Enhancement of Superconductivity near a Nematic Quantum Critical Point

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We consider a low T_c metallic superconductor weakly coupled to the soft fluctuations associated with proximity to a nematic quantum critical point (NQCP). We show that (1) a BCS-Eliashberg treatment remains valid outside of a parametrically narrow interval about the NQCP, (2) the symmetry of the superconducting state (d wave, s wave, p wave) is typically determined by the noncritical interactions, but T_c is enhanced by the nematic fluctuations in all channels, and (3) in 2D, this enhancement grows upon approach to criticality up to the point at which the weak coupling approach breaks down, but in 3D, the enhancement is much weaker.

In both the hole-doped cuprate [1–9] and Fe-based [10–13] high temperature superconductors, there is evidence of a nematic quantum critical point (associated with the breaking of point group symmetry) at a critical doping, x_c, which is close to the "optimal doping" at which the superconducting T_c is maximal. These materials are complicated, strongly coupled systems with many intertwined ordering tendencies [14–17] in which quenched disorder plays a role in some aspects of the physics [6,18].

Thus, motivated by experiments, but without pretense that the theory is directly applicable to these materials, we study the situation in which a low T_c metallic superconductor is weakly coupled, with coupling constant α, to collective modes representing the soft fluctuations of a system in the neighborhood of a nematic quantum critical point (NQCP). Here, the effective interaction in the Cooper channel consists of the sum of a nonretarded, noncritical piece $V^{(0)}$, and a critical piece, $V^{(\text{ind})}$, which is increasingly peaked at small momentum and energy transfer the closer one approaches to the NQCP. The peak width as a function of wave number and frequency is, respectively, $\kappa \equiv \xi^{-1}$ and $\Omega \sim \xi^{-z}$, where ξ is the nematic correlation length, z is the dynamical critical exponent, and where, on the ordered side of the NQCP, the nematic transition temperature is comparable to Ω. For small α, outside of a parametrically narrow regime about criticality, the induced interactions among the electrons can be computed without needing to worry about the feedback effect of the fermions on the collective modes.

We thus gain analytic control of the problem in a parametrically broad quantum critical regime, though not in a small window of metallic quantum criticality, (see Fig. 1). In the regime of control, $V^{(\text{ind})}$ is weak ($T_c \ll \Omega$) and so can be treated in the context of BCS-Eliashberg theory, or equivalently [19,20], perturbative renormalization group (RG). The nematic modes play a role similar to that of phonons in a conventional superconductor, with

the difference that $V^{(\text{ind})}$ is strongly k dependent in such a way that it is attractive in all pairing channels, and so enhances T_c in whatever channel is favored by the noncritical interactions. The enhancement grows rapidly upon approach to criticality in 2D, and somewhat more slowly in 3D.

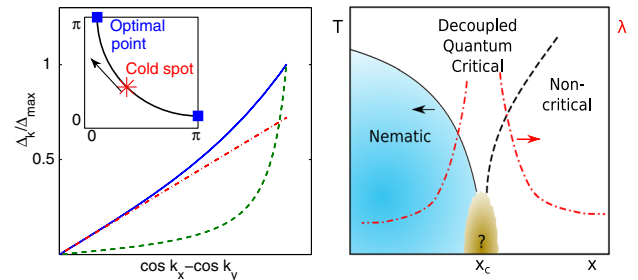*Model.*—We consider a system described by the Euclidean effective action

FIG. 1 (color online).   (a) Gap function vs $\cos k_x - \cos k_y$ along the Fermi surface, obtained by solving Eq. (8) in 2D. The noncritical interaction $\Gamma^{(0)}$ is taken to favor a d-wave gap with form factor $\cos k_x - \cos k_y$ (dotted-dashed line) and has a strength such that $\lambda^{(0)} = 10\alpha^2 \approx 0.05$. The solid and dashed lines are, respectively, for $k_F\xi = 10$ ("weak enhancement") and $k_F\xi = 100$ ("strong enhancement"). Inset: Cartoon of the Fermi surface of the cuprates; the star and squares indicate, respectively, a "cold spot" of the nematic $d_{x^2-y^2}$ form factor where $\Gamma^{(\text{ind})}$ vanishes by symmetry, and "optimal points" $\hat{k}_{\text{opt}}$ where $\Gamma^{(\text{ind})}$ is strongest. (b) Schematic phase diagram. The solid line shows the nematic transition temperature. Outside of the central shaded region, the quantum critical regime can be described in terms of a Wilson-Fisher fixed point weakly coupled to a Fermi liquid. Within the shaded region, this picture breaks down, and a different description (possibly in terms of a different, strongly coupled fixed point) is needed. T_c is much smaller than any temperature scales pictured, but varies dramatically. The dashed-dotted line shows the behavior of λ, the pairing eigenvalue [Eq. (7)].

$$S[\phi, \bar{\psi}, \psi] = S_{el}[\bar{\psi}, \psi] + S_{nem}[\phi] + S_{int}[\phi, \bar{\psi}, \psi], \quad (1)$$

where S_{el} is the action of itinerant electrons with an assumed weak interaction in the Cooper channel, $V^{(0)}(\vec{k}, \vec{k}')$, S_{nem} is the action of the nearly critical nematic mode ϕ, and

$$S_{int} = \alpha \int d\tau \frac{d\vec{k}}{(2\pi)^d} \frac{d\vec{q}}{(2\pi)^d} f(\vec{q}, \vec{k}) \phi_{\vec{q}} \bar{\psi}_{\vec{k}+\vec{q}/2} \psi_{\vec{k}-\vec{q}/2}, \quad (2)$$

where we have suppressed the spin index on the fermion fields. We consider Ising nematic order of $d_{x^2-y^2}$ symmetry in a system with tetragonal symmetry, which implies that $f(\vec{q}, \vec{k})$ is odd under rotation by $\pi/2$ and under reflection through $(1, \pm 1, 0)$ mirror planes, but even under inversion, time reversal, and reflection through $(1,0,0)$ and $(0,1,0)$ mirror planes. Since the physics near criticality is dominated by long-wavelength nematic fluctuations, the coupling constant can be replaced by its value at $|\vec{q}| \to 0$, $f(\vec{0}, \vec{k}) \equiv f(\vec{k}) \sim [\cos(k_x) - \cos(k_y)]$. (Note: this form factor reflects the symmetry of the nematic order, and is unrelated to the symmetry of the pair wave function.)

This effective action already represents a coarse-grained version of the microscopic physics. In particular, since the nematic phase breaks the point-group symmetry of the crystal, ϕ generally involves collective motion of both the electron fluid and the lattice degrees of freedom, with relative weights that depend on microscopic details. In the absence of coupling to low energy electronic degrees of freedom ($\alpha = 0$), we suppose that, as a function of an externally controlled parameter x (which could be doping concentration, pressure, etc.), there is a quantum phase transition from a nematic phase for $x < x_c$, in which ϕ is condensed, to an isotropic phase for $x > x_c$. Thus, the dynamics of ϕ are characterized by $d+1$ dimensional Ising exponents with $z = 1$.

In the fermion sector, we introduce a cutoff, W, defined as the energy scale of the noncritical portion of the electron-electron interaction, $V^{(0)}$. For instance, if $V^{(0)}$ is mediated by short-range spin fluctuations [21], the cutoff energy is proportional to the exchange coupling J. Restricting fermion energies to lie below W justifies both neglecting all irrelevant couplings (other than those in the Cooper channel) and treating the remaining interactions as non-retarded. (More generally, we should include Fermi liquid parameters in S_{el}, but we will neglect these for simplicity.)

Effective interactions.—In the small α limit, beyond a parametrically narrow interval about criticality, one can integrate out the nematic modes perturbatively to produce an effective action for the electrons alone. In the disordered phase ($x > x_c$), the leading order effect is an additive four-fermion term proportional to $\alpha^2 \chi(\vec{q}, \omega)$, which, in the Cooper channel, results in the net interaction

$$V(\vec{k}_+, \vec{k}_-, \omega) = V^{(0)}(\vec{k}_+, \vec{k}_-) - \frac{1}{4}\alpha^2 |f(\vec{q}, \vec{k})|^2 \chi(\vec{q}, \omega), \quad (3)$$

where $\vec{k}_{\pm} = \vec{k} \pm \vec{q}/2$ and $\chi(\vec{q}, \omega)$ is the nematic susceptibility, which is peaked at $\vec{q} = \vec{0}$ and $\omega = 0$. With the usual definition of the critical exponents, $\chi(\vec{0}, 0) \equiv \chi_0$ diverges as $\delta x \equiv (x - x_c) \to 0$ as $\chi_0 \sim |\delta x|^{-\gamma}$, and falls as a function of increasing $|\vec{q}|$ and $|\omega|$ as $\chi(\vec{q}, \omega) \sim \chi_0[\Omega^2/(c^2 q^2 + \omega^2 + \Omega^2)]^{1-\eta/2}$. The $\vec{q}$-space width of $\chi(\vec{q}, 0)$ is, thus, $\kappa = \xi^{-1} \sim \Omega^{1/z} \sim |\delta x|^{\nu}$. (From scaling, $1 - \eta/2 = \gamma/2\nu$.) The Ising critical exponents are $\{\nu, \eta, \gamma\} = \{1/2, 0, 1\}$ for $d = 3$, and $\{\nu, \eta, \gamma\} \approx \{0.63, 0.03, 1.23\}$ for $d = 2$ [22].

There are also other (mostly irrelevant) four-fermion interactions generated at order α^2, but these become appreciable only where the assumptions of our BCS approach break down, so we ignore them here [23]. On the other hand, they can give rise to observable effects, notably corrections to Fermi liquid theory such as quasi-particle mass renormalization, which, while small in the perturbative regime, diverges as a power law approaching the NQCP.

We have also neglected the effects of higher order terms in the effective action, generated at order α^4 and beyond. Among others things, these terms include the backaction of the fermions on the quantum critical dynamics of the nematic modes, i.e., Landau damping. These effect are unimportant so long as $1 \gg \alpha^2 \chi_0 \rho(E_F)$ where $\rho(E_F)$ is the density of states at the Fermi energy, i.e., for $|\delta x| \gg \alpha^{2/\gamma}$. When this inequality is violated, the apparent critical exponents and critical amplitudes that characterize the nematic fluctuations may deviate from their $d + 1$ dimensional Ising values at the decoupled NQCP.

As in the electron-phonon problem, we adopt a perturbative RG approach to account for the retarded nature of $V^{(ind)}$ [20]. We define dimensionless vertex operators in terms of the interactions and the Fermi velocities, $v_{\hat{k}}$ (where $\hat{k}$ denotes a point on the Fermi surface). We then integrate out the Fermionic modes with frequencies between W and Ω. This results in a new effective action with a high energy cutoff set by Ω and a renormalized, but now instantaneous, vertex in the Cooper channel

$$\underline{\Gamma} = \underline{\Gamma}^* + \underline{\Gamma}^{(ind)}, \quad (4)$$

where the the noncritical (instantaneous) piece of the vertex operator has been replaced by

$$\underline{\Gamma}^* = \underline{\Gamma}^{(0)}[\underline{1} + \underline{\Gamma}^{(0)} \log(W/\Omega)]^{-1}, \quad (5)$$

where $\Gamma_{\hat{k}, \hat{k}'}^{(0)} \equiv V(\vec{k}, \vec{k}')/\sqrt{v_{\hat{k}} v_{\hat{k}'}}$. However, the induced interaction is unaffected by this process, so

$$\Gamma_{\vec{k},\vec{k}'}^{(\text{ind})} \approx -\frac{\alpha^2}{4}\left|f\left(\frac{\vec{k}+\vec{k}'}{2}\right)\right|^2 \frac{\chi(\vec{k}-\vec{k}',0)}{\sqrt{v_{\hat{k}}v_{\hat{k}'}}}. \qquad (6)$$

This reflects the familiar feature of BCS-Eliashberg theory that only the instantaneous interaction gets renormalized by the high energy fermionic modes.

In addition to being highly peaked at small momentum transfer, i.e., small $|\vec{k}-\vec{k}'|$, $\Gamma^{(\text{ind})}$ has a significant dependence on the position of $\hat{k}$ and $\hat{k}'$ on the Fermi surface: $f(\hat{k})$ vanishes at symmetry related "cold-spots" [24] on the Fermi surface, $|k_x| = |k_y|$, and takes on its maximal value, $f(\hat{k}_{\text{opt}})| = 1$, at a set of "optimal pairing points," $\hat{k}_{\text{opt}}$. For example, for a cupratelike Fermi surface, these points correspond to the "antinodal points" on the Fermi surface, as illustrated in Fig. 1(a). Not surprisingly, we will find that the strongest pairing occurs for $\vec{k}$ near $\hat{k}_{\text{opt}}$.

Solution of the gap equation.—We are now left with the problem of fermions with energies within Ω of the Fermi surface, interacting by an instantaneous interaction vertex Γ—i.e., the BCS problem with a $\vec{k}$ dependent interaction. Thus, as usual, the superconducting T_c (so long as the weak-coupling condition $T_c \ll \Omega$ is satisfied) is determined as

$$T_c \sim \Omega \exp[-1/\lambda], \qquad (7)$$

where, in terms of the eigenstates of Γ,

$$\int d\hat{k}' \Gamma_{\hat{k},\hat{k}'} \phi_{\hat{k}'}^{(a)} = -\lambda_a \phi_{\hat{k}}^{(a)}, \qquad (8)$$

and λ is the largest positive value of λ_a.

As a function of x, $\Gamma^{(0)}$ is smooth and analytic (neglecting small corrections in the ordered state which we shall discuss), but $\Gamma_{\hat{k},\hat{k}}^{(\text{ind})}$ grows in magnitude upon approach to criticality in proportion to $\alpha^2 \chi_0 \rho(E_F) \sim \alpha^2 |\delta x|^{-\gamma}$. However, as we shall see, the pair wave function ϕ in Eq. (8) is always a more slowly varying function of $\hat{k}'$ than is $\Gamma_{\hat{k},\hat{k}'}^{(\text{ind})}$, so the contribution of the induced interactions to λ always involves the integrated weight

$$\lambda^{(\text{ind})} \equiv \frac{\alpha^2}{4}\int d\hat{k} v_{\hat{k}}^{-1}\chi(\hat{k},0) \sim \alpha^2 \rho(E_F)\chi_0(k_F\xi)^{1-d}, \qquad (9)$$

where k_F is the Fermi wave vector. Therefore, in $d=2$, $\lambda^{(\text{ind})}$ grows in proportion to $\alpha^2 |\delta x|^{\nu-\gamma}$, so the weak coupling BCS approach is valid only for $|\delta x| > \mathcal{O}(\alpha^{2/(\gamma-\nu)})$. However, in $d=3$, $\gamma-2\nu=0$ so $\lambda^{(\text{ind})}$ grows only logarithmically, $\lambda^{(\text{ind})} \sim -\alpha^2 \log|\delta x|$. Our principal remaining task is to analyze the eigenvalue problem in Eq. (8). This is readily done numerically, given an explicit form of $\underline{\Gamma}$. We begin, however, by discussing certain limiting cases which can be approximately analyzed analytically.

Regime of "weak enhancement."—The most straightforward regime to analyze is that in which the coupling to the nematic mode makes a subdominant contribution to the pairing interaction, i.e., where $\Gamma^{(\text{ind})}$ is small compared to Γ^*. Such a regime always exists sufficiently far from criticality provided that $\lambda^{(0)} \gg \alpha^2$, (where $\lambda^{(0)}$ is the largest positive eigenvalue of $-\Gamma^{(0)}$), a condition we, henceforth, assume.

In this regime, the form of the gap function is largely determined by the noncritical interactions, but T_c is enhanced (possibly by a large factor) by coupling to the nematic modes. This enhancement can be estimated using first order perturbation theory

$$\lambda_a = \lambda_a^* + \delta\lambda_a^{(\text{ind})};$$
$$\lambda_a^* = \lambda_a^{(0)}\{1 - \lambda_a^{(0)}\log[W/\Omega]\}^{-1},$$
$$\delta\lambda_a^{(\text{ind})} = -\int d\hat{k}d\hat{k}'(\phi_{\hat{k}}^{(a,0)})^*\Gamma_{\hat{k},\hat{k}'}^{(\text{ind})}\phi_{\hat{k}'}^{(a,0)}, \qquad (10)$$

where $\phi^{(a,0)}$ and $\lambda_a^{(0)}$ are, respectively, a (normalized) eigenstate and eigenvalue of $\underline{\Gamma}^{(0)}$. In the neighborhood of the NQCP, $\Gamma^{(\text{ind})}$ is peaked about small $|\hat{k}-\hat{k}'|$; hence,

$$\delta\lambda_a^{(\text{ind})} \approx \lambda^{(\text{ind})}\int d\hat{k}|f(\vec{k})|^2|\phi_{\hat{k}}^{(a,0)}|^2. \qquad (11)$$

The degree of the enhancement of pairing, thus, is larger the more the gap function is peaked near $\vec{k}_{\text{opt}}$. This result is valid so long as $1 \gg \lambda^{(0)} \gg \lambda^{(\text{ind})}$. Even so, the enhancement of $T_c \sim T_c^{(0)}\exp[\delta\lambda^{(\text{ind})}/(\lambda^{(0)})^2]$ can be large if $\delta\lambda^{(\text{ind})} \gg [\lambda^{(0)}]^2$, and grows larger the closer one approaches to the NQCP.

We can also estimate the changes to the form of the gap function, $\Delta_{\vec{k}}$, perturbatively in powers of $\Gamma^{(\text{ind})}$. The gap function is proportional to the pair wave function, $\Delta_{\hat{k}} \propto \phi_{\hat{k}}\sqrt{v_{\hat{k}}}$. The leading correction to the pair wave function is given by

$$\phi_{\hat{k}} \approx \phi_{\hat{k}}^{(0)}\left[1 + \left(\frac{\delta\lambda^{(\text{ind})}}{\lambda^{(0)}}\right)\frac{|f(\vec{k})|^2 - \overline{|f|^2}}{\overline{|f|^2}}\right], \qquad (12)$$

where $\overline{|f|^2}$ is the suitably weighted average of $|f(\vec{k})|^2$ over the Fermi surface [25]. As a result, the form of the gap function is little affected by the nematic fluctuations near the cold spots where $f(\vec{k})$ vanishes but is enhanced far from them. For example, if $\Delta_{\hat{k}}^{(0)}$ has the simplest d-wave form, $\Delta_{\hat{k}}^{(0)} \propto [\cos(k_x) - \cos(k_y)]$, the leading effect of the nematic fluctuations from Eq. (12) is to admix an increasing component proportional to $(\delta\lambda^{(\text{ind})}/\lambda)[\cos(k_x) - \cos(k_y)]^3$, as seen in Fig. 1(b). In addition, as derived in the Supplemental Material [26], the gap is renormalized by

$\exp[\delta\lambda^{(\text{ind})}/(\lambda^{(0)})^2]$ (i.e., the same enhancement factor as T_c) compared to its $\alpha = 0$ value, but retains a BCS-like T dependence.

Regime of "strong enhancement."—In 2D, since $\lambda^{(\text{ind})}$ grows rapidly with decreasing $|\delta x|$, there is a crossover to a regime [43] in which $\lambda^{(\text{ind})} \gg \lambda^*$. In 3D, such a regime is not generically encountered where our approximations are controlled, so we specialize to 2D for the present discussion. So long as $\lambda^{(\text{ind})} \ll 1$, weak coupling BCS theory still applies, but now the pair wave function is dominantly determined by $\Gamma^{(\text{ind})}$, while the effects of Γ^* can, in turn, be computed perturbatively. With the cuprates in mind, as illustrated in Fig. 1(a), we consider a single large closed Fermi surface with four cold spots along the zone diagonals, $|k_x| = |k_y|$, and four optimal points at $\hat{k}_{\text{opt}} = q\hat{x} + \pi\hat{y}$ and symmetry related points, although the discussion is readily generalized to more complex Fermi surfaces.

The asymptotic properties of the eigenvalues and eigenstates of $\Gamma^{(\text{ind})}$ can be derived analytically, as shown explicitly in the Supplemental Material [26]. The leading eigenstates are peaked about the positions $\hat{k}_{\text{opt}}$ with an extent in momentum space $\tilde{\kappa} \sim k_F(\kappa/k_F)^w$, where $w = (1 - \eta)/(3 - \eta) \approx 1/3$. Since $w < 1$, the eigenstates of $\Gamma^{(\text{ind})}$ vary on a parametrically larger momentum scale than $\Gamma^{(\text{ind})}$ itself, as previously stated.

Since both κ/k_F and $\tilde{\kappa}/k_F \ll 1$, to a first approximation, the relative phase of $\phi_{\hat{k}}$ in the neighborhood of the four optimal points is unimportant, and the eigenfunctions are fourfold degenerate. This degeneracy is lifted by the large momentum transfer portions of Γ: The contribution from $\Gamma^{(\text{ind})}$ is proportional to $\alpha^2(\tilde{\kappa}/k_F)$, which is parametrically smaller than $\delta\lambda^* \sim \lambda^*(\tilde{\kappa}/k_F)$, the perturbative eigenvalue shift produced by Γ^*. Accordingly, the four leading eigenvalues are $\lambda_a \approx \lambda^{(\text{ind})} + g_a\lambda^*(\tilde{\kappa}/k_F)$ where $|g_a| \sim 1$ depends on the relative phase of the gap function at the different optimal points. Even where the noncritical interactions make a small contribution to the pairing energy, they still determine the relative phase of the pair wave function at the different optimal points and, hence, the symmetry of the superconducting state.

For a given symmetry, the splitting between the largest and next-to-largest eigenvalue is of order $\lambda^{(\text{ind})}(\tilde{\kappa}/k_F)^2 \ll \lambda^{(\text{ind})}$. Within the strong enhancement regime, there are several subregimes depending on the size of this splitting relative to $[\lambda^{(\text{ind})}]^2$ and to $\delta\lambda^*$. We defer discussion of subregimes to a later paper, but note two salient limits, both within the strong enhancement regime. (1) Sufficiently far from criticality, the form of the gap function at all temperatures below T_c is determined by the solution of Eq. (8); even the eccentric shape of the gap function [shown in Fig. 1(b)] results in only modest enhancement of $|\Delta_{\text{max}}(T = 0)|/T_c$. (2) Sufficiently close to criticality, the form of the gap function becomes strongly temperature dependent. In particular, the gap function

becomes less strongly peaked at $\hat{k}_{\text{opt}}$ ($\tilde{\kappa}$ increases) with decreasing T. In addition, beyond mean field theory, the near-degeneracy among different symmetry channels within the strong enhancement regime leads to a new class of fluctuations involving the relative phase of the order parameter on different portions of the Fermi surface, as previously explored in Ref. [44].

Approaching the NQCP from the ordered phase.—Until now, we have considered the approach to criticality from the disordered side. Unlike the case of an antiferromagnetic quantum critical point [45], in which the opening of a gap on the ordered side of the transition results in a strong suppression of superconductivity, in the case of a NQCP, the physics is largely similar when approached from the ordered side. The major difference is in band structure, i.e., the distortion of the Fermi surface by an amount $\delta k_F \equiv k_{F,x} - k_{F,y} \propto \alpha k_F\langle\phi\rangle \sim \alpha|\delta x|^\beta$. $\Gamma^{(\text{ind})}$ is qualitatively affected, because, under orthorhombic symmetry, there are now only two fermi surface positions $\hat{k}_{\text{opt}}$ of optimal pairing rather than four. The leading eigenstates of $\Gamma^{(\text{ind})}$ consist of a singlet state of extended s-wave ("$s + d$") symmetry and a triplet state of either p_x or p_y symmetry.

The distortion of the Fermi surface also alters the eigenvalues of both Γ^* and $\Gamma^{(\text{ind})}$ by corrections in powers of δk_F, but these corrections are negligible near criticality. The major difference between the ordered and disordered sides comes through the critical amplitude ratio for the quantity $\chi_0\xi^{1-d}$. This is a universal number of order one associated with the decoupled NQCP, and gives the ratio of $\lambda^{(\text{ind})}$ on the two sides of the transition. It is greater than one for $d = 2$ and equal to one for $d = 3$ [22], implying that, for fixed $|\delta x|$, T_c is greater on the disordered side in $d = 2$ and comparable on both sides in $d = 3$.

Relation to previous work.—The importance of the form factor of the coupling between the electrons and the quantum critical modes has been explored in the context of intra-unit-cell orbital current antiferromagnetism in Ref. [46]. However, there the collective modes were assumed to have an essential $\vec{k}$ independent susceptibility. The effects on Fermi liquids of boson-mediated interactions with strong forward scattering have been treated extensively in various related contexts [47–53].

We were also inspired by two sets of studies which address superconducting instabilities at a NQCP, Refs. [54–56]. Both address the issue of superconducting pairing asymptotically close to criticality, which is the regime we have avoided in the present approach. In this regime, the different fields are intrinsically strongly coupled to each other. Thus, in order to obtain theoretical control of the problem, both works involve large N extensions of the model. References [54] and [55] introduce an artificially large number N_F of fermion flavors and a much larger number of boson flavors, $N_B = (N_F)^2$; no pairing tendency is found to leading order in $1/N_B$ for

$d \leq 3$. Reference [56] treats $d = 2$ and $N_B = 1$, but extends the model by introducing both a large N_F and a nonlocal interaction characterized by an exponent, ϵ, assumed small (the physically relevant limit is $N_F = 2$, $N_B = 1$, and $\epsilon = 1$). In contrast to the results of Ref. [55], they conclude that T_c at criticality is proportional to a power of the coupling constant, which is what we would find were we to extrapolate our results to where $\lambda^{(\text{ind})} \sim 1$.

Relation to experiment.—The present results provide a rationale to associate the anomalous stability [7,8] of the superconducting dome in near-optimally doped YBCO in high magnetic fields with the proximity of a putative NQCP at doped hole concentration $x \approx 0.18$. The simple d-wave (nearly $\cos k_x - \cos k_y$) character of the pairing around this doping, at least in the related material Bi-2212 [57], then suggests that nematic fluctuations play a subdominant role, enhancing a broader tendency to d-wave pairing (presumably associated with noncritical magnetic fluctuations). The fact that recent evidence indicates that a NQCP occurs at near-optimal doping in some Fe-based superconductors [13,58,59] is further evidence that such enhancement may be a more general feature of high temperature superconductivity. Moreover, the much stronger enhancement of T_c that arises near a NQCP in 2D may provide some insight as to why T_c is considerably enhanced in single layer films of FeSe [53,60,61].

Note added.—Recently, we received a paper by Maier and Scalapino [62] reporting a more microscopically realistic study of the enhancement of T_c by nematic fluctuations— the conclusions are complementary and in broad agreement with the present results.

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