Impact of η_c Hadroproduction Data on Charmonium Production and Polarization within the Nonrelativistic QCD Framework

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With the recent LHCb data on η_c production and based on heavy quark spin symmetry, we obtain the long-distance matrix elements for both η_c and J/ψ productions, among which, the color-singlet one for η_c is obtained directly by the fit of experiment for the first time. Using our long-distance matrix elements, we can provide good description of the η_c and J/ψ hadroproduction measurements. Our predictions on J/ψ polarization are in good agreement with the LHCb data, explain most of the CMS data, and pass through the two sets of CDF measurements in the medium p_t region. Considering all the possible uncertainties carefully, we obtained quite narrow bands of the J/ψ polarization curves.

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The nonrelativistic QCD (NRQCD) factorization framework [1] has gained its reputation from the success in many processes (see, e.g., [2,3]), among which, heavy quarkonia hadroproduction [4–6] is one of the most remarkable examples. Moreover, several groups have accomplished their computer programs for the calculation of QCD corrections to quarkonium related processes. QCD next-to-leading order (NLO) predictions [7–12] based on NRQCD achieved good agreement with almost all the experimental measurements on quarkonia hadroproduction. However, for the J/ψ case, one is still suffering from the ambiguity caused by the freedom in the determination of the color-octet (CO) long-distance matrix elements (LDMEs) [7–9,12–15]. In addition, the J/ψ polarization puzzle is another challenge that NRQCD is facing. Despite that three groups [13–15] have made great efforts to proceed the calculation to NLO in α_s, none of their CO LDMEs can reproduce the recent LHCb data [16,17] with good precision. On the other hand, many works [5,6,18,19] have proceeded with their concerns to the processes in which no experimental data can be used to extract the LDMEs. There, they estimate these LDMEs based on heavy quark spin symmetry (HQSS) and the velocity scaling rule (VSR). Nevertheless, the proof of NRQCD factorization does not require the two rules [20,21]]; hence, the phenomenological test of them is urgent.

Recently, LHCb data [22] on η_c production came out and provided an opportunity to further investigate these problems. This measurement is based on the tools established by the authors in their previous work [23]. References [24,25] studied direct η_c hadroproduction at leading order (LO) in α_s within the NRQCD framework, however, missing the ${}^1S_0^{[8]}$ channel. Since only the inclusive and prompt η_c production rate has been measured, one should also consider contributions from the h_c feed down, the asymptotic behavior of which, in large transverse momentum (p_t) limit, scales as p_t^{-6}. According to our previous work [19], the contribution of this part is negligible comparing with experimental data. Feed down contributions from other excited $c\bar{c}$ bound states are even smaller than that from h_c, so they are not under our consideration. For direct η_c production, up to the order of v^4, where v is the typical charm-quark velocity in the charmonium rest frame, four channels (${}^1S_0^{[1]}$, ${}^1S_0^{[8]}$, ${}^3S_1^{[8]}$, ${}^1P_1^{[8]}$) are involved. Among them, the ${}^3S_1^{[8]}$ channel scales as p_t^{-4} in the large p_t limit, while p_t^{-6} behavior dominates the other three in the medium p_t region [11,26]. Moreover, the NLO QCD corrections to all the channels are not significant, which indicates good convergence in α_s expansion. Therefore, it is possible to determine $\langle O^{\eta_c}({}^3S_1^{[8]})\rangle$ precisely by the fit of the experimental data. Further, we can assume HQSS and fix the other two CO LDMEs for η_c production as well as those for J/ψ production, and see whether they are able to provide reasonable descriptions of J/ψ production and polarization. Noticing that, in Refs. [8,14,15], the LDMEs obtained by minimizing χ^2 do not indicate the VSR, we give up employing this rule as the basis of our argument.

We should also notice that the values of the production LDMEs $\langle O^{\eta_c}({}^1S_0^{[1]})\rangle$ and $\langle O^{J/\psi}({}^3S_1^{[1]})\rangle$ have never been obtained directly from the fit of experiment; only the values of the decay ones have been extracted from experiment. The production LDMEs are considered to be the same as the decay ones in the sense of the VSR, the importance of the higher order effects of which is not clear. Since the absolute values of the color-singlet (CS) LDMEs play a very important role in the exclusive double charmonia production in e^+e^- collisions, high-precision determination of them would be urgent. LHCb data on η_c the production rate provide an opportunity to obtain the value of $\langle O^{\eta_c}(^1S_0^{[1]})\rangle$ by fitting experimental data. As a result, precise evaluation of the short-distance coefficient (SDC) of the $^1S_0^{[1]}$ channel is necessary, and our calculation will be accurate to NLO in α_s as well as in v^2, while higher order corrections are neglected. We should also consider the uncertainty caused by the possible large logarithmic terms involving E_{η_c} (the energy of η_c), which is brought in by the large rapidity (denoted as y) in the LHCb experimental condition.

Since there are only seven experimental data points on the p_t distribution of the η_c production rate, it is impossible to determine all the LDMEs without further constraints. So, we base our work on HQSS, and employ the relations of the LDMEs for direct J/ψ production obtained in Refs. [7,12],

$$\begin{aligned} M_0 &= \langle O^{J/\psi}(^1S_0^{[8]})\rangle + r_0\frac{\langle O^{J/\psi}(^3P_0^{[8]})\rangle}{m_c^2}, \\ M_1 &= \langle O^{J/\psi}(^3S_1^{[8]})\rangle + r_1\frac{\langle O^{J/\psi}(^3P_0^{[8]})\rangle}{m_c^2}, \end{aligned} \tag{1}$$

where

$$\begin{aligned} &M_0 = (7.4 \pm 1.9)\times 10^{-2}\ \mathrm{GeV}^3, \quad r_0 = 3.9, \\ &M_1 = (0.05 \pm 0.02)\times 10^{-2}\ \mathrm{GeV}^3, \quad r_1 = -0.56. \end{aligned} \tag{2}$$

Notice that the universally used definitions of the LDMEs are spin and color summed; the relations between the LDMEs for η_c and J/ψ based on HQSS are

$$\begin{aligned} \langle O^{\eta_c}(^1S_0^{[n]})\rangle &= \frac{1}{3}\langle O^{J/\psi}(^3S_1^{[n]})\rangle, \\ \langle O^{\eta_c}(^3S_1^{[8]})\rangle &= \langle O^{J/\psi}(^1S_0^{[8]})\rangle, \\ \langle O^{\eta_c}(^1P_1^{[8]})\rangle &= 3\times\langle O^{J/\psi}(^3P_0^{[8]})\rangle, \\ \langle O^{h_c}(^1P_1^{[1]}/^1S_0^{[8]})\rangle &= \langle O^{\chi_{c1}}(^3P_1^{[1]}/^3S_1^{[8]})\rangle, \end{aligned} \tag{3}$$

where n denotes 1 or 8, corresponding to CS or CO, respectively. To determine the value of $\langle O^{\eta_c}(^3S_1^{[8]})\rangle$, we translate the relation in Eq. (1) into the η_c version as

$$\begin{aligned} \frac{\langle O^{\eta_c}(^1P_1^{[8]})\rangle}{m_c^2} &= \frac{3}{r_0}[M_0 - \langle O^{\eta_c}(^3S_1^{[8]})\rangle], \\ \langle O^{\eta_c}(^1S_0^{[8]})\rangle &= \frac{M_1}{3} - \frac{r_1}{3r_0}[M_0 - \langle O^{\eta_c}(^3S_1^{[8]})\rangle]. \end{aligned} \tag{4}$$

Then we obtain the equation for fit,

$$\begin{aligned} &f_{^1S_0^{[1]}}\langle O^{\eta_c}(^1S_0^{[1]})\rangle + \left(f_{^3S_1^{[8]}} + \frac{r_1}{3r_0}f_{^1S_0^{[8]}} - \frac{3m_c^2}{r_0}f_{^1P_1^{[8]}}\right) \\ &\quad\times\langle O^{\eta_c}(^3S_1^{[8]})\rangle = \sigma_{\mathrm{exp}} - \sigma_{h_c} \\ &\quad + M_0\left(\frac{r_1}{3r_0}f_{^1S_0^{[8]}} - \frac{3}{r_0}m_c^2 f_{^1P_1^{[8]}}\right) - \frac{M_1}{3}f_{^1S_0^{[8]}}, \end{aligned} \tag{5}$$

where σ_{exp}, σ_{h_c}, and f_n denote the experimental data for prompt η_c production, the contribution from h_c feed down, and the SDC for the state n, respectively. Without Eq. (5), the CS LDME cannot be determined precisely. Since, on the one hand, the SDCs of $^1S_0^{[8]}$ and $^1P_1^{[8]}$ have the same p_t behavior with the CS one, only the summation of the LDMEs of the three channels can be fixed. On the other hand, M_0 and M_1 also have uncertainties which might affect those of the CO LDMEs to be obtained. Equation (5) separates the CS SDC from the CO ones, at the same time, the errors from M_0, M_1, and σ_{h_c} are combined with the experimental ones naturally.

To obtain the SDCs, we employ the FDC package [27]. In the numerical calculation, we have the following common choices. $|R'_{h_c}(0)|^2 = 0.075\ \mathrm{GeV}^5$ [28] for both the LO and NLO calculation, $m_c = 1.5\ \mathrm{GeV}$, and $v^2 = 0.23$. We employ CTEQ6M [29] as the parton distribution function and two-loop α_s running for the up-to-NLO calculation, and CTEQ6L1 [29] and one-loop α_s running for LO. The branching ratio [30] of h_c to η_c is $\mathcal{B}(h_c \to \eta_c\gamma) = (51 \pm 6)\%$. Having got the SDCs, after a short calculation, we find that, in Eq. (5), the terms involving h_c, $^1S_0^{[8]}$, and $^1P_1^{[8]}$ are negligible (less than 2% of the dominant terms). Eventually, Eq. (5) reduces to

$$f_{^1S_0^{[1]}}\langle O^{\eta_c}(^1S_0^{[1]})\rangle + f_{^3S_1^{[8]}}\langle O^{\eta_c}(^3S_1^{[8]})\rangle = \sigma_{\mathrm{exp}}. \tag{6}$$

Equation (6) provides an excellent opportunity to determine both of the LDMEs, $\langle O^{\eta_c}(^1S_0^{[1]})\rangle$ and $\langle O^{\eta_c}(^3S_1^{[8]})\rangle$. First, the p_t behaviors of $f_{^1S_0^{[1]}}$ and $f_{^3S_1^{[8]}}$ are different (leading power dominates the $^3S_1^{[8]}$ channel, while next-to-leading power dominates the $^1S_0^{[1]}$ channel [11,26]), which is unlike the J/ψ case where the $^3S_1^{[8]}$ and $^3P_J^{[8]}$ channels are entangled. Further, higher order terms in α_s expansion of both of the SDCs might not be significant [26]. We can expect NLO results to give reliable predictions.

To fix the values of the LDMEs in Eq. (6), we should first make sure that our SDCs are evaluated properly. For the CS channel, we would like to obtain the absolute value of the LDME, to this end, the corresponding SDC should be evaluated precisely. Hence, both QCD and relativistic corrections are considered here, while higher order corrections are dropped. Since the rapidity for the LHCb experimental condition is large, i.e., $2 < y < 4.5$, we also consider the uncertainty coming from possible large logarithmic terms brought in by the large scale, $E_{\eta_c} \approx m_t e^y/2$, where $m_t = \sqrt{m_{\eta_c}^2 + p_t^2}$. Therefore, we calculate the SDC for ${}^1S_0^{[1]}$ at both $\mu_R = \mu_F = m_t$ and $\mu_R = \mu_F = E_{\eta_c}$, and investigate the corresponding uncertainty, where μ_R and μ_F denote the renormalization and factorization scales, respectively. For the ${}^3S_1^{[8]}$ channel, we should be careful. Equations (1) and (2) are obtained in the absence of relativistic corrections, where only QCD corrections are considered. To be consistent, at the same time, noticing that the relativistic correction contributes a part proportional to the QCD LO (as well as NLO) SDC, when p_t is larger than about 7 GeV [31], we should also give up the relativistic-correction contributions to the ${}^3S_1^{[8]}$ channel. For the same reason, we fix μ_R and μ_F to be m_t in the calculation of the ${}^3S_1^{[8]}$ SDC. The relativstic-correction contribution and the difference coming from employing another scale are considered to be absorbed into the corresponding LDME. Throughout the rest of this Letter, when referring to CO channels, we adopt the same scheme.

Now, we fit our theoretical predictions to the LHCb data on the p_t distribution of the prompt η_c production rate at both 7 and 8 TeV presented in Ref. [22], and obtain the LDMEs in Eq. (6). For $\mu_R = \mu_F = m_t$, the LDMEs are given as

$$\begin{aligned} \langle O^{\eta_c}({}^1S_0^{[1]})\rangle &= (0.16 \pm 0.08)\ \mathrm{GeV}^3, \\ \langle O^{\eta_c}({}^3S_1^{[8]})\rangle &= (0.74 \pm 0.30) \times 10^{-2}\ \mathrm{GeV}^3, \end{aligned} \tag{7}$$

and the $\chi^2/\mathrm{d.o.f.} = 0.15$. For $\mu_R = \mu_F = E_{\eta_c}$, they are

$$\begin{aligned} \langle O^{\eta_c}({}^1S_0^{[1]})\rangle &= (0.23 \pm 0.12)\ \mathrm{GeV}^3, \\ \langle O^{\eta_c}({}^3S_1^{[8]})\rangle &= (0.84 \pm 0.28) \times 10^{-2}\ \mathrm{GeV}^3, \end{aligned} \tag{8}$$

and the $\chi^2/\mathrm{d.o.f.} = 0.17$. We get a relatively large uncertainty of the LDMEs in Eqs. (7) and (8), which is due to the large error of the experimental data. We simply estimate the possible range of $\langle O^{\eta_c}({}^1S_0^{[1]})\rangle$ to be from 0.08 to 0.35 GeV^3, which is comparable with the values obtained in most of the other existing works [e.g., $\langle O^{\eta_c}({}^1S_0^{[1]})\rangle = (3/2\pi)|R(0)|^2 = 0.39\ \mathrm{GeV}^3$ for $|R(0)|^2 = 0.81\ \mathrm{GeV}^3$ in Ref. [28] and $\langle O^{\eta_c}({}^1S_0^{[1]})\rangle = 0.437^{+0.111}_{-0.105}\ \mathrm{GeV}^3$ in Ref. [32]]. Our smaller value of the CS LDME also leaves room for the CO mechanism in the J/ψ production experiment at B factories [33].

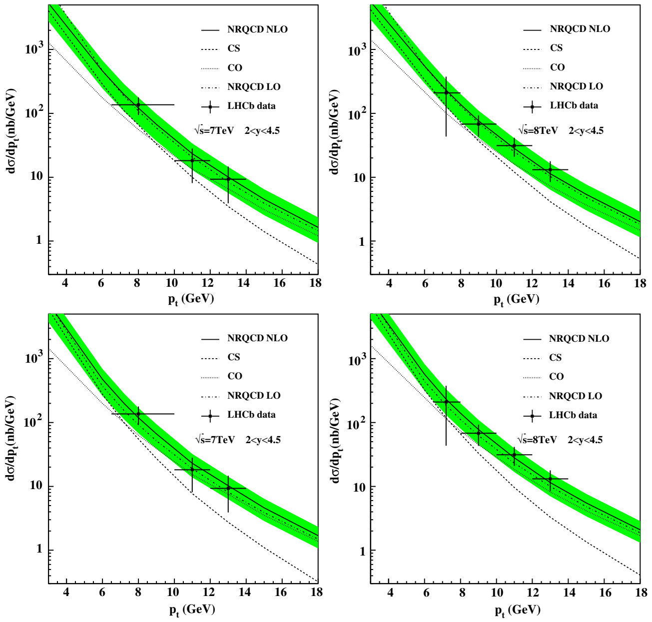

FIG. 1 (color online). p_t distribution of η_c hadroproduction. The upper and lower plots correspond to $\mu_R = \mu_F = m_t$ and $\mu_R = \mu_F = E_{\eta_c}$, respectively. The experimental data are taken from Ref. [22].

The p_t distribution of the η_c hadroproduction rate is shown in Fig. 1. We can see that our theoretical prediction can explain the experimental data for both of the choices of the scales. Also, we can evaluate the integrated cross sections for η_c hadroproduction in the kinematic range $p_t > 6.5$ GeV and $2 < y < 4.5$ at the center-of-mass energy of 7 and 8 TeV as $(\sigma_{\eta_c(1s)})_{\sqrt{s}=7\ \mathrm{TeV}} = (0.53 \pm 0.24)\ \mu\mathrm{b}$ and $(\sigma_{\eta_c(1s)})_{\sqrt{s}=8\ \mathrm{TeV}} = (0.62 \pm 0.28)\ \mu\mathrm{b}$, respectively, which are consistent with the LHCb measurement [22], where $(\sigma_{\eta_c(1s)})_{\sqrt{s}=7\ \mathrm{TeV}} = (0.52 \pm 0.08 \pm 0.09 \pm 0.06)\ \mu\mathrm{b}$ and $(\sigma_{\eta_c(1s)})_{\sqrt{s}=8\ \mathrm{TeV}} = (0.59 \pm 0.11 \pm 0.09 \pm 0.08)\ \mu\mathrm{b}$, respectively. The comparison to LO curves indicates that the QCD and relativistic corrections cancel and higher order corrections might not be significant.

Using HQSS and Eq. (1), we can derive the LDMEs for J/ψ production using the second equation in Eq. (3) and

$$\begin{aligned} \langle O^{J/\psi}({}^3S_1^{[8]})\rangle &= M_1 + \frac{r_1}{r_0}\langle O^{J/\psi}({}^1S_0^{[8]})\rangle - \frac{r_1}{r_0}M_0, \\ \frac{\langle O^{J/\psi}({}^3P_0^{[8]})\rangle}{m_c^2} &= \frac{M_0}{r_0} - \frac{1}{r_0}\langle O^{J/\psi}({}^1S_0^{[8]})\rangle. \end{aligned} \tag{9}$$

We find that the values of $\langle O^{J/\psi}({}^3S_1^{[8]})\rangle$ and $\langle O^{J/\psi}({}^3P_0^{[8]})\rangle$ are not sensitive to the values of $\langle O^{\eta_c}({}^3S_1^{[8]})\rangle$ and M_1. The major uncertainty of the two LDMEs comes from the uncertainty of M_0. The large errors in Eqs. (7) and (8) only affect the other two CO LDMEs for J/ψ production slightly. And we obtain

$$\begin{aligned} 0.24\ \mathrm{GeV}^3 < \langle O^{J/\psi}({}^3S_1^{[1]})\rangle &< 1.05\ \mathrm{GeV}^3, \\ 0.44 \times 10^{-2}\ \mathrm{GeV}^3 < \langle O^{J/\psi}({}^1S_0^{[8]})\rangle &< 1.12 \times 10^{-2}\ \mathrm{GeV}^3, \\ \langle O^{J/\psi}({}^3S_1^{[8]})\rangle &= (1.0 \pm 0.3) \times 10^{-2}\ \mathrm{GeV}^3, \\ \frac{\langle O^{J/\psi}({}^3P_0^{[8]})\rangle}{m_c^2} &= (1.7 \pm 0.5) \times 10^{-2}\ \mathrm{GeV}^3. \end{aligned} \tag{10}$$

The LDMEs obtained here are consistent with the VSR, while in most of the existing versions of the CO LDMEs

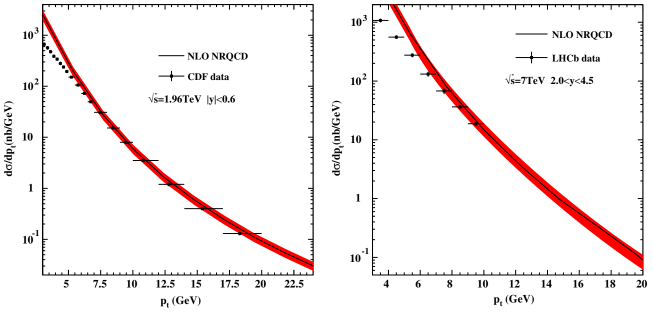

FIG. 2 (color online). p_t distribution of the J/ψ production rate. The CDF and LHCb data are taken from Refs. [34–37].

for NLO calculation, the values of $\langle O^{J/\psi}({}^1S_0^{[8]})\rangle$ are 1 order of magnitude larger than the values of the corresponding $\langle O^{J/\psi}({}^3S_1^{[8]})\rangle$.

Using the LDMEs in Eq. (10), we present the results for the J/ψ yield in Fig. 2. The LHCb data are obtained by subtracting feed down contributions of $\psi(2s)$ [34] and χ_c [35] from the prompt one [36]. As for the CDF data [37], lacking measurements on χ_c feed down contributions, we also calculate the production rate of J/ψ coming from χ_c feed down based on our previous work [10,15], while the $\psi(2s)$ part is omitted [38]. Even though the values of our LDMEs are quite different from those in Refs. [8,14,15], they are also able to explain the CDF and LHCb data for J/ψ production well. This indicates that the three CO SDCs are linear correlated and only two linear combinations of the three CO LDMEs can be fixed stably through the hadroproduction experiment [7]. To present the uncertainty, we should be careful and look at Eq. (9). $\langle O^{J/\psi}({}^3S_1^{[8]})\rangle$ and $\langle O^{J/\psi}({}^3P_0^{[8]})\rangle$ should vary their values accordingly and reach their maximum or minimum at the same time, since all their uncertainties have the same origin, M_0. Hence, we rewrite the expression of the cross section as

$$\sigma(J/\psi) = \langle O^{J/\psi}({}^3S_1^{[1]})\rangle f_{{}^3S_1^{[1]}} + \langle O^{J/\psi}({}^1S_0^{[8]})\rangle(f_{{}^1S_0^{[8]}} - f_0) + M_0 f_0 + M_1 f_1, \tag{11}$$

where

$$f_0 = \frac{1}{r_0}(m_c^2 f_{{}^3P_J^{[8]}} - r_1 f_{{}^3S_1^{[8]}}), f_1 = f_{{}^3S_1^{[8]}}. \tag{12}$$

The uncertainties of the redefined LDMEs, say $\langle O^{J/\psi}({}^1S_0^{[8]})\rangle$, M_0, and M_1, are now independent. The bands presented in Fig. 2 come from the uncertainties of the LDMEs in this sense.

In Fig. 3, we present the results for J/ψ polarization and compare them with the LHCb [16], CMS [39], and CDF [40,41] data. Our predictions can reproduce the LHCb data in both helicity and the Collins-Soper frame at $p_t > 7$ GeV, below which, perturbative calculations are believed not able to give reliable predictions. For the CMS experiment, our predictions exhibit the same behavior with the measurement and explain most of the data. The polarization curve for the Tevatron experimental condition passes through the two sets of CDF measurements in the medium p_t region. Adopting the expression in Eq. (11), the large errors in Eq. (10) finally result in very narrow bands of the J/ψ polarization parameter, which is actually easy to understand: the p_t distribution of f_0 is proportional to that of the ${}^1S_0^{[8]}$ channel [42]; even though the uncertainty of M_0 is large, its contribution is small and does not affect the results for J/ψ polarization very much. However, before we constrain $\langle O^{J/\psi}({}^1S_0^{[8]})\rangle$ by the η_c data, as is shown in Ref. [12], the possible values of the polarization parameter can differ from the LHCb experiment to a great extent. This is evidence not only to support the LDMEs obtained in this Letter, but also for the capability of NRQCD to deal with J/ψ polarization problems.

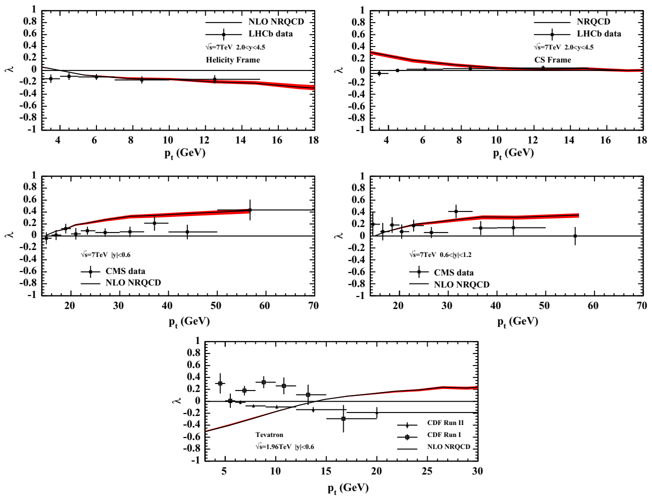

FIG. 3 (color online). J/ψ polarization parameter λ as a function of p_t. The CDF, LHCb, and CMS data are taken from Refs. [16,39–41].

In summary, with the recent LHCb data of the η_c production rate, along with HQSS, we obtained the CS LDMEs for both J/ψ and η_c directly by the fit of the experiment for the first time, which is based on a thorough analysis of the uncertainties. Our results are comparable with the values obtained in most of the other existing works. Using the relations of the LDMEs for J/ψ production in Ref. [12], we also obtained the CO LDMEs for both η_c and J/ψ production, which are consistent with the VSR. Employing these LDMEs, our predictions on η_c and J/ψ hadroproduction rates are in good agreement with the CDF and LHCb data. We also calculated the polarization of prompt J/ψ at hadron colliders. Our predictions can explain the LHCb and CMS data for J/ψ polarization, and pass through the two sets of CDF measurements in the medium p_t region. Our work provides another example to support NRQCD and evidence for the HQSS and VSR. It also helps to clarify the ambiguity of the determination of the CO LDMEs for J/ψ production and, at the same time, opens a door to the solution to the long-standing J/ψ polarization puzzle.

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Note added.—When our calculation was finished and the manuscript was being prepared for publication, we noticed two independent citations [43,44] about the same topic. However, our work contains something new and interesting. On the one hand, we noticed that the η_c hadroproduction process provided an excellent opportunity to fix the CS LDMEs, and through a thorough analysis of the uncertainties, we obtained a reasonable range of these LDMEs. On the other hand, our work not only supported the CO mechanism, but also suggested evidence for the HQSS and VSR. The CO LDMEs obtained in our work are consistent with the VSR, and are able to explain the J/ψ and η_c production experiment as well as the LHCb data on J/ψ polarization in good precision. Besides, the p_t behavior of our prediction on η_c hadroproduction is consistent with the experiment.

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[1] G. T. Bodwin, E. Braaten, and G. P. Lepage, Phys. Rev. D **51**, 1125 (1995).
[2] N. Brambilla, S. Eidelman, B. Heltsley, R. Vogt, G. Bodwin *et al.*, Eur. Phys. J. C **71**, 1534 (2011).
[3] N. Brambilla, S. Eidelman, P. Foka, S. Gardner, A. Kronfeld *et al.*, Eur. Phys. J. C **74**, 2981 (2014).
[4] E. Braaten and S. Fleming, Phys. Rev. Lett. **74**, 3327 (1995).
[5] P. L. Cho and A. K. Leibovich, Phys. Rev. D **53**, 150 (1996).
[6] P. L. Cho and A. K. Leibovich, Phys. Rev. D **53**, 6203 (1996).
[7] Y.-Q. Ma, K. Wang, and K.-T. Chao, Phys. Rev. Lett. **106**, 042002 (2011).
[8] M. Butenschoen and B. A. Kniehl, Phys. Rev. D **84**, 051501 (2011).
[9] M. Butenschön and B. A. Kniehl, Phys. Rev. Lett. **106**, 022003 (2011).
[10] L. Jia, L. Yu, and H.-F. Zhang, arXiv:1410.4032.
[11] G. T. Bodwin, H. S. Chung, U.-Rae Kim, and J. Lee, Phys. Rev. Lett. **113**, 022001 (2014).
[12] H. Shao, H. Han, Y. Ma, C. Meng, Y. Zhang *et al.* arXiv:1411.3300.
[13] M. Butenschoen and B. A. Kniehl, Phys. Rev. Lett. **108**, 172002 (2012).
[14] K.-T. Chao, Y.-Q. Ma, H.-S. Shao, K. Wang, and Y.-J. Zhang, Phys. Rev. Lett. **108**, 242004 (2012).
[15] B. Gong, L.-P. Wan, J.-X. Wang, and H.-F. Zhang, Phys. Rev. Lett. **110**, 042002 (2013).
[16] R. Aaij *et al.* (LHCb Collaboration), Eur. Phys. J. C **73**, 2631 (2013).
[17] R. Aaij *et al.* (LHCb Collaboration), Eur. Phys. J. C **74**, 2872 (2014).
[18] L.-K. Hao, F. Yuan, and K.-T. Chao, Phys. Rev. Lett. **83**, 4490 (1999).
[19] J.-X. Wang and H.-F. Zhang, J. Phys. G **42**, 025004 (2015).
[20] G. C. Nayak, J.-W. Qiu, and G. F. Sterman, Phys. Rev. D **72**, 114012 (2005).
[21] G. C. Nayak, J.-W. Qiu, and G. F. Sterman, Phys. Lett. B **613**, 45 (2005).
[22] R. Aaij *et al.* (LHCb Collaboration), arXiv:1409.3612.
[23] S. Barsuk, J. He, E. Kou, and B. Viaud, Phys. Rev. D **86**, 034011 (2012).
[24] S. S. Biswal and K. Sridhar, J. Phys. G **39**, 015008 (2012).
[25] A. K. Likhoded, A. V. Luchinsky, and S. V. Poslavsky, Mod. Phys. Lett. A **30**, 1550032 (2015).
[26] Y.-Q. Ma, J.-W. Qiu, G. Sterman, and H. Zhang, Phys. Rev. Lett. **113**, 142002 (2014).
[27] J.-X. Wang, Nucl. Instrum. Methods Phys. Res., Sect. A **534**, 241 (2004).
[28] E. J. Eichten and C. Quigg, Phys. Rev. D **52**, 1726 (1995).
[29] J. Pumplin, D. R. Stump, J. Huston, H.-L. Lai, P. Nadolsky and W.-K. Tung, J. High Energy Phys. 07 (2002) 012.
[30] K. Olive *et al.* (Particle Data Group), Chin. Phys. C **38**, 090001 (2014).
[31] G.-Z. Xu, Y.-J. Li, K.-Y. Liu, and Y.-J. Zhang, Phys. Rev. D **86**, 094017 (2012).
[32] G. T. Bodwin, H. S. Chung, D. Kang, J. Lee, and C. Yu, Phys. Rev. D **77**, 094017 (2008).
[33] P. Pakhlov *et al.* (Belle Collaboration), Phys. Rev. D **79**, 071101 (2009).
[34] R. Aaij *et al.* (LHCb Collaboration), Eur. Phys. J. C **72**, 2100 (2012).
[35] R. Aaij *et al.* (LHCb Collaboration), Phys. Lett. B **718**, 431 (2012).
[36] R. Aaij *et al.* (LHCb Collaboration), Eur. Phys. J. C **71**, 1645 (2011).
[37] D. Acosta *et al.* (CDF Collaboration), Phys. Rev. D **71**, 032001 (2005).
[38] T. Aaltonen *et al.* (CDF Collaboration), Phys. Rev. D **80**, 031103 (2009).
[39] S. Chatrchyan *et al.* (CMS Collaboration), Phys. Lett. B **727**, 381 (2013).
[40] T. Affolder *et al.* (CDF Collaboration), Phys. Rev. Lett. **85**, 2886 (2000).
[41] A. Abulencia *et al.* (CDF Collaboration), Phys. Rev. Lett. **99**, 132001 (2007).
[42] Y.-Q. Ma, K. Wang, and K.-T. Chao, Phys. Rev. D **84**, 114001 (2011).
[43] M. Butenschön, Z.-G. He, and B. A. Kniehl, this issue, Phys. Rev. Lett. **114**, 092004 (2015).
[44] H. Han, Y.-Q. Ma, C. Meng, H.-S. Shao, and K.-T. Chao, this issue, Phys. Rev. Lett. **114**, 092005 (2015).