurate semiclassical approximation may be developed that is valid in the case of large numbers of atoms. Taking advantage of the fact that all expectation values are symmetric with respect to atom exchange, we find from Eq. (3),

$$\frac{d}{dt} \langle \hat{\sigma}_j^+ \rangle = i\Delta\nu \langle \hat{\sigma}_j^+ \rangle - \frac{\Gamma_t}{2} \langle \hat{\sigma}_j^+ \rangle + \frac{\Gamma_C}{2} (N-1) \langle \hat{\sigma}_j^+ \hat{\sigma}_k^z \rangle, \quad (5)$$

where $j \neq k$ and $\Gamma_t = 2\Gamma_S + w + \Gamma_C$ is the total decay rate of the atomic coherence. We first point out that instead of calculating the population difference measured at the end of the RS sequence, it is equivalent to calculate $2\text{Im}[\langle \hat{\sigma}_j^+ \rangle]$ just before the second $\pi/2$ pulse. The decay rate of $\langle \hat{\sigma}_j^+ \rangle$ during the interrogation time T is therefore the same as that of the Ramsey fringe visibility. As seen in Fig. 2(b), the quantities $\alpha(t) = \langle \hat{\sigma}_j^+ \hat{\sigma}_k^z \rangle / (\langle \hat{\sigma}_j^+ \rangle \langle \hat{\sigma}_k^z \rangle)$ and $\langle \hat{\sigma}_j^z(t) \rangle$ rapidly approach the steady state on the short time scale of the phase locking, w^{-1} . We therefore substitute the steady-state values α_{ss} and $\langle \hat{\sigma}_j^z \rangle_{ss}$ into Eq. (5). This produces the exponentially

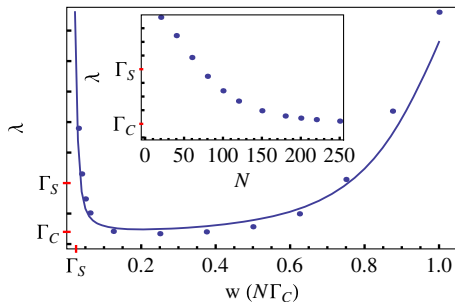


FIG. 3 (color online). The decay rate of the visibility of Ramsey fringes at $\Gamma_C = 0.2/T_1$ and $T_2 = T_1$ as a function of repumping for $N = 200$ and as a function of N for $w = N\Gamma_C/2$ (Inset). The dots are numerical solutions of Eq. (3), and the solid blue line is the semiclassical approximation for comparison.

decaying sine function solution noted earlier with decay constant

$$\lambda = \frac{1}{2}[\Gamma_t - (N-1)\Gamma_C\alpha_{ss}\langle\hat{\sigma}_j^z\rangle_{ss}]. \quad (6)$$

Furthermore $\alpha_{ss} \approx 1$, see Fig. 2(b). At the level of the mean field [29], $\langle\hat{\sigma}_j^z\rangle_{ss} \approx \Gamma_t/(N-1)\Gamma_C$ giving the trivial result $\lambda = 0$. It is therefore necessary to develop a semiclassical expression for $\langle\hat{\sigma}_j^z\rangle_{ss}$ that goes beyond the mean field, as shown in Ref. [29]. Figure 3 compares λ from the semiclassical expression with the quantum master equation solution, showing good agreement over the full range of pumping rates.

All of these results consider the ensemble that is formed from a statistical average of independent trials. The decay of the fringe visibility is really due to the averaging itself, as we will now see. In each trial, the quantum phase diffuses as a function of interrogation time. This means that as time goes on, different trials begin to add out of phase, and so the fringe visibility decays.

This motivates us to consider the properties of a single experimental run, where the behavior is qualitatively different. Although in a single run, the fringe undergoes a quantum phase diffusion, it does so with nondecaying visibility. This quantum phase diffusion has a simple physical interpretation in terms of quantum measurements. Since the cavity field follows the atomic coherence through adiabatic elimination, measuring the phase of the cavity output field, for example by homodyne measurement, is equivalent to a continuous nondestructive measurement on which information is gathered about the evolving collective atomic phase. The backaction of this measurement introduces fluctuations that cause the collective atomic phase to undergo a random walk [35].

We demonstrate this in Fig. 4(a), where we show a typical Ramsey fringe for a single experimental trial by using the method of quantum state diffusion [44,45] to yield conditional evolution of the system subject to continuous measurements of the cavity field. The phase diffusion of the synchronized atoms is evident from the phase fluctuation of the Ramsey fringe. To find the phase diffusion coefficient, Fig. 4(b) shows the statistics of the positions of the zero crossings of the fringe for 4000 trials. They fit well to Gaussian distributions with variance given by $T\Gamma_C$, clearly demonstrating that it is a diffusion process and that the diffusion coefficient is $\sqrt{\Gamma_C}$. Note that this is the same mechanism that also sets the quantum limited linewidth in a superradiant laser to be Γ_C [32], observed here in the time rather than frequency domain.

We should emphasize that the quantum phase diffusion does not itself provide a fundamental limit to the performance of conditional RS, since the collective atomic phase can be tracked by measuring the light output from the cavity. This opens up the exciting possibility of observing conditional Ramsey fringes (meaning an experimental trial

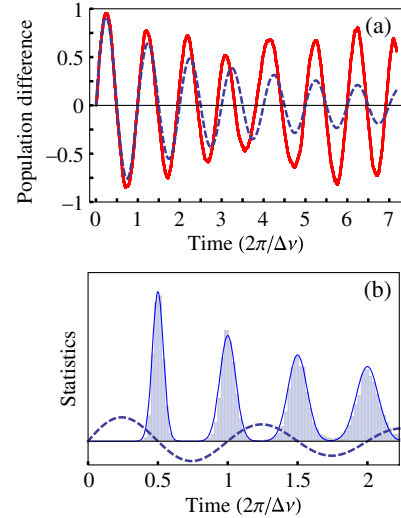


FIG. 4 (color online). Quantum state diffusion calculations of conditional Ramsey fringes subject to continuous homodyne measurement of the cavity output field for $N = 10$ and $w = N\Gamma_C/2$. The blue dashed lines are the ensemble average for reference. (a) A typical Ramsey fringe for a single experimental trial (red solid line). (b) Histograms are the statistics of the positions of zero crossings of each fringe for 4000 trials. The blue solid lines are fitted Gaussian distributions with variance of $T\Gamma_C$ centered on the zero crossing of the ensemble average.

conditioned on the measurement record of the output field) of near maximum fringe visibility for as long as the atoms can be stored, even in the presence of T_1 and T_2 processes. Of course a practical limit is also set by the length of time for which the local oscillator can remain phase coherent. In principle, if experimentally achieved, this work could lead to dramatic advances in the sensitivity of RS, since the entire measurement interval could then be used to determine frequency at the Fourier limit.

In conclusion, we have proposed and analyzed RS with synchronized atoms where we have shown that the interrogation time can be extended beyond the T_1 and T_2 times that limit conventional RS. Because of the rephasing effect, we have demonstrated that synchronized atoms are potentially robust against local decoherence. However, we have also found that the synchronization process itself intrinsically generates quantum phase diffusion through the quantum fluctuations that arise due to the cavity dissipation. This implies that the quantum phase of the atomic ensemble relative to the local oscillator must be tracked in real time by observation of the output light from the cavity in order to achieve the optimal precision for the RS with synchronized atoms.

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