Central Charges from the $\mathcal{N}=1$ Superconformal Index

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We present prescriptions for obtaining the central charges, a and c, of a four-dimensional superconformal quantum field theory from the superconformal index. At infinite N, for holographic theories dual to Sasaki-Einstein 5-manifolds the prescriptions give the $\mathcal{O}(1)$ parts of the central charges. This allows us, among other things, to show the exact AdS/CFT matching of a and c for arbitrary toric quiver CFTs without adjoint matter that are dual to smooth Sasaki-Einstein 5-manifolds. In addition, we include evidence from nonholographic theories for the applicability of these results outside of a holographic setting and away from the large-N limit.

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Introduction.—Given a possibly strongly interacting quantum field theory, one of the basic questions that can be asked is what are its degrees of freedom. In general, this appears to be a difficult problem. However, with the addition of conformal symmetry, there is a growing body of evidence that universal information on the spectrum of operators is contained in the central charges. In two dimensions, this is evident from the Cardy formula [1], which relates the asymptotic density of states to the central charge c, as well as the Zamolodchikov c-theorem [2] governing flows between fixed points. In four dimensions, the central charges a and c control the entanglement entropy [3], while the a-theorem [4] suggests that a is a proxy for the number of degrees of freedom at conformal fixed points.

Additional support for a four-dimensional connection between central charges and the spectrum comes from the recent observation [5,6] that the difference $c-a$ can be obtained from the four-dimensional $\mathcal{N}=1$ superconformal index [7,8]. This index counts the number of shortened states in the spectrum, and for the right-handed index is given by

$$\mathcal{I}^R(t,y;a_i)=\mathrm{Tr}(-1)^F e^{-\beta\delta}t^{-2(E+j_2)/3}y^{2j_1}\prod a_i^{2s_i}, \quad (1)$$

where $\delta=E-\frac{3}{2}r-2j_2$, and $\{E,j_1,j_2,r\}$ are the quantum numbers of the superconformal group $\mathrm{SU}(2,2|1)$. Here β regulates the infinite sum but otherwise drops out of the index since only states with $\delta=0$ contribute. The final factor above encodes global flavor symmetries with quantum numbers $\{s_i\}$ and corresponding fugacities $\{a_i\}$. The left-handed index $\mathcal{I}^L$ is similarly defined with the replacement $r\to -r$ and $j_1\leftrightarrow j_2$. The insertion of $(-1)^F$ is what ensures that only the shortened spectrum contributes to the index, and the result of [5,6] is consistent with the central charges a and c being among the unrenormalized (protected) information in the theory [9].

There have been other attempts in the literature to relate the central charges to the index. In [10], the central charge c was noticed to play a role in the modular properties of the $\mathcal{N}=2$ index, while in [11] a relation was obtained for $2a-c$ of a CFT with $\mathcal{N}=2$ supersymmetry (see also [12]). Moreover, in [13] the central charges were related to the so-called *single-letter* index, and in [14] it was observed that the central charges dictate a specific relation between the supersymmetric partition function on Hopf surfaces and the index. These results suggest that it ought to be possible to obtain both of the central charges a and c independently from the index.

In this Letter we demonstrate that the superconformal index indeed provides information about a and c separately. This follows from the recent work by Beccaria and Tseytlin [15] that demonstrated that the one-loop corrections to a and c in the holographic dual only receive contributions from the shortened spectrum. Since it is precisely this information that is captured by the index, it is then possible to extract the corrections to a and c from the index. Following a similar approach as in [6], we find that the central charges are encoded in the $t,y\to 1$ limit of the functions

$$\hat{a}=\frac{1}{32}(t\partial_t+1)\left(-\frac{9}{2}t\partial_t(t\partial_t+2)+\frac{9}{2}(y\partial_y)^2-3\right)\hat{I}(t,y),$$
$$\hat{c}=\frac{1}{32}(t\partial_t+1)\left(-\frac{9}{2}t\partial_t(t\partial_t+2)-\frac{3}{2}(y\partial_y)^2-2\right)\hat{I}(t,y), \quad (2)$$

where $\hat{I}=(1-yt^{-1})(1-y^{-1}t^{-1})\mathcal{I}^{+}_{s.t.}$ is the single-trace index with descendants removed and $\mathcal{I}^{+}_{s.t.}\equiv\frac{1}{2}(\mathcal{I}^{R}_{s.t.}+\mathcal{I}^{L}_{s.t.})$. [The single-trace index is obtained from Eq. (1) by restricting the sum to the single-trace spectrum and is natural from a holographic point of view.] The fugacities are taken to one after acting with the differential operator on $\hat{I}$ and the central charges are extracted as

$$a = \lim_{t\to 1} \hat{a}(t, y = 1), \qquad c = \lim_{t\to 1} \hat{c}(t, y = 1). \tag{3}$$

Note also that the difference of these equations reproduces the $c - a$ prescription of [6].

Since Eq. (2) was derived from a one-loop computation in the holographic dual, it only computes the subleading $\mathcal{O}(1)$ parts of a and c in holographic theories. Curiously, however, it is possible to recover the full values of a and c from these expressions for some classes of large-N non-holographic theories. In any case, the result obtained from these equations may be divergent when working in the large-N limit, in which case the appropriate prescription is to take the finite term in the Laurent expansions of $\hat{a}$ and $\hat{c}$ about $t = 1$.

In order to highlight the potential divergences in a and c, we consider a series expansion of $\mathcal{I}^+_{s.t.}$, first around $y = 1$ and then around $t = 1$. Generically, the expansion takes the form (see Sec. IV of [6])

$$\mathcal{I}_{s.t.} = \left(\frac{a_0}{t-1} + a_1 + a_2(t-1) + \cdots\right) + (y-1)^2\left(\frac{b_0}{(t-1)^3} + \frac{b_1}{(t-1)^2} + \frac{b_2}{t-1} + \cdots\right) + \cdots. \tag{4}$$

We have dropped the $+$ superscript of $\mathcal{I}_{s.t.}$ assuming that we are dealing with CP invariant theories; this will be the running assumption in the rest of this Letter. Applying Eq. (2) to this expression gives

$$\hat{a}|_{y=1} = \frac{9(a_0 - b_0)}{32(t-1)^2} - \frac{3(a_0 + 12a_2) - 9(b_0 - b_1 + b_2)}{32} + \cdots,$$
$$\hat{c}|_{y=1} = -\frac{3(a_0 - b_0)}{32(t-1)^2} - \frac{2(a_0 + 12a_2) + 3(b_0 - b_1 + b_2)}{32} + \cdots. \tag{5}$$

Provided the single-trace index has the structure of (4), this demonstrates that $\hat{a}$ and $\hat{c}$ have at most a double pole and no single pole. The prescription for removing the divergence then amounts to dropping the double pole.

Large-N theories with holographic dual.—We first examine the holographic case, since that is the framework in which the expressions for a and c were derived. More specifically, we focus on four-dimensional Superconformal Field Theories (SCFTs) dual to IIB theory on $\mathrm{AdS}_5 \times SE_5$ (and leave the study of other holographic settings to future work). For these examples Eq. (2) gives only an $\mathcal{O}(1)$ subleading correction to the central charges, so in this section we expect to only reproduce this subleading contribution which we denote by δa and δc. Of course, for such theories the computation of the $\mathcal{O}(1)$ part of a and c from the large-N single-trace index of the SCFT (or equivalently, the single-particle index of the gravity side) follows directly from the work of Beccaria and Tseytlin [15] on the one-loop contributions of bulk one-particle states to the boundary central charges. The use of the superconformal index in Eq. (2) is at one level simply a rewriting of the sum of the contributions over all bulk states. However, the index does provide an alternative method for regularizing the divergent sum over the Kaluza-Klein (KK) towers in terms of keeping the finite term in an expansion about $t = 1$.

In principle, the application of Eq. (2) to a holographic SCFT can also be viewed as a one-loop test of AdS/CFT. In this sense, the result of [15] can be interpreted as a test for the $\mathcal{N} = 4$ theory, confirming and refining the earlier results of [16,17]. This can be easily generalized to the case of arbitrary toric quiver CFTs without adjoint matter that are dual to smooth Sasaki-Einstein 5-manifolds. The index of such a toric theory is [18,19]

$$\mathcal{I}_{s.t.} = \sum_i \frac{1}{t^{r_i/3} - 1}, \tag{6}$$

where r_i are the R charges of extremal Bogomol'nyi-Prasad-Sommerfield (BPS) mesons. Applying (2) to (6) gives

$$\hat{a} = -\frac{27}{32(t-1)^2}\sum_{i=1}^{n_v}\frac{1}{r_i} - \frac{1}{32}\sum_{i=1}^{n_v} r_i + \cdots \tag{7}$$

in an expansion about $t = 1$. Keeping the finite piece and noting that $\sum r_i = 6(\#\,\text{nodes in the quiver})$, we obtain

$$\delta a = -\frac{3}{16}(\#\,\text{nodes in the quiver}). \tag{8}$$

This matches the expected result for the $\mathcal{O}(1)$ part of a based on the decoupling of a U(1) at each node in the quiver; since there are no adjoint matter fields in the quiver, there are no additional $\mathcal{O}(1)$ contributions to a in the field theoretical computation through $a = \frac{1}{32}(9\mathrm{Tr}R^3 - 3\mathrm{Tr}R)$. The successful matching for the $\mathcal{O}(1)$ part of c can now be deduced either from a similar application of Eq. (2) to (6) or from the successful matching of $c - a$ reported in [6].

We have also checked that Eq. (2) successfully reproduces the $\mathcal{O}(1)$ part of the central charges of all the other holographic theories discussed in [6]. These include the $\mathcal{N} = 4$ theory which has adjoint matter, the singular $\mathbb{Z}_2$ orbifold, and the nontoric Suspended-Pinch-Point (SPP) and del Pezzo theories.

It is of course possible to perform a one-loop test by directly performing the KK sum, and not going through the index as a regulator. In particular, one could proceed along the lines of [20–22] by introducing a z^p regulator where p is the KK level, and then taking the limit $z \to 1$. This type of regulator was recently justified for the $\mathcal{N} = 4$ theory in [15] in terms of the ten-dimensional spectral ζ function, and we have verified that it continues to provide a successful $\mathcal{O}(1)$ matching of both a and c for all the $\mathcal{N} = 1$ cases discussed in [20–22].

The second order pole in a and c: As seen in (5), the coefficients of the second order pole in $\hat{a}$ and $\hat{c}$, and hence $\hat{c} - \hat{a}$ are all determined by the combination $a_0 - b_0$. A relation was given in [6] for the pole in $\hat{c} - \hat{a}$ in terms of curvature invariants of the dual geometry. Therefore similar relations may be obtained for the coefficients of the pole terms that Eq. (2) gives for $\hat{a}$ and $\hat{c}$ of a holographic SCFT. The relation proposed in [6] implies a negative coefficient for the pole in $\hat{c} - \hat{a}$, and hence a positive one for $\hat{a}$ and a negative one for $\hat{c}$.

Because of the universal behavior of the second order pole, for all SCFTs dual to IIB theory on $\mathrm{AdS}_5 \times SE_5$ the combination

$$3\hat{c} + \hat{a} = -\frac{9}{32}(t\partial_t + 1)[2t\partial_t(t\partial_t + 2) + 1]\hat{I}(t, y) \tag{9}$$

is finite at $t = 1$. In fact, assuming the expansion (4) for the index, this combination is always finite. The finiteness can be traced to the absence of the y-dependent operator in (9). This particular combination of a and c has been shown to be proportional to a supersymmetric Casimir energy in [13] and further discussed and argued to be regularization scheme independent in [14,23]. Here we find explicit evidence for these statements of scheme independence. In particular, we see that this quantity receives no contributions from states with arbitrarily large dimension in the large-N limit which would give rise to the second order pole in $\hat{a}$ and $\hat{c}$ individually. It would be interesting to understand this behavior more completely.

Nonholographic SCFTs.—Although the expression (2) was derived from a holographic computation of the $\mathcal{O}(1)$ contributions to a and c, we can nevertheless ask whether it can apply to nonholographic SCFTs as well. Since the single-trace index is inherently a large-N construct, we start the discussion with large-N SCFTs.

Our primary example are the A_k theories, the simplest of which has $k = 1$; this is SQCD without adjoint matter. The single-trace index can be obtained in the Veneziano limit [6,24], and application of Eq. (2) then gives

$$\begin{aligned} \hat{a} &= \frac{9(2k-1)(k+1)}{128k(t-1)^2} + \frac{-3 + 3k - 12k^2 + N_c^2(6 + 3k + 15k^2) - 36N_c^4/N_f^2}{8(k+1)^3} + \cdots, \\ \hat{c} &= -\frac{3(2k-1)(k+1)}{128k(t-1)^2} + \frac{-2 + 5k - 11k^2 + N_c^2(7 + 5k + 16k^2) - 36N_c^4/N_f^2}{8(k+1)^3} + \cdots, \end{aligned} \tag{10}$$

with the finite terms giving the full values of a and c, and not just their $\mathcal{O}(1)$ components [25].

This example demonstrates that the divergence at $t = 1$ remains, presumably as a large-N effect, regardless of holography. However here the finite term recovers the full $\mathcal{O}(N^2)$ values of both a and c in contrast with the holographic examples where the result only gave the $\mathcal{O}(1)$ contributions. The difference presumably lies in the type of large-N limit taken. For the A_k theories we have taken the Veneziano limit, i.e., $N_c \gg 1$ with N_c/N_f fixed. To emphasize one reason why this is different from a holographic 't Hooft limit, a distinction should be made in the number of types (or flavors) of single-trace operators. In the holographic setting this corresponds to the number of Kaluza-Klein towers that exist in the reduction so we will refer to each flavor as an individual tower. In the Veneziano limit there are an infinite number of towers of single-trace operators, as opposed to a finite number of towers arising in the holographic examples. This feature is presumably what allows the index to capture the full expressions for a and c, including the N^2 terms.

Moving away from large-N: Since the index is well defined even away from the large-N limit, Eq. (2) ought to be applicable to finite-N theories as well. However, in this case the single-trace index is not well defined, and a natural choice is to replace it by the plethystic log [26] of the full index.

Obtaining tractable analytic expressions for the superconformal index of interacting theories at finite N is generally difficult, so instead we first comment on the general structure. Following [6], we assume that the result of the plethystic log gives a reduced index $\hat{I}$ that is a regular function with a first order zero at $t = 1$ when $y = 1$. In this case, one can Taylor expand around $t = y = 1$

$$\begin{aligned} \hat{I}(t, y) = {} & f_1(t-1) + f_2(t-1)^2 + f_3(t-1)^3 + \cdots \\ & + (y-1)^2[g_0 + g_1(t-1) + \cdots] + \cdots, \end{aligned} \tag{11}$$

where we have kept only the terms relevant for the calculation of $\hat{a}$ and $\hat{c}$ in (5). Comparison with (4) then yields $a_0 = b_0 = f_1$. Examination of (5) then demonstrates that the expressions for $\hat{a}$ and $\hat{c}$ in (2) remain finite.

Consistency with the result of Di Pietro and Komargodski [5] then gives $a_0 = b_0 = 32(c-a)$, along with a further condition $g_0 + g_1 = 0$ that was obtained in [6]. Combining this information with (5) we find

$$\hat{a} = -\frac{3}{32}(a_0 + 9a_2) + \mathcal{O}(t-1),$$
$$\hat{c} = -\frac{1}{32}(2a_0 + 27a_2) + \mathcal{O}(t-1). \tag{12}$$

Remarkably, only the a_0 and a_2 coefficients enter the expressions for $\hat{a}$ and $\hat{c}$. This means, in particular, that the central charges can be obtained from $\mathcal{I}^{\text{finite-}N}_{s.t.}(t,1)$ where y is set to unity. Since $y=1$ corresponds to a computation of the supersymmetric partition function on the round S^3 [27–29], we see that no squashing is needed to have a and c separately encoded. One can also turn Eq. (12) around and write it as an expansion of the finite-N single-trace index. The result is

$$\mathcal{I}^{\text{finite-}N}_{s.t.}(t,1) = \frac{32(c-a)}{t-1} + a_1 - \frac{32}{27}(3c-2a)(t-1) + \cdots. \tag{13}$$

Note that the Hofman-Maldacena bound $3c \geq 2a$ [30] guarantees that the coefficient of $t-1$ in the above expansion is never positive.

As an example, consider the family of theories with a $\mathrm{U}(1)^N$ gauge group and N_χ neutral chiral multiplets (along with their conjugates) having R charges R_i. This class includes the magnetic dual description of SQCD with $N_f = N_c + 1$. The index is

$$\mathcal{I}^{\text{finite-}N}_{s.t.} = N\left(1 - \frac{1-t^{-2}}{(1-t^{-1}y)(1-t^{-1}y^{-1})}\right) + \sum_{i=1}^{N_\chi} \frac{t^{-R_i} - t^{R_i-2}}{(1-t^{-1}y)(1-t^{-1}y^{-1})}. \tag{14}$$

Expanding $\mathcal{I}^{\text{finite-}N}_{s.t.}(t,1)$ around $t=1$ yields

$$\mathcal{I}^{\text{finite-}N}_{s.t.} = \frac{-2(N + \sum(R_i - 1))}{t-1} - \sum(R_i - 1) - \frac{1}{3}\left(\sum(R_i-1)^3 - \sum(R_i-1)\right)(t-1) + \cdots. \tag{15}$$

Comparing with (4), it is now easy to see that $a_0 = -2\mathrm{Tr}R = 32(c-a)$ and $a_2 = -\frac{1}{3}(\mathrm{Tr}R^3 - \mathrm{Tr}R) = -\frac{32}{27}(3c-2a)$, thus confirming (13).

Taking the expression (13) one step further, we now consider the plethystic exponential of the finite-N single-trace index near $t=1$. For this it is convenient to define $t = e^\beta$ and expand near $\beta = 0$. From (13) we find

$$\mathcal{I}^{\text{finite-}N}(e^\beta, 1) = \exp\left(\sum_{n=1}^{\infty} \frac{1}{n}\mathcal{I}^{\text{finite-}N}_{s.t.}(e^{n\beta},1)\right) = \exp\left(\sum_{n=1}^{\infty} \frac{32(c-a)}{n^2\beta} + \frac{a_1'}{n} - \frac{8}{27}(3c+a)\beta + \cdots\right) = \exp\left(\frac{16\pi^2(c-a)}{3\beta} + \frac{4}{27}(3c+a)\beta + \cdots\right), \tag{16}$$

where in the final equality we have replaced the infinite sums on n with their ζ-function regularized values and thrown away the divergent harmonic series, i.e.,

$$\sum_{n=1}^{\infty} \frac{1}{n^2} = \frac{\pi^2}{6}, \qquad \sum_{n=1}^{\infty} \frac{1}{n} \to 0, \qquad \sum_{n=1}^{\infty} 1 \to -\frac{1}{2}. \tag{17}$$

Note that with this regularization we are neglecting potential $\mathcal{O}(\beta^0)$ and $\mathcal{O}(\log\beta)$ terms in the exponent of the index [see Eq. (4.9) in [5] which demonstrates the existence of such terms in the index of a free vector multiplet]. Nonetheless the $\mathcal{O}(1/\beta)$ and $\mathcal{O}(\beta)$ terms in (16) appear to be unambiguous. In particular, the leading behavior of the result (16) is consistent with the generic results of [5] on supersymmetric partition functions. Furthermore, the $\mathcal{O}(\beta)$ term in the exponential of (16) is precisely the supersymmetric Casimir energy (9) which was originally obtained in [13,14] from the single-letter index.

The above discussion provides evidence that the expression (2), when applied to finite-N theories, yields a and c directly, without any needed subtraction. With the assumption in (11) on the form of the single-trace index (whose validity is worth exploring), we can turn our conjecture into one for the coefficient of the linear term in the expansion of the single-trace index around $t=1$; this is shown as the last term in Eq. (13). The fact that this term reproduces the precise behavior in [13,14] provides a nontrivial test of this statement.

At large-N, the expression for $\hat{a}$ and $\hat{c}$ formally diverges at $t=1$. This divergence is related to the infinite sum encountered when computing the single-trace index, which at finite-N would terminate at $\mathcal{O}(N)$ due to trace identities. In the holographic examples, Eq. (2) computes the $\mathcal{O}(1)$ contribution to a and c, while in the A_k theories in the Veneziano limit, it yields the complete $\mathcal{O}(N^2)$ behavior. The distinction between these two cases appears to be related to the number of types (or flavors) of single-trace operators present in the theory, with the holographic cases having a finite $\mathcal{O}(1)$ number and the A_k theories having an infinite $\mathcal{O}(N^2)$ number.

It would be interesting to explore the pole structure and validity of our prescriptions in (2) and (3) for the indices of strongly coupled theories with a six-dimensional origin [31]. These theories have $\mathcal{O}(N^3)$ degrees of freedom and also admit a dual holographic description in the large-N

limit [32]. Results on the indices are already available [33,34], although their behavior near $t = 1$ remains to be explored.

Finally, for the holographic examples, the leading $\mathcal{O}(N^2)$ contributions to a and c are well understood from the gravity dual in terms of the geometry of the internal manifold [35,36]. In the field theory they appear in the behavior of the Hilbert series for mesonic operators in the CFT [37]. Therefore, the leading order central charges are encoded in the spectrum as well; our results suggest that they are not, however, encoded in the large-N superconformal index. A proper understanding of this distinction may shed light on the manifestation of a holographic dual directly within field theory.

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