Two Copies of the Einstein-Podolsky-Rosen State of Light Lead to Refutation of EPR Ideas

Krzysztof Rosołek,[1] Magdalena Stobińska,[1,2,*] Marcin Wieśniak,[1] and Marek Żukowski[1]
[1]Institute of Theoretical Physics and Astrophysics, University of Gdańsk, Wita Stwosza 57, 80-952 Gdańsk, Poland
[2]Institute of Physics, Polish Academy of Sciences, Aleja Lotników 32/46, 02-668 Warsaw, Poland
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Bell's theorem applies to the normalizable approximations of original Einstein-Podolsky-Rosen (EPR) state. The constructions of the proof require measurements difficult to perform, and dichotomic observables. By noticing the fact that the four mode squeezed vacuum state produced in type II down-conversion can be seen both as two copies of approximate EPR states, and also as a kind of polarization supersinglet, we show a straightforward way to test violations of the EPR concepts with direct use of their state. The observables involved are simply photon numbers at outputs of polarizing beam splitters. Suitable chained Bell inequalities are based on the geometric concept of distance. For a few settings they are potentially a new tool for quantum information applications, involving observables of a nondichotomic nature, and thus of higher informational capacity. In the limit of infinitely many settings we get a Greenberger-Horne-Zeilinger-type contradiction: EPR reasoning points to a correlation, while quantum prediction is an anticorrelation. Violations of the inequalities are fully resistant to multipair emissions in Bell experiments using parametric down-conversion sources.

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Introduction.—Quantum phenomena are counterintuitive and the formalism is even more so. Predictions of quantum mechanics (QM) are of statistical nature: there is no deterministic theory of response of individual systems to all possible experiments. Some quantum predictions seem paradoxical [1].

The Einstein-Podolsky-Rosen (EPR) paradox [2] was an attempt to show that the quantum description of reality cannot be complete. Elements of reality, properties of a system, which can be established with perfect accuracy without in any way disturbing it, were suggested to be the missing component of the theory. EPR used perfectly correlated systems to argue that such elements are derivable from quantum predictions and the principle of relativistic locality. There were some additional tacit assumptions in the reasoning of EPR, like the freedom of the experimentalist to choose the observable to be measured, and the equivalence of the actual experimental situation realized for the given individual system, and a complementary one [3]. The second of these was challenged by Bohr [4]: "… there is essentially the question of an influence on the very conditions which define the possible types of predictions regarding the future behavior of the system… In fact, it is the mutual exclusion of any two experimental procedures, permitting unambiguous definition of complementary physical quantities, which provides room for new physical laws, the coexistence of which at first sight appear irreconcilable with the basic principles of science."

50 years ago, Bell showed a technical flaw in the EPR reasoning [5]: in Bohm's version of the paradox [6], for a two-spin $1/2$ singlet, elements of reality are incompatible with QM. They must satisfy Bell's inequalities, while quantum predictions violate them. A more striking contradiction is shown by Greenberger, Horne, and Zeilinger (GHZ) [7,8]: for three spin $1/2$ particles, if elements of reality exist, then $1 = -1$. These results led to an "industry" that uses violations of Bell inequalities in practical applications: e.g., reduction of communication complexity [9], randomness generation [10], *device-independent* quantum cryptography [11], and as entanglement "witnesses" [12,13].

A question remained unresolved for many years: Does Bell's theorem hold true also for the EPR state? The momentum representation of it is $\Psi(p_1, p_2) = \delta(p_1 + p_2)$, where p_i is the momentum of the ith "particle." Such singular objects do not exist in the Hilbert space. Nevertheless, they can be approximated by well-behaved functions, which in some limit give $\delta(p_1 + p_2)$. In Ref. [14], Bell shows that the Wigner distribution for the EPR state is non-negative in the entire phase space; thus, there is no chance for a Bell inequality violation, as we have the explicit local hidden variable model.

Meanwhile, Reid and Drummond [15,16] showed that the state emitted by a nondegenerate optical parametric amplifier, *two mode* squeezed vacuum, is an optical approximation of the EPR state. This opened prospects for observing approximate "original" EPR correlations.

Bell's theorem for approximate EPR states was finally given in [17] and [18]. The idea was to use different observables than the ones discussed by EPR. Cohen [17] used an approach which requires a highly specific interferometer, or coupling of the EPR state to a pair of spin $1/2$

ancillas. In Ref. [18], observables with singular Wigner representations were used (parity operators, or no count events, highly dependent on losses). In both cases displacement was involved. It requires auxiliary coherent states, and thus necessary filtering to get indistinguishability of photons from different sources, which introduces losses [19,20].

Below we review and reveal properties of the four-mode squeezed vacuum state (SV) related with EPR paradox. This leads us to formulation of specific chained Bell inequalities, violated by the SV state. The nonclassical phenomena related with SV can be used in quantum information and communication, and allow for a GHZ-like argument. The SV can be interpreted both as approximate two copies of the EPR state or a polarization supersinglet. We conclude with a discussion and interpretation of our results, and remarks on the feasibility of their experimental demonstration. We emphasize that we do not aim at seeking robust phenomena leading to loophole-free Bell tests, but rather to reveal exciting phenomena linked with the four-mode SV state. It constitutes both a realistic resource for quantum technologies, and can lead to exciting case studies in foundations of quantum theory.

Four-mode SV singlet state.—The standard method of its generation employs a type II parametric down-conversion (PDC) in a nonlinear crystal pumped by a laser beam [13]. This process is described by the Hamiltonian $\mathcal{H} = ig(a_H^\dagger b_V^\dagger + e^{i\phi} a_V^\dagger b_H^\dagger) + \mathrm{H.c.}$, where in the notation for creation operators letters a, b stand for distinct spatial beams, and subscripts H, V for linear polarizations; the coupling g is proportional to the pumping field. We assume $e^{i\phi} = -1$. The output state is a superposition of maximally entangled $2N$-photon polarization singlet states

$$|\Psi^{(-)}\rangle = \sum_{N=0}^{\infty} \lambda_N |\psi_N^{(-)}\rangle, \tag{1}$$

where $\lambda_N = \cosh^{-2}\Gamma\sqrt{N+1}\tanh^N\Gamma$, $\sum_{N=0}^{\infty}\lambda_N^2 = 1$,

$$\begin{aligned} |\psi_N^{(-)}\rangle &= \frac{1}{\sqrt{N+1}N!}(a_H^\dagger b_V^\dagger - a_V^\dagger b_H^\dagger)^N|0\rangle \\ &= \frac{1}{\sqrt{N+1}}\sum_{n=0}^{N}(-1)^n|n_H,(N-n)_V\rangle_a|(N-n)_H,n_V\rangle_b. \end{aligned} \tag{2}$$

The symbol $|n_H,(N-n)_V\rangle_a$ denotes n horizontally and $N-n$ vertically polarized photons in beam a, similarly for beam b. The states $|\psi_N^{(-)}\rangle$ contain N photons in each beam. Polarization of each beam is undefined. However, due to equal photon numbers in the orthogonal polarizations beams are anticorrelated. The effective strength of the interaction is $\Gamma = gt$, where t is the interaction time.

The unitary transformation generating $\Psi^{(-)}$ is given by $e^{i\mathcal{H}t}$, and can be factorized as $e^{i\mathcal{H}_{H,V}t}e^{i\mathcal{H}_{V,H}t}$, where $\mathcal{H}_{H,V} = ig(a_H^\dagger b_V^\dagger) + \mathrm{H.c.}$ and $\mathcal{H}_{V,H} = -ig(a_V^\dagger b_H^\dagger) + \mathrm{H.c.}$ The initial state is vacuum. We get two approximate EPR states, two squeezed two-mode vacua: one for modes a_H and b_V and the second one, for a_V and b_H.

EPR elements of reality vs $|\Psi^{(-)}\rangle$.—Consider a Bell experiment in Fig. 1. Two spatially separated observers, Alice and Bob observe radiation of a pulse pumped source producing the SV state. They control the orientation of their local polarizing beam splitters, θ_A and θ_B, respectively, and count photons at their outputs. The result of the local measurement for run k is a certain number of θ_A linearly polarized photons counted at Alice's side $n^{(k)}(\theta_A)$ and at Bob's side $m^{(k)}(\theta_B)$. Since the Hamiltonian is invariant with respect to the choice of pairs of orthogonal (generally elliptic) polarizations: $\mathcal{H} = ig(a_\theta^\dagger b_{\theta^\perp}^\dagger - a_{\theta^\perp}^\dagger b_\theta^\dagger) + \mathrm{H.c.}$, where $\theta^\perp = \theta + \pi/2$, if $\theta_B = \theta_A + \pi/2$ then $n^{(k)}(\theta_A) = m^{(k)}(\theta_A + \pi/2)$. In the above notation $\theta = 0$ denotes horizontal polarization H, etc. Recall that the two-photon polarization singlet state of Bohm [6], $(1/\sqrt{2})\times(a_H^\dagger b_V^\dagger - a_H^\dagger b_V^\dagger)|0\rangle$, is invariant with respect to $U \otimes U$ polarization rotations. The four mode SV possess the same invariance. Thus, it is a kind of polarization supersinglet, with an undefined number of photons.

This feature of $|\Psi^{(-)}\rangle$ allows for an EPR-like reasoning with different observables than the ones considered in earlier works. A distant measurement at Alice's side with setting θ_A can fix Bob's value for the kth run for his setting $\theta_B = \theta_A + \pi/2$, without measuring it, and vice versa. Here, we use the property $n^{(k)}(\theta_A) = m^{(k)}(\theta_A + \pi/2)$. This suggest that $n^{(k)}(\theta_A)$ and $m^{(k)}(\theta_B)$ are *elements of reality*. They seem to exist for any θ_A and θ_B.

This EPR-like reasoning is inconsistent. A Bell inequality is satisfied by the elements of reality, and violated by quantum predictions. The double EPR-like supersinglet $|\Psi^{(-)}\rangle$ leads to predictions that disagree with the ideas of EPR.

Chained Bell inequalities.—The inequalities are based on the concept of distance. Any properly defined distance

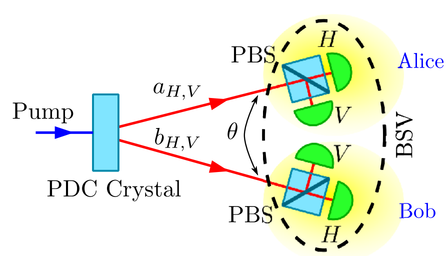

FIG. 1 (color online). Test of inequality (4) with four-mode squeezed vacuum state. Parametric down-conversion crystal (PDC) and polarizing beam splitter (PBS). The detectors measure photon numbers.

satisfies polygon inequalities. Take two stochastic variables $V(\lambda)$ and $W(\lambda)$, governed by a joint probability $\rho(\lambda)$. Their "separation" can be measured by $D(V,W)=\int|V(\lambda)-W(\lambda)|\rho(\lambda)d\lambda$. This function satisfies all defining properties of a distance $D(V,V)=0$, $D(V,W)=D(W,V)\geq 0$ and the triangle inequality $D(V,Z)\leq D(V,W)+D(W,Z)$. The last property is due to the fact that for any three numbers a,b,c one has $|a-c|\leq|a-b|+|b-c|$.

Alice and Bob choose freely between several local settings of their polarizers, θ_{A_i} and θ_{B_j}, respectively. For a concise notation, we denote the elements of reality associated with the kth run of the experiment by $n_i^{(k)}=n^{(k)}(\theta_{A_i})$ and $n_j^{(k)}=n^{(k)}(\theta_{B_j})$.

The triangle inequality implies polygon inequalities, illustrated in Fig. 2. Let $i,j=1,\ldots,L$. A polygon inequality for numbers representing the elements of reality takes the form

$$\sum_{i=1}^{L}|m_i^{(k)}-n_i^{(k)}|+\sum_{i=1}^{L-1}|m_{i+1}^{(k)}-n_i^{(k)}|\geq|m_1^{(k)}-n_L^{(k)}|. \quad (3)$$

For averages, $\langle|m_i-n_j|\rangle=(1/R)\sum_{k=1}^{R}|m_i^{(k)}-n_j^{(k)}|$, where R is the number of runs, we get

$$\sum_{i=1}^{L}\langle|m_i-n_i|\rangle+\sum_{i=1}^{L-1}\langle|m_{i+1}-n_i|\rangle\geq\langle|m_1-n_L|\rangle. \quad (4)$$

Thus, we arrive at distance-based Bell inequalities (for different chained inequalities, see [21]).

Inequality (4) also holds for local hidden variable (LHV) approaches. If variables m_i and n_j depend on some hidden parameters λ, and their "distance" equals

$$\langle|m_i-n_j|\rangle=\int d\lambda\rho_{hv}(\lambda)|m_i(\lambda)-n_j(\lambda)|, \quad (5)$$

where $\rho_{hv}(\lambda)$ is a probability distribution.

Within quantum theory, in (4) we shall use as observables photon number operators $a_i^\dagger a_i$ (Alice) and $b_j^\dagger b_j$ (Bob). The measurement settings by Alice and Bob will be defined by $a_i=\cos\theta_{A_i}a_H+\sin\theta_{A_i}a_V$, and $b_i=-\sin\theta_{B_i}b_H+\cos\theta_{B_i}b_V$. The inequality (4) requires the following holds:

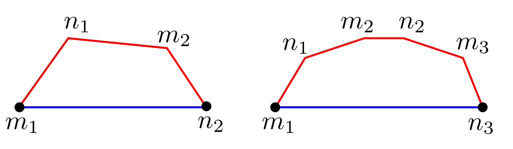

FIG. 2 (color online). Polygon inequalities for distance. The sum of the lengths of the red segments is greater than the length of the blue segment.

$$\begin{aligned}\text{lhs}&=\sum_{i=1}^{L}\langle|a_i^\dagger a_i-b_i^\dagger b_i|\rangle+\sum_{i=1}^{L-1}\langle|a_{i+1}^\dagger a_{i+1}-b_i^\dagger b_i|\rangle\\&\geq\langle|a_1^\dagger a_1-b_L^\dagger b_L|\rangle=\text{rhs}.\end{aligned} \quad (6)$$

Violations of (4) by the supersinglet $\Psi^{(-)}$.—The measurements for Alice and Bob are displayed in Fig. 3. We set $\theta_{A_1}=0$ and $\theta_{B_1}=\theta=\pi/(4L)$ The relative angle between the polarization settings by Alice θ_{A_i} and Bob θ_{B_i} we put as constant, equal to θ. Each subsequent setting of Alice and Bob changes by 2θ. Thus, the angle between $\theta_{A_{i+1}}$ and θ_{B_i} is also θ. The angle between the first Alice's setting θ_{A_1} and the last of Bob's setting θ_{B_L} is set to $\theta'=(2L-1)\pi/(4L)$. Because of the $U\otimes U$ invariance of $\Psi^{(-)}$, the quantum predictions for counts in a and b depend only on the relative angle, θ or θ'. Note that, for $\theta=0$, perfect correlations (2) between the orthogonal polarizations in beams a and b are observed.

In the lossless case, Alice and Bob always measure altogether, in the two outputs of local polarizers, N photons each (we shall analyze losses later). For the settings $\theta_{A_1}=0$ and $\theta_{B_1}=\theta$, the probability $p_Q^N(n,m|\theta)$ to register n photons in Alice's channel H and m in Bob's channel $\theta_{B_1}^{\perp}$, denoted below as $V+\theta$, reads

$$|\langle\psi_N^{(-)}|(|n_H,(N-n)_V\rangle_a|(N-m)_{H+\theta},m_{V+\theta}\rangle_b)|^2. \quad (7)$$

As the components $|\psi_N^{(-)}\rangle$ do not mix up, we can consider (6) for each component separately, as effectively we have

$$\begin{aligned}\text{lhs}&=\sum_{n,m=0}^{N}|m-n|(2L-1)p_Q^N(n,m|\theta)\\&\geq\sum_{n,m=0}^{N}|m-n|p_Q^N(n,m|\theta')=\text{rhs}.\end{aligned} \quad (8)$$

Let us estimate the rhs of (8) for a large number of settings L (a long chain). Then, $\theta'\approx\pi/2$. We have $\pi/2$ in the limit $L\to\infty$, and Bob's H is now V. Perfect anticorrelation is observed, $m=N-n$; one has $p_Q^N(n,N-n|\pi/2)=1/(N+1)$. Taking into account the

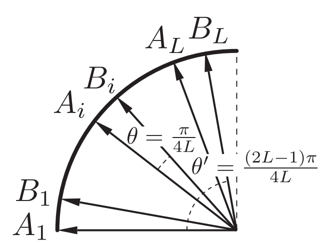

FIG. 3. Measurement settings for Alice (A) and Bob (B) for testing the distance Bell inequality (6).

summation over n and m, the rhs grows linearly with N.

To estimate the lhs of (8), notice that $|n_H, (N-n)_V\rangle_a \otimes |(N-m)_{H+\theta}, m_{V+\theta}\rangle_b$ is proportional to $a_H^{\dagger n} a_V^{\dagger (N-n)} \otimes b_{H+\theta}^{\dagger (N-m)} b_{V+\theta}^{\dagger m}|0\rangle$. Since $b_{H+\theta}^\dagger = b_H^\dagger \cos\theta + b_V^\dagger \sin\theta$ and $b_{V+\theta}^\dagger = b_V^\dagger \cos\theta - b_H^\dagger \sin\theta$, for $\theta = 0$ the perfect singlet correlations are recovered: $p_Q^N(n, m|\theta = 0)$ is nonzero only for $n = m$, and the average of $|m - n|$ vanishes. For $\theta \neq 0$, all "new" terms in (7) are proportional to *even* powers of $\sin\theta$. The "old" term proportional to $\cos^{2N}\theta$ does not contribute to $p_Q^N(n \neq m|\theta)$. Thus, the difference between $p_Q^N(n \neq m|0) = 0$ and $p_Q^N(n \neq m|\theta) = 0$ is a polynomial in $\sin\theta$ with the lowest power equal to 2. As $\theta = \pi/(4L)$, the lowest order terms in the lhs of (8) behave as $(2L-1)\sin^2[\pi/(4L)]$ and tend to zero for large L. Higher order terms vanish even quicker. Therefore, the lhs approaches zero and, in the limit $L \to \infty$, we have an "all-versus-nothing" conflict with the prediction for the rhs.

We may define a Bell parameter for the SV state as follows: $B_Q = \sum_{N=0}^{\infty} \lambda_N^2 B_Q^N$, where $B_Q^N = \text{lhs} - \text{rhs}$ is computed for the $|\psi_N^{(-)}\rangle$ state. For $L \to \infty$ and an odd N, $B_Q^N = -(\frac{1}{2}N^2 + N + \frac{1}{2})/(N+1)$ and for even, $B_Q^N = -(\frac{1}{2}N^2 + N)/(N+1)$ (for details see the Supplemental Material [22]). According to LHV theories, B_Q is positive. Figure 4 shows that for sufficiently large L, the values of B_Q become negative. The mean number of photons in the SV state is $2\sinh^2\Gamma$. The value of B_Q decreases with population, $B_Q \approx -e^{2\Gamma}$, and in the macroscopic limit of $\Gamma \to \infty$, in the case of $L \to \infty$, we obtain a striking contradiction: $0 \geq \infty$.

In *the case of inefficient detection*, Alice and Bob measure unequal total photon numbers. These various components $|\psi_N^{(-)}\rangle$ of the SV state contribute to the same detection event. We assume that losses in each polarization mode are independent but equal and model them using the Bernoulli distribution with probability of success η corresponding to detection efficiency. Probability $p_Q^N(n, m|\theta)$ in (8) is replaced with a modified one, $P_Q(n, m|\theta, \eta)$, which includes all events contributing to a measurement of n photons by Alice and m by Bob with efficiency η, and the summation over n and m extends to infinity. Figure 5 displays a numerically computed violation of (8) as a function of gain and efficiency for the fixed number of settings $L = 2$. Violation for the higher gains occurs for larger L's.

Summary and feasibility.—We show that for an arbitrary pump power, the *four-mode* SV state $\Psi^{(-)}$ involving two propagation and polarization modes, is both an approximation of *two copies* of the EPR state and a polarization supersinglet. It has all invariance properties of a two-photon singlet state, although it is a superposition of multiphoton components. We introduce a family of chained Bell inequalities based on the concept of distance, which are violated by $\Psi^{(-)}$ for all (nonzero) values of squeezing (gain). Our inequalities employ straightforward local observables: merely photon numbers at outputs of polarization analyzers, which do not require auxiliary fields, or ancillas; just beam splitting, no interferometric overlaps. For low pump powers the inequalities do not give results which differ much from the traditional CHSH-like chained inequalities. However, for high powers they are robustly violated because multiphoton emissions do not decrease the contrast of the interference effect which defines the terms of the inequalities (averaged moduli of differences of photon numbers). Note, that standard correlation functions $\langle a_i^\dagger a_i b_j^\dagger b_j\rangle$, which behave as

$$\sinh^2\Gamma\cosh^2\Gamma\cos^2(\theta_{A_i} - \theta_{B_j}) + \sinh^4\Gamma,$$

lose their interferometric contrast for increasing Γ, eventually reaching the value $1/3$, characteristic for thermal fields; see, e.g., Ref. [23]. This renders CHSH-like approaches, based on such correlations, useless. Thus, our chained inequalities are better suited for high gain parametric down-conversion experiments.

The "short" inequalities (8), involving 2 to 4 settings at each side, can be useful in quantum information tasks, cryptography, and reduction of communication complexity,

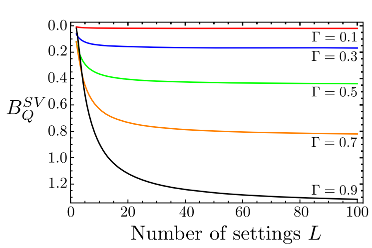

FIG. 4 (color online). The Bell parameter B_Q as a function of the number of settings L evaluated for the entangled squeezed vacuum state. For $\Gamma = 0.8$ the mean number of photons equals 1.6.

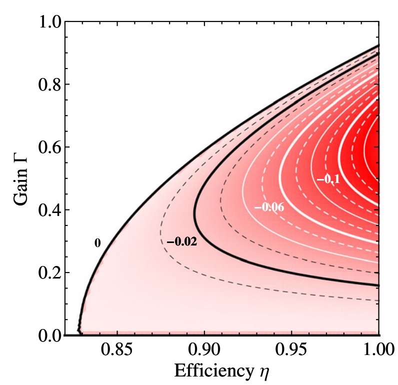

FIG. 5 (color online). Violation of the inequality (8) by four mode squeezed vacuum as a function of gain and efficiency, for two measurement settings.

in device-independent protocols. They are violated also for final efficiencies, Fig. 5. Note that as the PDC process now produces entangled pairs with fidelity approaching 100%, the main distortions in production the SV, which involves multipair emissions, are due to losses. Thus our efficiency analysis covers also the imperfections in the generation of SV.

The inequalities involving large numbers of settings are impractical, but they lead to an all-versus-nothing direct GHZ-like refutation of EPR concepts, for states that are close approximations of EPR states and share the basic properties with Bohm's singlets. Thus, the four-mode SV emerges as a versatile state in studies of both quantum information and foundational problems.

The SV states with mean photon number of the order of 10 are accessible in laboratories [13,24]. Violations of the presented Bell inequalities may be soon feasible for a small number of settings and for pump intensities in Fig. 5. Experiments could employ the techniques of Ref. [25] and integrated optics setups equipped with superconducting transition-edge sensors (TESs) [26], which reach photon-counting efficiencies near 100% and have extremely well-resolved photon-number peaks, up to around ten photons [27]. Therefore, the efficiency required for the chained Bell inequality violation with the four-mode SV is, in principle, achievable with state-of-the-art techniques. However, our work is rather a motivation for new research, than a blueprint for an experiment.

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*Corresponding author.
magdalena.stobinska@gmail.com

[1] Y. Aharonov and D. Rohrlich, *Quantum Paradoxes: Quantum Theory for the Perplexed* (Wiley-VCH, New York, 2005).
[2] A. Einstein, B. Podolsky, and N. Rosen, Phys. Rev. **47**, 777 (1935).
[3] M. Zukowski and C. Brukner, J. Phys. A **47**, 424009 (2014).
[4] N. Bohr, Phys. Rev. **48**, 696 (1935).
[5] J. S. Bell, Physics **1**, 195 (1964).
[6] D. Bohm, *Quantum Theory* (Prentice-Hall, Englewood Cliffs, NJ, 1951).
[7] D. M. Greenberger, M. A. Horne, and A. Zeilinger, *Bell's Theorem, Quantum Theory, and Conceptions of the Universe*, edited by M. Kafatos (Kluwer, Dordrecht, 1989), p. 69.
[8] N. D. Mermin, Am. J. Phys. **58**, 731 (1990).
[9] C. Brukner, M. Żukowski, J.-W. Pan, and A. Zeilinger, Phys. Rev. Lett. **92**, 127901 (2004)
[10] S. Pironio *et al.*, Nature (London) **464**, 1021 (2010).
[11] A. Acín, N. Brunner, N. Gisin, S. Massar, S. Pironio, and V. Scarani, Phys. Rev. Lett. **98**, 230501 (2007).
[12] R. Horodecki, P. Horodecki, M. Horodecki, and K. Horodecki, Rev. Mod. Phys. **81**, 865 (2009).
[13] J.-W. Pan, Z.-B. Chen, C.-Y. Lu, H. Weinfurter, A. Zeilinger, and M. Żukowski, Rev. Mod. Phys. **84**, 777 (2012).
[14] J. S. Bell, in *New Techniques and Ideas in Quantum Measurement Theory*, edited by D. M. Greenberger (New York Acad. Sci., New York, 1986) p. 263; see also U. Leonhardt, Phys. Lett. A **182**, 195 (1993); L. M. Johansen, Phys. Lett. A **236**, 173 (1997).
[15] M. D. Reid and P. D. Drummond, Phys. Rev. Lett. **60**, 2731 (1988).
[16] M. D. Reid, Phys. Rev. A **40**, 913 (1989).
[17] O. Cohen, Phys. Rev. A **56**, 3484 (1997).
[18] K. Banaszek and K. Wódkiewicz, Phys. Rev. A **58**, 4345 (1998); Phys. Rev. Lett. **82**, 2009 (1999).
[19] K. Laiho, K. N. Cassemiro, D. Gross, and C. Silberhorn, Phys. Rev. Lett. **105**, 253603 (2010).
[20] T. J. Bartley, G. Donati, X.-M. Jin, A. Datta, M. Barbieri, and I. A. Walmsley, Phys. Rev. Lett. **110**, 173602 (2013).
[21] P. M. Pearle, Phys. Rev. D **2**, 1418 (1970); S. L. Braunstein and C. M. Caves, Ann. Phys. (N.Y.) 202, 22 (1990); J. Pykacz and E. Santos, J. Math. Phys. (N.Y.) **32**, 1287 (1991); A. Cabello, Phys. Rev. A **58**, 1687 (1998); L. Hardy, Phys. Lett. A **161**, 21 (1991); J. Barrett, A. Kent, and S. Pironio, Phys. Rev. Lett. **97**, 170409 (2006); P. Kurzynski and D. Kaszlikowski, Phys. Rev. A **89**, 012103 (2014); M. Żukowski and A. Dutta, Phys. Rev. A **90**, 012106 (2014).
[22] See Supplemental Material at http://link.aps.org/supplemental/10.1103/PhysRevLett.114.100402 for details of calculations presented in the main text and a description of some properties of $\psi_N^{(-)}$ states.
[23] W. Laskowski, M. Wiesniak, M. Zukowski, M. Bourennane, and H. Weinfurter, J. Phys. B **42**, 114004 (2009).
[24] T. Sh. Iskhakov, M. V. Chekhova, G. O. Rytikov, and G. Leuchs, Phys. Rev. Lett. **106**, 113602 (2011); T. Sh. Iskhakov, I. N. Agafonov, M. V. Chekhova, and G. Leuchs, Phys. Rev. Lett. **109**, 150502 (2012).
[25] M. Radmark, M. Żukowski, and M. Bourennane, Phys. Rev. Lett. **103**, 150501 (2009).
[26] A. E. Lita, A. J. Miller, and S. W. Nam, Opt. Express **16**, 3032 (2008).
[27] T. Gerrits, B. Calkins, N. Tomlin, A. E. Lita, A. Migdall, R. Mirin, and S. W. Nam, Opt. Express **20**, 23798 (2012).