Loophole to the Universal Photon Spectrum in Electromagnetic Cascades and Application to the Cosmological Lithium Problem

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(Received 13 November 2014; revised manuscript received 8 January 2015; published 2 March 2015)

The standard theory of electromagnetic cascades onto a photon background predicts a quasiuniversal shape for the resulting nonthermal photon spectrum. This has been applied to very disparate fields, including nonthermal big bang nucleosynthesis (BBN). However, once the energy of the injected photons falls below the pair-production threshold the spectral shape is much harder, a fact that has been overlooked in past literature. This loophole may have important phenomenological consequences, since it generically alters the BBN bounds on nonthermal relics; for instance, it allows us to reopen the possibility of purely electromagnetic solutions to the so-called "cosmological lithium problem," which were thought to be excluded by other cosmological constraints. We show this with a proof-of-principle example and a simple particle physics model, compared with previous literature.

DOI: 10.1103/PhysRevLett.114.091101 PACS numbers: 95.30.Cq, 14.60.St, 26.35.+c

Introduction.—Electromagnetic cascades, namely, the evolution of $\gamma, e^{\pm}$ particle numbers and energy distribution following the injection of an energetic γ or e in a medium filled with radiation, magnetic fields, and matter, are one of the physical processes most frequently encountered in astroparticle physics, in domains as disparate as high-energy gamma-ray astrophysics, ultra-high-energy cosmic ray propagation, or the physics of the early Universe. In particular, the elementary theory of such a cascade onto a photon background has been well known for decades and can be shown via a textbook derivation (see Chapter VIII in Ref. [1], for instance) to lead to a universal "metastable" spectrum—attained on time scales much shorter than the thermodynamical equilibration scale—of the form

$$\frac{dN_\gamma}{dE_\gamma} = \begin{cases} K_0\left(\frac{E_\gamma}{\epsilon_X}\right)^{-3/2} & \text{for } E_\gamma < \epsilon_X, \\ K_0\left(\frac{E_\gamma}{\epsilon_X}\right)^{-2} & \text{for } \epsilon_X \le E_\gamma \le \epsilon_c, \\ 0 & \text{for } E > \epsilon_c. \end{cases} \quad (1)$$

In the above expression, $K_0 = E_0\epsilon_X^{-2}[2+\ln(\epsilon_c/\epsilon_X)]^{-1}$ is a normalization constant enforcing the condition that the total energy is equal to the injected electromagnetic energy E_0, the characteristic energy $\epsilon_c = m_e^2/\epsilon_\gamma^{\max}$ denotes the effective threshold for pair production ($\epsilon_\gamma^{\max}$ being the highest energy of the photon background onto which pairs can be effectively created), and $\epsilon_X \lesssim \epsilon_c/3$ is the maximum energy of up-scattered inverse Compton (IC) photons. Natural units with $c = k_B = 1$ are used throughout.

A notable application of this formalism concerns the possibility of a nonthermal nucleosynthesis phase in the early Universe (for recent review on this and other aspects of primordial nucleosynthesis, or BBN, see Refs. [2,3]). The determination of the baryon energy density of the Universe Ω_b inferred from the cosmic microwave background (CMB) acoustic peaks measurements can be used, in fact, to turn the standard BBN into a parameter-free theory. The resulting predictions for the deuterium abundance (or ^{2}H, the most sensitive nuclide to Ω_b) are in remarkable agreement with observations, providing a tight consistency check for the standard cosmological scenario. The ^{4}He and ^{3}He yields too are, broadly speaking, consistent with this value, although affected by larger uncertainties. The ^{7}Li prediction, however, is a factor $\sim$3 above its determination in the atmosphere of metal-poor halo stars. If this is interpreted as reflecting a cosmological value —as opposed to a postprimordial astrophysical reprocessing, a question which is far from settled [4,5]—it requires a nonstandard BBN mechanism, for which a number of possibilities have been explored [2,3].

In particular, cosmological solutions based on electromagnetic cascades have been proposed in the last decade; see, for instance, Ref. [6]. However, typically they do not appear to be viable [3], as also confirmed in recent investigations (see for instance Fig. 4 in Ref. [7], dealing with massive "paraphotons") due to the fact that whenever the cascade is efficient in destroying enough ^{7}Li, the destruction of ^{2}H is too extreme and spoils the agreement with the CMB observations mentioned above. Actually, this tension also affects some nonelectromagnetic (non-em) nonthermal BBN models; see, for instance, Ref. [8].

This difficulty can be evaded if one exploits the property that ^{7}Be (from which most of ^{7}Li come from for the currently preferred value of Ω_b, via late electron capture decays) has the lowest photodissociation threshold among light nuclei, of about 1.59 vs 2.22 MeV for next to most fragile, ^{2}H. Hence, to avoid any constraint from ^{2}H while being still able to photodisintegrate some ^{7}Be, it is sufficient to inject photons with energy $1.6 < E_\gamma/\mathrm{MeV} < 2.2$, with a "fine-tuned" solution (see e.g., the remark in Ref. [3] or the discussion in Ref. [9]). Nonetheless, it turns out to be hard or impossible to produce a sizable reduction of the final ^{7}Li yield, while respecting other cosmological bounds, such as those coming from extra relativistic degrees of freedom (N_{eff}) or spectral distortions of the CMB. A recent concrete example of these difficulties was illustrated in Ref. [10], which tried such a fine-tuned solution by studying the effects of O(10) MeV sterile neutrino decays.

In this Letter, we point out that, depending on the epoch, at sufficiently low energies of injection the cascade develops differently and the final spectrum is significantly altered with respect to Eq. (1), which had been incorrectly used until recently; see e.g., Refs. [9,10]. As a concrete application, we show how this reopens a broad window to a cosmological solution to the ^{7}Li problem via em decays. Additionally, one expects peculiar signatures associated to such scenarios, which can be probed with cosmological observations. We will discuss this both in a proof-of-principle example and in the context of a particle physics model, involving one sterile neutrino. This was chosen for its simplicity and to allow for a direct comparison with the results of Ref. [10], which was a study of a similar model. Further considerations on some additional implications of our insight are finally outlined.

Em cascades and universal nonthermal spectrum.—Our argument is the following. Let us assume that one injects photons at some time (or corresponding plasma temperature T) whose energy E_0 is below the pair-production threshold at that epoch, which can be estimated for the CMB plasma to be $\epsilon_c \sim m_e^2/(22T) \sim 10T_{\mathrm{keV}}^{-1}$ MeV [11]. Note that as long as $T <$ few keV, this is compatible with the typical nuclear photodisintegration energies relevant for BBN. It is clear that the spectrum of Eq. (1) cannot stay valid in this regime; there is no pair-production cutoff, of course, but even the lower-energy part cannot be correctly captured by Eq. (1). Unless one considers *other physical processes* for the photon interactions, not included in the derivation of Eq. (1), there are no nonthermal electrons that can up-scatter CMB photons. Since the photon interaction probability is much smaller below pair-production threshold, at leading level the injected spectrum below ϵ_c stays the same—apart for redshifting, which happens on very long time scales with respect to particle photon interactions and, hence, we neglect. Accounting for the finite probability for the photons to scatter—via $\gamma\gamma$, via Compton scattering off the background electrons, or via Bethe-Heitler $e^\pm$ production onto background protons and helium nuclei—one does end up with a suppression of the injected spectrum, plus a lower energy tail due to downgraded energy γ particles as well as γ particles produced via IC by the secondary electrons. The resulting secondary or tertiary photons, on the other hand, are typically at too low energies to contribute to photodissociations and will be neglected. Within this approximation, the Boltzmann equation describing the evolution of the distribution function f_γ reads

$$\frac{\partial f_\gamma(E_\gamma)}{\partial t} = -\Gamma_\gamma(E_\gamma, T(t)) f_\gamma(E_\gamma, T(t)) + \mathcal{S}(E_\gamma, t), \quad (2)$$

where $\mathcal{S}(E_\gamma, t)$ is the source injection term, Γ is the total interaction rate, and we neglected the Hubble expansion rate [12], since interaction rates are much faster and rapidly drive f_γ to a quasistatic equilibrium $\partial f_\gamma(\epsilon_\gamma)/\partial t = 0$. Thus, we simply have

$$f_\gamma^S(E_\gamma, t) = \frac{\mathcal{S}(E_\gamma, t)}{\Gamma_\gamma(E_\gamma)}, \quad (3)$$

where the term $\mathcal{S}$ is for an exponentially decaying species with lifetime τ_X and density $n_X(t)$, whose total em energy injected per particle is E_0, can be written as

$$\mathcal{S}(E_\gamma, t) = \frac{n_\gamma^0 \zeta_X (1 + z(t))^3 e^{-t/\tau_X}}{E_0 \tau_X} p_\gamma(E_\gamma), \quad (4)$$

with $z(t)$ being the redshift at time t, and the energy parameter ζ_X (conventionally used in the literature) is simply defined in terms of the initial comoving density of the X particle n_X^0 and the actual one of the CMB, n_γ^0, via $n_X^0 = n_\gamma^0 \zeta_X / E_0$. A monochromatic emission line would then correspond to $p_\gamma(E_\gamma) = \delta(E_\gamma - E_0)$. For a two-body decay $X \to \gamma U$ into a monochromatic line plus another not better specified (quasi)massless particle U, one would have $E_0 = m_X/2$, where m_X is the mass of the particle. Here, we will be interested in multi-MeV values for the mass m_X and at temperatures of order few keV or lower; hence, the thermal broadening is negligible, and a Dirac delta spectrum as the one above is appropriate.

The interaction rate Γ_γ is computed by accounting for (i) Compton scattering over thermal electrons $\gamma + e_{\mathrm{th}} \to \gamma + e$, taken from Ref. [11], (ii) scattering off CMB photons $\gamma + \gamma_{\mathrm{th}} \to \gamma + \gamma$, for which we follow Ref. [13], and (iii) Bethe-Heitler pair creation $\gamma + N \to X + e^\pm$, for which we use the formulas of Ref. [15]. Note that we neglect the small effect due to the finite probability for the secondary or tertiary photons to induce some dissociations; i.e., once a photon interacts it is "lost." The results that we obtain are in this respect slightly conservative, by an amount which we estimated to be of the order of a few %.

Nonthermal nucleosynthesis.—At temperatures of few keV or lower, the standard BBN is over, and the additional nucleosynthesis can be simply dealt with as a postprocessing of the abundances computed in the standard scenario. The nonthermal nucleosynthesis due to electromagnetic cascades can be described by a system of coupled differential equations of the type

$$\frac{dY_A}{dt} = \sum_T Y_T \int_0^\infty dE_\gamma f_\gamma(E_\gamma)\sigma_{\gamma+T\to A}(E_\gamma) - Y_A \sum_P \int_0^\infty dE_\gamma f_\gamma(E_\gamma)\sigma_{\gamma+A\to P}(E_\gamma), \quad (5)$$

where $Y_A \equiv n_A/n_b$ is the ratio of the number density of the nucleus A to the total baryon number density n_b (this factors out the trivial evolution due to the expansion of the Universe), $\sigma_{\gamma+T\to A}$ is the photodissociation cross sections onto the nuclei T into the nucleus A, i.e., the production channel for A, and $\sigma_{\gamma+A\to P}$ is the analogous destruction channel (both cross sections are actually vanishing below the corresponding thresholds). In general, one also needs to follow secondary reactions of the nuclear byproducts of the photodissociation, which can spallate on or fuse with background thermalized target nuclei (see for instance Ref. [6]), but none of that is relevant for the problem at hand. If the injected energy is $1.59 < E_0/\mathrm{MeV} < 2.22$, the only open nonthermal BBN channel is $\gamma + {}^7\mathrm{Be} \to {}^3\mathrm{He} + {}^4\mathrm{He}$, whose cross section [16] we denote with $\sigma_\star$; there are no relevant source terms and only one evolving species (since $Y_7 \ll Y_{3,4}$), thus yielding for the final (at z_f) to initial (at z_i) abundance ratio

$$\ln\left(\frac{Y_{^7\mathrm{Be}}(z_i)}{Y_{^7\mathrm{Be}}(z_f)}\right) = \int_{z_f}^{z_i} \frac{n_\gamma^0 \zeta_X \sigma_\star(E_0) e^{-1/[2H_r^0\tau_X(z'+1)^2]}}{E_0 H_r^0 \tau_X \Gamma(E_0, z)} dz'. \quad (6)$$

To obtain Eq. (6), we transformed Eq. (5) into redshift space, defining $H(z) = H_r^0(1+z)^2$ as appropriate for a universe dominated by radiation, with $H_r^0 \equiv H_0\sqrt{\Omega_r^0}$, H_0 and Ω_r^0 being the present Hubble expansion rate and fractional radiation energy density, respectively. By construction, equating the suppression factor given by the RHS of Eq. (6) to $\sim 1/3$ provides a solution to the ^{7}Li problem, which is in agreement with all other constraints from BBN. In Fig. 1, the lower band shows for each τ_X the range of ζ_X corresponding to a depletion goes from 40% to 70%, for the case $E_0 = 2$ MeV. Similar results would follow by varying E_0 by 10% about this value, i.e., provided one is not too close to the reaction threshold. The upper band represents the analogous region if we had distributed the same injected energy according to the spectrum of Eq. (1), up to min $[\epsilon_c, E_0]$. It is clear that in the correct treatment a large portion of this region survives other cosmological constraints, described below, while none survives in the incorrect treatment.

CMB constraints.—We mentioned that the baryon abundances Ω_b inferred from CMB and BBN (notably ^{2}H) probes are consistent within errors. This implies that no major injection of entropy took place between the BBN time and the CMB epoch, otherwise the baryon-to-photon ratio (proportional to Ω_b) would have changed; see for instance, Ref. [21]. In a radiation-dominated universe, the change in entropy associated to a release of energy into *all* em particles characterized by parameter $\zeta_{X\to\mathrm{em}}$ and a lifetime τ_X can be estimated as

$$\frac{\Delta S}{S} \simeq \ln\frac{S_f}{S_i} = 2.14 \times 10^{-4} \frac{\zeta_{X\to\mathrm{em}}}{10^{-9}\ \mathrm{GeV}} \left(\frac{\tau_X}{10^6\ \mathrm{s}}\right)^{1/2}. \quad (7)$$

For illustration, in Fig. 1 the solid blue line represents the level of entropy release associated to a variation of 2σ around the best-fit measured value of Ω_b by Planck, $\Delta S/S \simeq 0.022$ [22]. It is clear that this constraint is very weak, but for very short lifetimes of the order of 10^4 s.

Another constraint comes from the level of spectral distortions in the CMB. For the relatively short lifetimes relevant for the problem, the Compton scattering is fast enough that energy redistribution is effective; no y-type distortion survives. On the other hand, processes that change the number of photons are relatively rare, and a residual distortion of the μ type is possible. This has been constrained by COBE-FIRAS to be $|\mu| \leq 9 \times 10^{-5}$ [17]. The level of spectral distortion produced by the decay process here has been estimated in the past (see for instance Ref. [19]), but a recent reevaluation [18] found significant improvements at short lifetimes, essentially due to a better

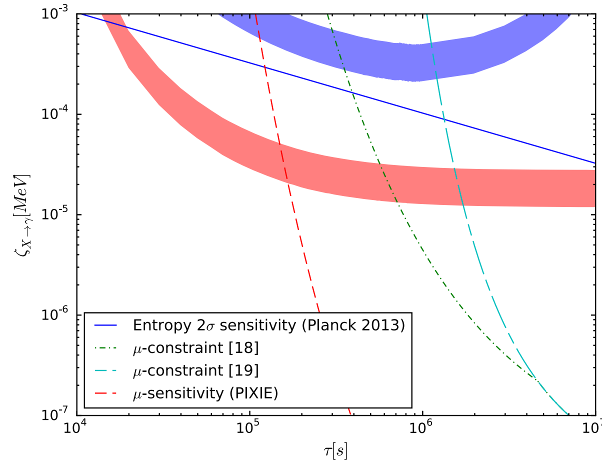

FIG. 1 (color online). Lower band represents the range of abundance parameter $\zeta_{X\to\gamma}$ vs lifetime τ_X, for which the primordial lithium is depleted from 40% to 70% of its standard value, for a monochromatic photon injection with energy $E_0 = 2$ MeV. The upper band represents the analogous region if we had distributed the same injected energy, up to $E_0 = 2$ MeV, according to the erroneous spectrum of Eq. (1). Above the solid blue curve, a change in entropy (and Ω_b) between BBN and CMB time larger than the 2σ error inferred from CMB would be obtained. The region to the right of the dot-dashed green curve is excluded by current constraints from μ distortions in the CMB spectrum [17] according to the computation of Ref. [18], while the dashed cyan curve illustrates the weaker bounds that would follow from the less accurate parametrization of Ref. [19]. The dotted red curve is the forecasted sensitivity of the future experiment PIXIE, corresponding to $|\mu| \sim 5 \times 10^{-8}$ [20].

treatment of the time dependence of the visibility function. The theoretical expectation for μ can be written as

$$\mu \simeq 8.01 \times 10^2 \left(\frac{\tau_X}{1\mathrm{s}}\right)^{1/2} \left(\frac{\zeta_{X\to\mathrm{em}}}{1\ \mathrm{GeV}}\right) \mathcal{J}(\tau_X), \qquad (8)$$

where the function $\mathcal{J}$ is taken from Ref. [18]. The bound excludes the region to the right of the dot-dashed, green curve in Fig. 1. For comparison, the dashed cyan curve reports the much weaker bound that would follow from the approximations in Ref. [19]. We also checked that the extra constraint due to extra "dark radiation" parametrized by N_{eff} is irrelevant as long as the branching ratio in extra relativistic species is not greater than a couple of orders of magnitude with respect to the photon one. We, thus, conclude that there is a significant interval of lifetimes ($10^4 \lesssim \tau_X/\mathrm{s} < 10^6$) and corresponding energy injection parameter $10^{-3} > \zeta_{X\to\gamma}/\mathrm{MeV} > 1.3 \times 10^{-6}$ for which a perfectly viable solution is possible. We remind once again that this possibility appeared to be closed due to the use of Eq. (1) beyond its regime of applicability.

One may wonder how realistic such a situation is in a concrete particle physics model. Although we refrain here from detailed model-building considerations, it is worth showing as a proof-of-principle that models realizing the mechanism described here while fulfilling the other cosmological constraints (as well as laboratory ones) can actually be constructed. Let us take the simplest case of a sterile Majorana neutrino with mass in the range $3.2 < M_s/\mathrm{MeV} < 4.4$, mixing with flavor α neutrinos via an angle θ_α. We also define $\Theta^2 \equiv \sum_\alpha \theta_\alpha^2$. The three main decay channels of this neutrino are (see e.g., Ref. [23] and refs. therein) (i) $\nu_s \to 3\nu$, with rate $\Gamma_{\nu_s\to 3\nu} \simeq (G_F^2 M_s^5 \Theta^2/192\pi^3)$, (ii) $\nu_s \to \nu_\alpha e^+ e^-$, with a rate depending on single θ_α values, (iii) $\nu_s \to \nu\gamma$, with a rate $\Gamma_{\nu_s\to\nu\gamma} \simeq (9G_F^2 \alpha M_s^5/256\pi^4)\Theta^2$. The resulting branching ratios for the masses of interest and $\theta_e \ll \Theta$ are of the level of 0.9:0.1:0.01, respectively. It is physically more instructive to normalize the abundance of the ν_s, n_s^0 in terms of one thermalized neutrino (plus antineutrino) flavor species n_ν^0. In Fig. 2, we show the corresponding range of parameters in the $\Theta - n_s^0/n_\nu^0$ plane, for $M_s = 4.4$ MeV, for which the ^{7}Li problem is solved, fulfills cosmological constraints and, provided that $\theta_e \ll \Theta$, also laboratory ones [24]. It is worth noting that (i) the entropy release bound is now close to the region of interest, since the decay mode $\nu_s \to \nu_\alpha e^+ e^-$, which is useless as far as the ^{7}Be dissociation is concerned, dominates the em energy injection. (ii) A non-negligible fraction of "dark radiation" is now injected, mostly via the dominant decay mode $\nu_s \to 3\nu$; hence, we added the current 1σ sensitivity of Planck to N_{eff} [22], with ΔN_{eff} computed similarly to what was done in Ref. [10]. The needed abundance could be obtained in scenarios with low reheating temperature [24].

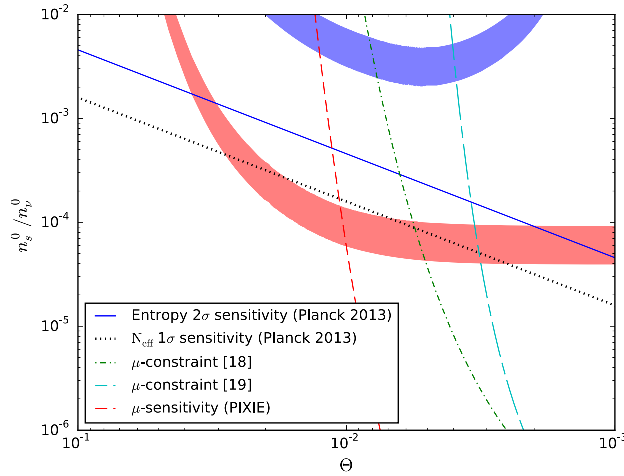

FIG. 2 (color online). Constraints for the sterile neutrino model discussed in the text. The legend is the same as for case (1).

Conclusions.—We have discussed the breaking of the universality of the photon spectrum in electromagnetic cascades, when the energy of the injected photons falls below the pair-production threshold. This may be of interest for a number of astroparticle applications, but in the specific case of the cosmological context, this happens when $E_\gamma \lesssim m_e^2/(22T) \sim 10T_{\mathrm{keV}}^{-1}$ MeV. We noted that the energies concerned are of the same level of the binding energies of light nuclei. This implies a potential large impact on nonthermal nucleosynthesis models, notably of electromagnetic type, but could be also relevant for models with late-time hadronic cascades. We provided an analytical estimate of the resulting (much harder) metastable spectrum of nonthermal photons and showed that the impact is so large that it can potentially reopen the possibility of electromagnetic cascade solutions to the so-called lithium problem, which were thought to be excluded by other cosmological constraints. We substantiated this point with a proof-of-principle example of a photon line injection at ~2 MeV from a particle decay, satisfying by construction all other BBN constraints but, not trivially, also all other cosmological bounds plaguing previous attempts. Although we did not indulge into particle model building, we proved that the right conditions can be actually satisfied in a simple scenario involving a ~4 MeV sterile neutrino mostly mixed with ν_τ and/or ν_μ with effective mixing angle $\Theta \sim 10^{-2}$.

The possibility to find new mechanisms to deplete the standard BBN prediction of lithium abundance in a consistent way is probably the most spectacular consequence of our investigation. In turn, this could stimulate more specific model-building activities. For instance, decays of relatively light new neutral fermionic particles X for which the $\nu + \gamma$ channel is the only two-body standard model channel opened—as is the case for the light gravitinos in supergravity models—constitute a natural class of candidates. Alternatively, one may think of decaying scenarios involving a pair of quasidegenerate mass states X and Y, which are potentially much heavier than the MeV scale. Some of these scenarios may be motivated by other astroparticle or particle physics reasons and certainly deserve further investigation. We also showed how improvements in the determination of μ-type spectral distortions bounds of the CMB might be crucial to test these scenarios: testing frameworks for the particle physics solutions to the lithium problem may thus provide additional scientific motivations for future instruments such as PIXIE [20]. Computations of distortions corresponding to specific injection histories may also be refined: for instance, for short lifetimes relativistic corrections to the double Compton and Compton scatterings may be important to improve the theoretical accuracy [25].

Finally, from a phenomenological perspective, an obvious spinoff of our work would be to recompute the BBN bounds to electromagnetic decaying particles in cases where the universality of the spectrum of Eq. (1) breaks down. Preliminary results indicate that bounds can be easily modified by 1 order of magnitude. These results will be reported in a forthcoming publication.

We thank J. Chluba and J. Pradler for comments on the manuscript. Support by the Labex Grant ENIGMASS is acknowledged.

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