Squeezed Optomechanics with Phase-Matched Amplification and Dissipation

Xin-You Lü,[1,2,*] Ying Wu,[1,†] J. R. Johansson,[2] Hui Jing,[3,4] Jing Zhang,[3,5] and Franco Nori[3,6]
[1]*School of Physics, Huazhong University of Science and Technology, Wuhan 430074, China*
[2]*iTHES, RIKEN, Saitama 351-0198, Japan*
[3]*CEMS, RIKEN, Saitama 351-0198, Japan*
[4]*Department of Physics, Henan Normal University, Xinxiang 453007, China*
[5]*Tsinghua National Laboratory for Information Science and Technology, Beijing 100084, China*
[6]*Department of Physics, The University of Michigan, Ann Arbor, Michigan 48109-1040, USA*
(Received 14 December 2014; published 4 March 2015)

We investigate the nonlinear interaction between a squeezed cavity mode and a mechanical mode in an optomechanical system (OMS) that allows us to selectively obtain either a radiation-pressure coupling or a parametric-amplification process. The squeezing of the cavity mode can enhance the interaction strength into the single-photon strong-coupling regime, even when the OMS is originally in the weak-coupling regime. Moreover, the noise of the squeezed mode can be suppressed completely by introducing a broadband-squeezed vacuum environment that is phase matched with the parametric amplification that squeezes the cavity mode. This proposal offers an alternative approach to control the OMS using a squeezed cavity mode, which should allow single-photon quantum processes to be implemented with currently available optomechanical technology. Potential applications range from engineering single-photon sources to nonclassical phonon states.

DOI: 10.1103/PhysRevLett.114.093602 PACS numbers: 42.50.-p, 07.10.Cm, 42.65.-k

Cavity optomechanics has progressed enormously in recent years [1], with achievements including cooling of mechanical modes to their quantum ground states [2,3], demonstration of optomechanically induced transparency [4,5], coherent state transfer between cavity and mechanical modes [6–9], and the realization of squeezed light [10–12]. In these experiments, a strong linearized optomechanical coupling is obtained under the condition of strong optical driving. However, the intrinsic nonlinearity of the radiation-pressure coupling in these optomechanical systems (OMSs) is negligible [13–19].

To explore the intrinsic nonlinearity of the optomechanical interaction, much theoretical research has recently focused on the single-photon strong-coupling regime, where the single-photon optomechanical-coupling strength g_0 exceeds the cavity decay rate κ. In this regime, several interesting single-photon quantum processes are predicted, for both the optical and the mechanical modes. For example: photon blockade, the preparation of the nonclassical states of the optical and mechanical modes, multiphonon sidebands, and quantum state reconstruction of the mechanical oscillator [20–34]. However, these effects have not yet been realized experimentally due to the intrinsically weak radiation-pressure coupling in current OMSs, i.e., $g_0 \ll \kappa$. To achieve $g_0 \sim \kappa$, it has been proposed to use the collective mechanical modes in transmissive scatter arrays [35,36]. The ratio g_0/κ may also be increased in superconducting circuits using the Josephson effect, but such devices are limited to electromechanical systems [37–39]. Moreover, postselected weak measurements [40] and optical coalescence [41] could also be used to increase the effective linear and quadratic optomechanical interactions, respectively.

Here, we present a method for reaching the single-photon strong-coupling regime in an OMS, which is originally in the weak-coupling regime. In contrast to normal optomechanics, we focus on the nonlinear interaction between a parametric-amplification-squeezed cavity mode and a mechanical mode. We obtain an optomechanical coupling that, selectively, can take the forms of a radiation-pressure or a parametric-amplification process. Physically, a single-photon state in the squeezed cavity mode corresponds to an exponentially growing number of photons in the original cavity mode as a function of increasing squeezing strength. Consequently, the optomechanical interaction in units of the squeezed-cavity-mode photons can be enhanced, e.g., into the single-photon strong-coupling regime by tuning the intensity (or frequency) of the driving field that induced the squeezing.

In addition, we show that the noise of the squeezed cavity mode can be suppressed by introducing a broadband-squeezed vacuum [42,43] with a reference phase matching the phase of the driving field. Under these conditions of enhanced coupling strength and suppressed noise, it should be feasible to implement single-photon quantum processes even in an originally weakly coupled OMS. Our proposal is also suitable for electromechanical systems with squeezed-vacuum reservoirs for superconducting resonators [44]. Note that a broadband-squeezed vacuum can also suppress the radiative decay of atoms [45–47] or artificial atoms [44] and can be used to squeeze the mechanical modes in OMSs [48,49].

System.—We consider an OMS depicted in Fig. 1(a) with the Hamiltonian ($\hbar = 1$)

$$H = H_c + H_m - g_0 a^\dagger a (b^\dagger + b), \tag{1}$$

where a ($a^\dagger$) and b ($b^\dagger$) are the annihilation (creation) operators of the cavity mode and the mechanical mode, respectively. The optical cavity (with resonance frequency ω_c) contains a $\chi^{(2)}$ nonlinear medium that is pumped with driving frequency ω_d, amplitude Λ, and phase Φ_d. Its Hamiltonian can be written as $H_c = \Delta_c a^\dagger a + \Lambda(a^{\dagger 2} e^{-i\Phi_d} + a^2 e^{i\Phi_d})$, with $\Delta_c = \omega_c - \omega_d/2$ in a frame rotating with $\omega_d/2$. The Hamiltonian of the mechanical mode $H_m = \omega_m b^\dagger b + F(b^\dagger + b)$ (with mechanical frequency ω_m) contains a constant force F that cancels a force induced by the parametric amplification (see below). The third term in Eq. (1) describes the radiation-pressure interaction between the cavity and the mechanical modes with coupling strength g_0 [50]. Here, the $\chi^{(2)}$ nonlinearity is used to induce a squeezed cavity mode. It could also be used to enhance optomechanical cooling [51], induce genuine tripartite entanglement [52], or impact the classical dynamics of OMSs [53].

As shown in Fig. 1(b), an optical parametric amplification (OPA) is introduced to generate a broadband-squeezed vacuum field c_k (with central frequency ω_c) which is injected into the cavity. Here, r_e and Φ_e are the squeezing parameter and reference phase of this squeezed environment, respectively, corresponding to the intensity and phase of the pump field. Experimentally, optical (microwave) light with squeezing bandwidth up to GHz [42] (tens of

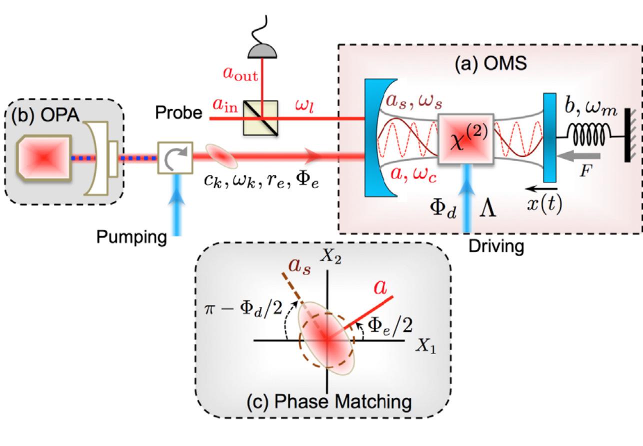

FIG. 1 (color online). (a) A schematic illustration of an OMS with mechanical mode b (driven by a force F), main cavity a and a squeezed cavity-mode a_s induced by driving a $\chi^{(2)}$ nonlinear medium with frequency ω_d, amplitude Λ, and phase Φ_d. Here, $a_{\rm in}$ and $a_{\rm out}$ are the input and output of a weak probe field with frequency ω_l. (b) A broadband squeezed-vacuum-field c_k with frequency ω_k (generated by an OPA) interacts with a. The squeezing parameter and reference phase are r_e and Φ_e. (c) The phase-matching condition $\Phi_e - \Phi_d = \pm n\pi$ ($n = 1, 3, 5, \ldots$) for suppressing the noise of a_s is indicated by the squeezing directions.

MHz [44]) has been realized. This is much larger than the typical linewidth of optical (microwave) cavities, i.e., MHz (hundreds of kHz). From the point of view of the cavity, the squeezed input field is well approximated as having infinite bandwidth [44]. The dissipation caused by the system-bath coupling can then be described by the Lindblad superoperators $\kappa(N+1)\mathcal{D}[a]\rho + \kappa N \mathcal{D}[a^\dagger]\rho - \kappa M \mathcal{G}[a]\rho - \kappa M^* \mathcal{G}[a^\dagger]\rho$ (cavity damping) and $\gamma(\bar{n}_{\rm th}^m + 1)\mathcal{D}[b]\rho + \gamma \bar{n}_{\rm th}^m \mathcal{D}[b^\dagger]\rho$ (mechanical damping) in the master equation. Here, $\mathcal{D}[o]\rho = o\rho o^\dagger - (o^\dagger o \rho + \rho o^\dagger o)/2$, $\mathcal{G}[o]\rho = o\rho o - (oo\rho + \rho oo)/2$, κ and γ are the cavity and mechanical decay rates, respectively, and $\bar{n}_{\rm th}^m$ is the thermal phonon number of the mechanical mode. The mean photon number of the broadband squeezed field is $N = \sinh^2(r_e)$, and $M = \sinh(r_e)\cosh(r_e)e^{i\Phi_e}$ describes the strength of the two-photon correlation [54].

Parametric-amplification-induced strong optomechanical coupling.—Parametric amplification in the cavity introduces a preferred squeezed cavity mode a_s that satisfies a squeezing transformation $a = \cosh(r_d)a_s - e^{-i\Phi_d}\sinh(r_d)a_s^\dagger$, with $r_d = (1/4)\ln[(\Delta_c + 2\Lambda)/(\Delta_c - 2\Lambda)]$. In terms of a_s, Hamiltonian (1) can be rewritten as

$$H = \omega_s a_s^\dagger a_s + \omega_m b^\dagger b - g_s a_s^\dagger a_s (b^\dagger + b) + \frac{g_p}{2}(a_s^{\dagger 2} + a_s^2)(b^\dagger + b), \tag{2}$$

where the cavity Hamiltonian H_c has been diagonalized by the squeezing transformation, and is expressed as an oscillator with a controllable frequency $\omega_s = (\Delta_c - 2\Lambda)\exp(2r_d)$. Here, we have chosen $F = g_0 \sinh^2(r_d)$ to cancel an induced force applied to the mechanical oscillator. The third and fourth terms in Eq. (2) describe the standard optomechanical radiation-pressure and parametric-amplification interactions, respectively, with the controllable strengths

$$g_s = \frac{g_0 \Delta_c}{\sqrt{\Delta_c^2 - 4\Lambda^2}} = g_0 \cosh(2r_d), \tag{3a}$$

$$g_p = \frac{2 g_0 \Lambda}{\sqrt{\Delta_c^2 - 4\Lambda^2}} = g_0 \sinh(2r_d). \tag{3b}$$

This provides an optomechanical interaction that can be tuned by adjusting the system parameters, such as the frequency detuning Δ_c and the driving strength Λ. [The small optical linewidth should be included in Eqs. (3) in the extremely narrow critical regime where $|\Delta_c|$ infinitely approaches 2Λ. The detailed discussion is omitted because it does not limit the efficiency of our mechanism significantly in practice.]

On one hand, the parametric interaction [last term of Eq. (2)] can be suppressed by adjusting Δ_c or Λ so that $\omega_s \gg g_p, \omega_m$. Under a rotating-wave approximation (RWA), we obtain a standard optomechanical Hamiltonian

$$H_{\mathrm{OMS}} = \omega_s a_s^\dagger a_s + \omega_m b^\dagger b - g_s a_s^\dagger a_s (b^\dagger + b), \quad (4)$$

by safely neglecting the terms that oscillate with high frequencies, $2\omega_s \pm \omega_m$. In this case, the single-photon optomechanical-coupling strength g_s could be significantly enhanced (approximately 3 orders of magnitude) and reach the strong-coupling regime, i.e., $g_s > \kappa$ [see Figs. 2(a) and 2(b)]. This enhancement is due to a single-photon state in the squeezed mode $|1\rangle_s$ corresponding to an exponentially growing number of photons in the original cavity, as a function of increasing squeezing strength, i.e., ${}_s\langle 1|a^\dagger a|1\rangle_s \to \cosh(2r_d)$. The radiation pressure of a single squeezed photon on the mechanical resonator is, therefore, correspondingly increased, which effectively enhances the optomechanical coupling between the mechanical mode and the squeezed cavity mode.

On the other hand, we could also suppress the radiation-pressure interaction by adjusting Δ_c or Λ, so that g_s/ω_m, $g_p/\omega_s \ll 1$ and $\omega_s \approx \omega_m/2$ [see Figs. 2(c) and 2(d)]. Under a RWA, Hamiltonian (2) is simplified to a resonant photon-phonon parametric interaction (PI), i.e., $H_{\mathrm{PI}} = \omega_s a_s^\dagger a_s + \omega_m b^\dagger b + g_p (a_s^2 b^\dagger + b a_s^{\dagger 2})$, in the strong-coupling regime $g_p > \kappa$. This could potentially be used for highly efficient down-conversion of a single phonon into an entangled photon pair.

Suppressing the cavity noise with phase matching.—Expressing the system-bath interaction in terms of a_s, the system master equation can be rewritten as

$$\dot{\rho} = -i[H,\rho] + \kappa(N_s+1)\mathcal{D}[a_s]\rho + \kappa N_s \mathcal{D}[a_s^\dagger]\rho - \kappa M_s \mathcal{G}[a_s]\rho - \kappa M_s^* \mathcal{G}[a_s^\dagger]\rho + \gamma \bar{n}_{\mathrm{th}}^m \mathcal{D}[b^\dagger]\rho + \gamma(\bar{n}_{\mathrm{th}}^m + 1)\mathcal{D}[b]\rho, \quad (5)$$

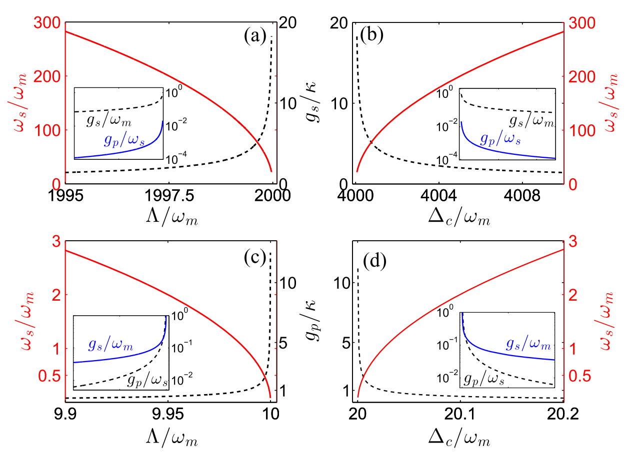

FIG. 2 (color online). The optomechanical coupling strengths g_s/κ, g_p/κ, and the cavity frequency ω_s/ω_m versus driving strength Λ and detuning Δ_c. The values g_s/ω_m and g_p/ω_s are presented in the insets. The parameters are $g_0 = 0.005\omega_m$, $\kappa = 0.05\omega_m$, $\gamma = 10^{-4}\omega_m$, and (a) $\Delta_c = 4000\omega_m$, (c) $\Delta_c = 20\omega_m$, (b) $\Lambda = 2000\omega_m$, (d) $\Lambda = 10\omega_m$.

where H is given by Eq. (2). Here N_s and M_s denote the effective thermal noise and two-photon-correlation strength, respectively, given by (setting $\Phi = \Phi_e - \Phi_d$)

$$N_s = \sinh^2(r_d)\cosh^2(r_e) + \cosh^2(r_d)\sinh^2(r_e) + \frac{1}{2}\cos(\Phi)\sinh(2r_d)\sinh(2r_e), \quad (6a)$$

$$M_s = e^{i\Phi_d}[\cosh(r_d)\cosh(r_e) + e^{-i\Phi}\sinh(r_d)\sinh(r_e)] \times [\sinh(r_d)\cosh(r_e) + e^{i\Phi}\cosh(r_d)\sinh(r_e)]. \quad (6b)$$

When $r_d = r_e = r$, N_s and M_s simplify to $N_s = \sinh^2(2r)[1+\cos(\Phi)]/2$ and $M_s = \exp(i\Phi_d)\sinh(2r)[1+\exp(i\Phi)][\cosh^2(r) + \exp(-i\Phi)\sinh^2(r)]/2$, respectively. This shows that the thermal noise and the two-photon correlation can be suppressed completely (i.e., $N_s, M_s = 0$) when $r_d = r_e$ and $\Phi = \pm n\pi$ ($n = 1, 3, 5, \ldots$). This result can be understood from the phase matching in Fig. 1(c). The reservoir of the original cavity is squeezed along the axis with angle $\Phi_e/2$, with a squeezing parameter r_e. In the basis of the squeezed cavity modes a_s, this effect is cancelled by the squeezing (along axis $\Phi_d/2$) induced by the parametric amplification of a, when $\Phi_e - \Phi_d = \pm n\pi$ and $r_e = r_d$. That is, the squeezed-vacuum reservoir (ellipse) of a corresponds to an effective vacuum reservoir (circle) of a_s.

In Fig. 3, we plot N_s as a function of the phase Φ and squeezing imbalance $\delta r = r_e - r_d$. Note that the amplitude of M_s has almost the same behavior as N_s, and is not plotted here. Figure 3 shows that the ideal parameters are $\Phi = \pm n\pi$ and $\delta r = 0$, which is consistent with our qualitative discussion. Deviating from these ideal parameters, N_s increases periodically (exponentially) with increasing Φ (δr). The inset of Fig. 3(b) also shows that the optimal point of δr shifts with changing Φ, which can be understood from the third term in Eq. (6a).

Applications.—To probe the radiation-pressure coupling, one can drive the original cavity mode using a weak probe field with frequency ω_l, amplitude ϵ_l, ($\epsilon_l \ll \kappa$). The Hamiltonian is $H_p = a^\dagger e^{-i\omega_l^s t} + a e^{i\omega_l^s t}$ in the frame rotating with $\omega_d/2$, and $\omega_l^s = \omega_l - \omega_d/2$ is the effective frequency of the probe field. Note that, only the squeezed mode a_s is excited when $\omega_l^s \approx \omega_s$, and this is achieved by a

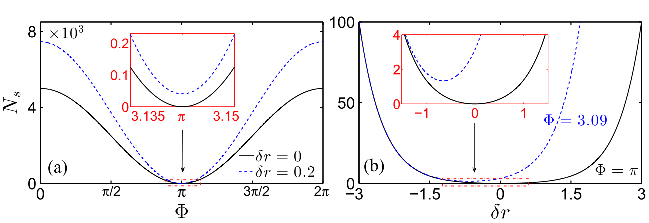

FIG. 3 (color online). The effective thermal noise N_s versus (a) Φ, (b) $\delta r = r_e - r_d$ for different (a) δr and (b) Φ. The inset corresponds to the vicinity of the ideal parameter regime.

joint effect of the probe and driving fields. In this case, the optomechanical coupling strength could be inferred by measuring the steady-state excitation spectrum, i.e., $S(\Delta_s) = [\mathrm{Lim}_{t\to\infty}\langle a_s^\dagger a_s\rangle(t) - N_s]/n_0$ $(n_0 = 4\epsilon_l^2/\kappa^2$, $\Delta_s = \omega_s - \omega_l^s)$ [23–25], which has been shifted by a constant N_s when $\Phi \neq \pi$ ($N_s = 0$ when $\Phi = \pi$).

The exact evolution of the system, including the probe field, is also governed by Eq. (5), but with the replacement $H \to H_t = H + H_p$, where

$$H_t = H + \epsilon_l[\cosh(r_d)a_s^\dagger e^{-i\omega_l^s t} - \sinh(r_d)a_s^\dagger e^{i\omega_l^s t - i\Phi_d} + \mathrm{H.c.}]. \tag{7}$$

Under the conditions of $\omega_l^s \approx \omega_s$ and $\omega_s \gg \omega_m$, g_p, $\epsilon_l \sinh(r_d)$, H_t simplifies to $H_{\mathrm{OMS}}^d = H_{\mathrm{OMS}} + \epsilon_l \cosh(r_d)$ $\{a_s^\dagger \exp[-i\omega_l^s t] + \mathrm{H.c.}\}$ by ignoring the terms oscillating with the high frequencies $2\omega_l^s$, $2\omega_s \pm \omega_m$. In Fig. 4(a), we present the excitation spectrum $S(\Delta_s)$ obtained by numerically solving Eq. (5) with Hamiltonian H_{OMS}^d. It shows that the coupling strength g_s could be obtained by measuring the position of the zero-phonon-transition peak δ, since $\delta = g_s^2/\omega_m$ [23–25]. Moreover, the appearance of phonon sidebands is another signature of the single-photon strong-coupling regime, i.e., $g_s > \kappa$. Figure 4(a) also shows that the spectral information is lost when Φ deviates too much from its optimal value π.

Strong radiation-pressure and parametric interactions at the single-photon level provide great potentials for single-photon quantum processes. As an example, we demonstrate the photon blockade, characterized by a vanishing equal-time second-order correlation function in the steady state, $g_{\mathrm{ss}}^2(0) = \mathrm{Lim}_{t\to\infty}\langle a_s^\dagger a_s^\dagger a_s a_s\rangle(t)/\langle a_s^\dagger a_s\rangle^2(t)$ and in the transient state, $g^2(0) = \langle a_s^\dagger a_s^\dagger a_s a_s\rangle(t)/\langle a_s^\dagger a_s\rangle^2(t)$, when only the a_s mode is weakly driven under the single-photon resonance $\Delta_s = g_s^2/\omega_m$. In Figs. 4(b) and 4(c), we plot the dependence of $g_{\mathrm{ss}}^2(0)$ on Φ and $\bar{n}_{\mathrm{th}}^m$, respectively, using H_{OMS}^d. They show that the photon blockade occurs in the vicinity of the phase matching $\Phi = \pi$ and for small $\bar{n}_{\mathrm{th}}^m$. The system is thermalized by the optical noise N_s (or the mechanical noise $\bar{n}_{\mathrm{th}}^m$) when Φ deviates too much from π (or the temperature of the mechanical bath is too high) even in the strong-coupling regime $g_s > \kappa$. Moreover, Fig. 4(c) also indicates the regime $g_{\mathrm{ss}}^2(0) < 0.1$, corresponding to a strong signature of photon blockade. It shows that photon blockade extends even out to $\bar{n}_{\mathrm{th}}^m \sim 10$ when $\tilde{\Delta}_c/\omega_m = 0.4$. The appearance of photon blockade can be understood qualitatively from the radiation-pressure-induced anharmonicity of the level spacing [see the inset of Fig. 4(d)]. Strong anharmonicity makes the probe photons go through the OMS one by one, because the two-photon transition is detuned under the condition of single-photon resonance. The validity of H_{OMS}^d is demonstrated in Fig. 4(d), where the evolution of $g^2(0)$ corresponding to H_{OMS}^d agrees well with the exact numerical solution using H_t [55]. We also note that $g^2(0)$ approaches a steady value when $t \approx 100/\omega_m$.

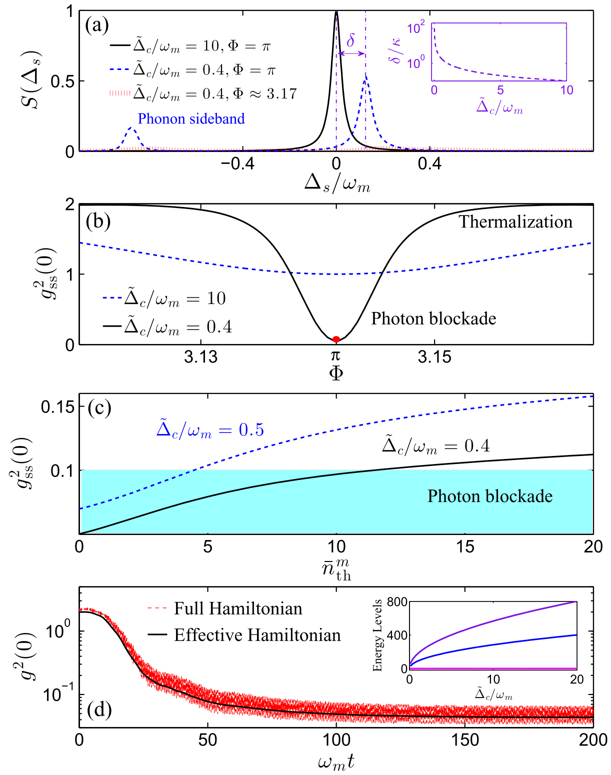

FIG. 4 (color online). (a) Cavity excitation spectrum $S(\Delta_s)$ for different $\tilde{\Delta}_c = \Delta_c - 2\Lambda$ and Φ. Inset: Shift of the zero-phonon-transition peak δ/κ versus $\tilde{\Delta}_c$. The correlation function $g_{\mathrm{ss}}^2(0)$ versus (b) Φ (c) $\bar{n}_{\mathrm{th}}^m$ for different $\tilde{\Delta}_c$. The shaded area in (c) corresponds to the regime $g_{\mathrm{ss}}^2(0) < 0.1$. (d) $g^2(0)$ versus time when $\Delta_s = g_s^2/\omega_m$, and modes a_s, b are initially in a thermal state and vacuum state, respectively. The black solid (red dashed) curve is obtained by numerically calculating Eq. (5) with H_{OMS}^d (H_t). Insert: the three lowest levels of OMS versus $\tilde{\Delta}_c$. The parameters are the same as in Fig. 2 except for $\epsilon_l = 10^{-3}\omega_m$, $\Lambda/\omega_m = 2000$, and (a),(b) $\bar{n}_{\mathrm{th}}^m = 0$, (c),(d) $\Phi = \pi$ corresponding to the red point in (b).

For $\omega_m = 100$ MHz, the relaxation time corresponds to 1 μs. This requires that the optical (or microwave) driving field has the stable frequency and phase during a time scale of μs, which is experimentally feasible with current laser technologies [56–58].

Strong radiation pressure is also useful for cooling a mechanical oscillator. In sideband cooling experiments, the phase and amplitude noise of the cooling laser induce radiation-pressure fluctuations that ultimately heat the mechanical mode [59], especially in the OMS with a "soft" mechanical oscillator [1]. This leads to an excess final occupancy $\bar{n}_f$ with a lowest value $\bar{n}_f^{\min} \propto 1/g_0$ [60]. Therefore, the enhancement of the radiation-pressure coupling g_0 could decrease the practical mechanical-cooling limit by suppressing the influence from the ubiquitous laser noise.

Conclusions.—We have presented a method for obtaining controllable optomechanical interactions between a squeezed cavity mode and a mechanical mode in an OMS. The squeezed cavity mode is generated by detuned parametric amplification of the original cavity mode, which also interacts with a broadband-squeezed vacuum. We showed that, by tuning the intensity or the frequency of the driving field, we can selectively obtain an optomechanical radiation-pressure coupling or a parametric-amplification interaction. Moreover, the effective interaction strengths can potentially be enhanced into the single-photon strong-coupling regime when originally in the weak-coupling regime. Photon blockade is demonstrated in the vicinity of a phase matching between the broadband squeezed vacuum and the parametric amplification, under which, the cavity noise is significantly suppressed. This study provides a promising route for implementing single-photon quantum processes with currently available optomechanical technology.

X. Y. L. thanks Dr. Jie-Qiao Liao, Professor Yu-xi Liu, and Professor Peter Rabl for valuable discussions. X. Y. L. is supported by NSFC-11374116. Y. W. is supported by NSFC-11375067, and the National Basic Research Program of China (Contracts No. 2012CB922103 and No. 2013CB921804). H. J. is supported by the NSFC (Contracts No. 11274098 and No. 11474087). J. Z. is supported by the NSFC (Contracts No. 61174084 and No. 61134008) and the National Basic Research Program of China (973 Program) under Grant No. 2014CB921401. F. N. is partially supported by the RIKEN iTHES Project, MURI Center for Dynamic Magneto-Optics, and a Grant-in-Aid for Scientific Research (S).

*xinyoulu@gmail.com
†yingwu2@126.com

[1] For recent reviews, see, e.g., M. Aspelmeyer, T. J. Kippenberg, and F. Marquardt, Rev. Mod. Phys. **86**, 1391 (2014); M. Aspelmeyer, P. Meystre, and K. Schwab, Phys. Today **65**, 29 (2012); P. Meystre, Ann. Phys. (Berlin) **525**, 215 (2013); F. Marquardt and S. M. Girvin, Physics **2**, 40 (2009); T. J. Kippenberg and K. J. Vahala, Science **321**, 1172 (2008).
[2] A. D. O'Connell *et al.*, Nature (London) **464**, 697 (2010).
[3] J. Chan, T. P. M. Alegre, A. H. Safavi-Naeini, J. T. Hill, A. Krause, S. Gröblacher, M. Aspelmeyer, and O. Painter, Nature (London) **478**, 89 (2011); J. D. Teufel, T. Donner, D. Li, J. W. Harlow, M. S. Allman, K. Cicak, A. J. Sirois, J. D. Whittaker, K. W. Lehnert, and R. W. Simmonds, *ibid.* **475**, 359 (2011).
[4] S. Weis, R. Riviére, S. Deléglise, E. Gavartin, O. Arcizet, A. Schliesser, and T. J. Kippenberg, Science **330**, 1520 (2010).
[5] A. H. Safavi-Naeini, T. P. Mayer Alegre, J. Chan, M. Eichenfield, M. Winger, Q. Lin, J. T. Hill, D. E. Chang, and O. Painter, Nature (London) **472**, 69 (2011).
[6] V. Fiore, Y. Yang, M. C. Kuzyk, R. Barbour, L. Tian, and H. Wang, Phys. Rev. Lett. **107**, 133601 (2011).
[7] X. Zhou, F. Hocke, A. Schliesser, A. Marx, H. Huebl, R. Gross, and T. J. Kippenberg, Nat. Phys. **9**, 179 (2013).
[8] T. A. Palomaki, J. W. Harlow, J. D. Teufel, R. W. Simmonds, and K. W. Lehnert, Nature (London) **495**, 210 (2013).
[9] M. Karuza, C. Biancofiore, M. Bawaj, C. Molinelli, M. Galassi, R. Natali, P. Tombesi, G. Di Giuseppe, and D. Vitali, Phys. Rev. A **88**, 013804 (2013).
[10] D. W. C. Brooks, T. Botter, S. Schreppler, T. P. Purdy, N. Brahms, and D. M. Stamper-Kurn, Nature (London) **488**, 476 (2012).
[11] A. H. Safavi-Naeini, S. Gröblacher, J. T. Hill, J. Chan, M. Aspelmeyer, and O. Painter, Nature (London) **500**, 185 (2013).
[12] T. P. Purdy, P.-L. Yu, R. W. Peterson, N. S. Kampel, and C. A. Regal, Phys. Rev. X **3**, 031012 (2013).
[13] S. Huang and G. S. Agarwal, Phys. Rev. A **80**, 033807 (2009).
[14] H. Xiong, L.-G. Si, A.-S. Zheng, X. Yang, and Y. Wu, Phys. Rev. A **86**, 013815 (2012).
[15] Y.-D. Wang and A. A. Clerk, Phys. Rev. Lett. **108**, 153603 (2012).
[16] L. Tian, Phys. Rev. Lett. **108**, 153604 (2012).
[17] J. Ma, C. You, L.-G. Si, H. Xiong, X. Yang, and Y. Wu, Opt. Lett. **39**, 4180 (2014).
[18] C. Jiang, Y. Cui, and K.-D. Zhu, Opt. Express, **22**, 13773 (2014).
[19] H. Jing, S. K. Özdemir, X.-Y. Lü, J. Zhang, L. Yang, and F. Nori, Phys. Rev. Lett. **113**, 053604 (2014).
[20] S. Bose, K. Jacobs, and P. L. Knight, Phys. Rev. A **56**, 4175 (1997).
[21] W. Marshall, C. Simon, R. Penrose, and D. Bouwmeester, Phys. Rev. Lett. **91**, 130401 (2003).
[22] X.-W. Xu, H. Wang, J. Zhang, and Y. X. Liu, Phys. Rev. A **88**, 063819 (2013).
[23] P. Rabl, Phys. Rev. Lett. **107**, 063601 (2011).
[24] A. Nunnenkamp, K. Børkje, and S. M. Girvin, Phys. Rev. Lett. **107**, 063602 (2011).
[25] J. Q. Liao, H. K. Cheung, and C. K. Law, Phys. Rev. A **85**, 025803 (2012).
[26] M. Ludwig, A. H. Safavi-Naeini, O. Painter, and F. Marquardt, Phys. Rev. Lett. **109**, 063601 (2012).
[27] J. Q. Liao and C. K. Law, Phys. Rev. A **87**, 043809 (2013); J. Q. Liao, K. Jacobs, F. Nori, and R. W. Simmonds, New J. Phys. **16**, 072001 (2014).
[28] X.-W. Xu, Y. J. Li, and Y. X. Liu, Phys. Rev. A **87**, 025803 (2013).
[29] A. Kronwald, M. Ludwig, and F. Marquardt, Phys. Rev. A **87**, 013847 (2013).
[30] X.-Y. Lü, W.-M. Zhang, S. Ashhab, Y. Wu, and F. Nori, Sci. Rep. **3**, 2943 (2013).
[31] K. Stannigel, P. Komar, S. J. M. Habraken, S. D. Bennett, M. D. Lukin, P. Zoller, and P. Rabl, Phys. Rev. Lett. **109**, 013603 (2012).
[32] P. Kómár, S. D. Bennett, K. Stannigel, S. J. M. Habraken, P. Rabl, P. Zoller, and M. D. Lukin, Phys. Rev. A **87**, 013839 (2013).
[33] M.-A. Lemonde, N. Didier, and A. A. Clerk, Phys. Rev. Lett. **111**, 053602 (2013); K. Børkje, A. Nunnenkamp, J. D. Teufel, and S. M. Girvin, Phys. Rev. Lett. **111**, 053603 (2013); A. Kronwald and F. Marquardt, Phys. Rev. Lett. **111**, 133601 (2013).
[34] J. Q. Liao and F. Nori, Phys. Rev. A **90**, 023851 (2014).
[35] A. Xuereb, C. Genes, and A. Dantan, Phys. Rev. Lett. **109**, 223601 (2012).

[36] A. Xuereb, C. Genes, and A. Dantan, Phys. Rev. A **88**, 053803 (2013).
[37] T. T. Heikkilä, F. Massel, J. Tuorila, R. Khan, and M. A. Sillanpää, Phys. Rev. Lett. **112**, 203603 (2014).
[38] J. R. Johansson, G. Johansson, and F. Nori, Phys. Rev. A **90**, 053833 (2014).
[39] A. J. Rimberg, M. P. Blencowe, A. D. Armour, and P. D. Nation, New J. Phys. **16**, 055008 (2014).
[40] G. Li, T. Wang, and H.-S. Song, Phys. Rev. A **90**, 013827 (2014).
[41] C. Genes, A. Xuereb, G. Pupillo, and A. Dantan, Phys. Rev. A **88**, 033855 (2013).
[42] S. Ast, M. Mehmet, and R. Schnabel, Opt. Express **21**, 13572 (2013).
[43] A. Kamal, A. Marblestone, and M. Devoret, Phys. Rev. B **79**, 184301 (2009).
[44] K. W. Murch, S. J. Weber, K. M. Beck, E. Ginossar, and I. Siddiqi, Nature (London) **499**, 62 (2013).
[45] C. W. Gardiner, Phys. Rev. Lett. **56**, 1917 (1986).
[46] N. P. Georgiades, E. S. Polzik, K. Edamatsu, H. J. Kimble, and A. S. Parkins, Phys. Rev. Lett. **75**, 3426 (1995).
[47] B. Dayan, A. Peer, A. A. Friesem, and Y. Silberberg, Phys. Rev. Lett. **93**, 023005 (2004).
[48] K. Jähne, C. Genes, K. Hammerer, M. Wallquist, E. S. Polzik, and P. Zoller, Phys. Rev. A **79**, 063819 (2009).
[49] W.-J. Gu, G.-X. Li, and Y.-P. Yang, Phys. Rev. A **88**, 013835 (2013).
[50] C. K. Law, Phys. Rev. A **51**, 2537 (1995).
[51] S. Huang and G. S. Agarwal, Phys. Rev. A **79**, 013821 (2009).
[52] A. Xuereb, M. Barbieri, and M. Paternostro, Phys. Rev. A **86**, 013809 (2012).
[53] F. Jiménez and C. Navarrete-Benlloch, arXiv:1412.2521.
[54] H.-P. Breuer and F. Petruccione, *The Theory of Open Quantum Systems* (Clarendon Press, Oxford, 2002), Chap. 3.
[55] J. R. Johansson, P. D. Nation, and F. Nori, Comput. Phys. Commun. **183**, 1760 (2012); **184**, 1234 (2013).
[56] B. C. Young, F. C. Cruz, W. M. Itano, and J. C. Bergquist, Phys. Rev. Lett. **82**, 3799 (1999).
[57] Y. Y. Jiang, A. D. Ludlow, N. D. Lemke, R. W. Fox, J. A. Sherman, L.-S. Ma, and C. W. Oates, Nat. Photonics **5**, 158 (2011).
[58] T. M. Fortier, M. S. Kirchner, F. Quinlan, J. Taylor, J. C. Bergquist, T. Rosenband, N. Lemke, A. Ludlow, Y. Jiang, C. W. Oates, and S. A. Diddams, Nat. Photonics **5**, 425 (2011).
[59] A. Schliesser, R. Rivière, G. Anetsberger, O. Arcizet, and T. J. Kippenberg, Nat. Phys. **4**, 415 (2008).
[60] P. Rabl, C. Genes, K. Hammerer, and M. Aspelmeyer, Phys. Rev. A **80**, 063819 (2009).