# Effects of Fermion Exchange on the Polarization of Exciton Condensates

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Exchange interaction is responsible for the stability of elementary boson condensates with respect to momentum fragmentation. This remains true for composite bosons when single fermion exchanges are included but spin degrees of freedom are ignored. Here, we show that their inclusion can produce a spin fragmentation of the dark exciton condensate, *i.e.*, an unpolarized condensate with an equal amount of spin $(+2)$ and $(-2)$ excitons not coupled to light. The composite boson many-body formalism allows us to predict that, for spatially indirect excitons, the condensate polarization switches from unpolarized to fully polarized when the distance between the layers confining electrons and holes increases. Importantly, the threshold distance for this switch lies in a regime fully accessible to experiments.

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Free elementary bosons undergo a quantum phase transition with a macroscopic amount of bosons condensed in the same ground state [1] when the boson number gets larger than $N_c(T) \propto (L/\lambda_T)^D$, where $D$ is the space dimension, $L$ the sample size, and $\lambda_T$ the thermal de Broglie wavelength, $k_BT = \hbar^2/2m\lambda_T^2$. Yet, it is surprising that the $\mathbf{k} = \mathbf{0}$ ground state plays such an important role compared to states with very small momentum because, in size $L$ sample, the energy difference between $N$ free bosons with momentum $k = 2\pi/L$ and with momentum $k = 0$ scales as $N/L^2$ which is underextensive.

Nozières [2] has pointed out that, when interactions are included, exchange processes between the two components of a fragmented condenstate $(\mathbf{k}, -\mathbf{k})$ produce an extensive energy difference. So, Bose-Einstein condensation with all elementary bosons in the same $\mathbf{k} = \mathbf{0}$ state is not driven by difference in free particle energy but by exchange interaction. For composite bosons made of two fermions [3], we have shown [4] that this conclusion remains true. This result is not obvious at first because, in addition to exchanging simultaneously the two fermions of a composite boson—which amounts to take the composite boson as a whole—one also has to consider exchanges of only one of the two fermions.

Semiconductor excitons, *i.e.*, electron-hole pairs bound by Coulomb interaction, are particularly attractive to study Bose-Einstein condensation [5–12]. Indeed, exciton condensation is enriched by the carrier spin degrees of freedom [13]. They allow for a multicomponent condensate similar to superfluid $^3$He phases [14], or to more recent spinor condensates of ultracold atomic Bose gases [15] where superfluid components with different internal degrees of freedom coexist and are coherent.

In this Letter, we show how fermion exchange affects the spin degree of freedom of exciton condensates. They lead to either a "spin-unfragmented" condensate, *i.e.*, a fully polarized condensate with all excitons having the same spin, or a "spin-fragmented" condensate with equal populations of the two accessible exciton spins. We also show that, in the thermodynamic limit, the unpolarized condensate is quasi-degenerate with the linearly polarized condensate.

To illustrate the effect of fermion exchanges on standard Bose-Einstein condensation, spatially indirect excitons are quite appropriate. Engineered by confining electrons and holes in two quantum wells [16–18] (see Fig. 1), these excitons have recently led to remarkable observations [19–27]. Here, we show that fermion exchanges between indirect excitons lead to a switch of the condensate polarization from unpolarized to fully polarized. This switch results from a sign change in the exchange Coulomb scattering when the spatial separation between electron and hole layers increases. Importantly, the threshold distance for this switch lies in the regime where experiments are performed. So, this polarization switch provides an experimentally accessible signature for exciton condensation.

*Elementary boson approach.*—In order to understand in a simple way, that the condensate polarization is controlled by interaction, we first consider elementary bosons.

(i) Let us start by recalling Nozières's argument on the "fragmentation" of Bose-Einstein condensates with respect to momentum [2]. We consider an elementary boson Hamiltonian with contact interactions

$$\bar{H} = \sum_{\mathbf{q}} \epsilon_{\mathbf{q}} \bar{B}^\dagger_{\mathbf{q}} \bar{B}_{\mathbf{q}} + \frac{V}{2} \sum_{\mathbf{q}_i} \bar{B}^\dagger_{\mathbf{q}_4} \bar{B}^\dagger_{\mathbf{q}_3} \bar{B}_{\mathbf{q}_2} \bar{B}_{\mathbf{q}_1} \delta_{\mathbf{q}_4+\mathbf{q}_3,\mathbf{q}_1+\mathbf{q}_2}, \qquad (1)$$

 

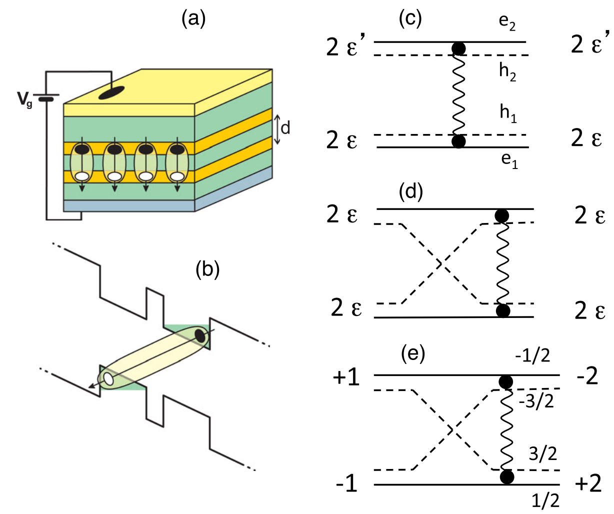


FIG. 1 (color online). (a) Field-effect device to engineer spatially indirect excitons in two quantum wells separated by a distance $d$: an external voltage $V_g$ is applied. The resulting internal electric field sets minimum energy states for electrons and holes (filled and open circles) in different quantum wells (b) thus yielding spatially indirect excitons. (c)–(e) Shiva diagrams representing direct Coulomb scattering between dark excitons (c), exchange Coulomb scatterings between same spin (d), and opposite spin (e) dark excitons, $(\epsilon, \epsilon') = \pm 1$.

and we look for the energy of the momentum-fragmented state $|\bar{\Phi}_{N,N'}\rangle = (\bar{B}^\dagger_{\mathbf{k}})^N (\bar{B}^\dagger_{-\mathbf{k}})^{N'}|0\rangle$. The $\bar{H}$ mean value in $|\bar{\Phi}_{N,N'}\rangle$ with norm $\sqrt{N!N'!}$ appears as

$$\langle \bar{H}\rangle_{N,N'} = \epsilon_{\mathbf{k}}(N+N') + \frac{V}{2}\{N(N-1) + N'(N'-1) + 4NN'\}. \quad (2)$$

The $N(N-1)$ term comes from interactions between two $(\mathbf{k})$ bosons. The $NN'$ term comes from interactions between $(\mathbf{k})$ and $(-\mathbf{k})$ bosons, with a coefficient 4 instead of 2 due to the existence of direct and exchange processes (see the Supplemental Material [28]). This gives the energy difference between momentum-fragmented and unfragmented condensates as

$$\langle \bar{H}\rangle_{N,N} - \langle \bar{H}\rangle_{2N,0} = VN^2. \quad (3)$$

For $V > 0$, as necessary to avoid density collapse, this difference is positive and extensive since $V$ scales as one over the sample volume. A similar result is found [14] for the coherent state $|\bar{\Phi}^{(L)}_{2N}\rangle = (\bar{B}^\dagger_{\mathbf{k}} + \bar{B}^\dagger_{-\mathbf{k}})^{2N}|0\rangle$.

The $\epsilon_{\mathbf{k}}$ part of Eq. (2) also shows that the *kinetic* energy difference between the fragmented state $|\bar{\Phi}_{N,N}\rangle$ taken for $k = 2\pi/L$ and the fully condensed state $|\bar{\Phi}_{2N,0}\rangle$ taken for $k = 0$, scales as $N/L^2$: it is underextensive and thus negligible in the thermodynamic limit. So, Bose-Einstein condensation with all bosons in the *same* $\mathbf{k} = \mathbf{0}$ state is driven by interactions, not by free-particle energy.

(ii) To study the condensate spin structure, we consider elementary bosons with up or down spin and same energy (taken as zero), their creation operators being $\bar{B}^\dagger_+$ or $\bar{B}^\dagger_-$. Let $V$ be the scattering independent of spin and $W$ the scattering between same-spin bosons only. The corresponding Hamiltonian reads as

$$\bar{H}_{\text{pol}} = \frac{V}{2}\sum_{s=\pm}\sum_{s'=\pm} \bar{B}^\dagger_s \bar{B}^\dagger_{s'} \bar{B}_{s'} \bar{B}_s + \frac{W}{2}\sum_{s=\pm} \bar{B}^\dagger_s \bar{B}^\dagger_s \bar{B}_s \bar{B}_s. \quad (4)$$

A calculation similar to the previous one gives the $\bar{H}_{\text{pol}}$ mean value in $|\bar{\Psi}_{N,N'}\rangle = (\bar{B}^\dagger_+)^N (\bar{B}^\dagger_-)^{N'}|0\rangle$ as

$$\langle \bar{H}_{\text{pol}}\rangle_{N,N'} = \frac{V}{2}\{N(N-1) + N'(N'-1) + 2NN'\} + \frac{W}{2}\{N(N-1) + N'(N'-1)\}. \quad (5)$$

The $NN'$ term comes from interaction between + and − spins, with a coefficient 2 instead of 4 because exchanges do not exist between + and − spin bosons. The first bracket, equal to $(N+N')(N+N'-1)$, depends on the total boson number $(N+N')$ but not on the degree of polarization, as expected since $V$ acts between arbitrary spins. By contrast, same-spin scatterings lead to an energy difference between the fully polarized state $|\bar{\Psi}_{2N,0}\rangle$ and the unpolarized state $|\bar{\Psi}_{N,N}\rangle$ equal to

$$\langle \bar{H}_{\text{pol}}\rangle_{2N,0} - \langle \bar{H}_{\text{pol}}\rangle_{N,N} = WN^2. \quad (6)$$

So, for $W < 0$, the lowest energy state is the polarized state $|\bar{\Psi}_{2N,0}\rangle$, degenerate with $|\bar{\Psi}_{0,2N}\rangle$, while for $W > 0$, it is the "spin-fragmented" unpolarized state $|\bar{\Psi}_{N,N}\rangle$.

We can also consider the linearly polarized state $|\bar{\Psi}^{(L)}_{2N}\rangle = (\bar{C}^\dagger_+)^{2N}|0\rangle$ with $\bar{C}^\dagger_\pm = (\bar{B}^\dagger_+ \pm \bar{B}^\dagger_-)/\sqrt{2}$. A simple way to calculate the Hamiltonian mean value in $|\bar{\Psi}^{(L)}_{2N}\rangle$ is to write $\bar{H}_{\text{pol}}$ in terms of $\bar{C}^\dagger_\pm$. We then find $\langle \bar{H}_{\text{pol}}\rangle^{(L)}_{2N} = (V/2 + W/4)2N(2N-1)$ from which we get $\langle \bar{H}_{\text{pol}}\rangle^{(L)}_{2N} - \langle \bar{H}_{\text{pol}}\rangle_{N,N} = WN/2$. So, for $W > 0$, the lowest energy state still is the unpolarized state. However, as $WN$ is underextensive, the energy difference goes to zero in the thermodynamic limit, the unpolarized and linearly polarized states being quasidegenerate.

*Composite boson approach.*—In the elementary boson approach, the possibility to have a sign change for same-spin scattering is not related to any microscopic physics. The composite boson many-body formalism [3] allows us to associate same-spin scattering with exchange interaction and to understand why such a sign change can occur. The attractive part of this scattering depends on the distance between electrons and holes. In the case of indirect excitons, it is possible to decrease this part by increasing the distance between electron plane and hole plane and to end with the repulsive electron-electron and hole-hole contributions only.

When taking composite bosons as elementary particles, we set to zero "deviation operators" $D_{mi} = [B_m, B_i^\dagger] - \delta_{m,i}$. As a result, the effects of fermion exchange in the absence of fermion-fermion interaction are omitted. Although it is commonly believed that the $D_{mi}$ operators can be neglected at small density, this is not correct: they control fermion exchanges and through them all semiconductor nonlinearities [3].

Motivated by experiments performed in two-dimensional zinc-blende heterostructures, we here concentrate on this class of materials. The lowest energy manifold then consists of four spin states: two optically active states ($\pm 1$) called "bright" since they are coupled to $\sigma_\pm$ photons and two optically inactive states ($\pm 2$) called "dark." These four states follow from the four different ways ($\pm 1/2$) spin electrons and ($\pm 3/2$) "spin" holes can be bound by Coulomb attraction. We have shown that dark excitons have a lower energy than bright excitons for the very same reason that they are not coupled to photon [29]. As a result, exciton condensation has to occur in the lowest energy dark exciton subspace [30] made of ($\pm 2$) states.

We study the polarization of the exciton condensate through the mean value of the electron-hole Hamiltonian $H$ in the partially polarized dark exciton state $|\Psi_{N,N'}\rangle = (B_{0,2}^\dagger)^N (B_{0,-2}^\dagger)^{N'}|0\rangle$, namely

$$\Delta_{N,N'} = \frac{\langle \Psi_{N,N'}|H - (N+N')E_0|\Psi_{N,N'}\rangle}{\langle \Psi_{N,N'}|\Psi_{N,N'}\rangle}. \qquad (7)$$

$B_{0,\pm 2}^\dagger$ creates a ground-state dark exciton with center-of-mass momentum $\mathbf{Q} = \mathbf{0}$, relative motion index $\nu_0$, and energy $E_0$. We will also consider $\Delta_N^{(L)}$ for the coherent linearly polarized state $|\Psi_N^{(L)}\rangle = (B_{0,2}^\dagger + B_{0,-2}^\dagger)^N|0\rangle$. It reads as $\Delta_{N,N'}$ with $|\Psi_{N,N'}\rangle$ replaced by $|\Psi_N^{(L)}\rangle$.

(i) Two excitons: To study carrier exchanges in a simple way, let us start with two excitons and compare the energy of the fully polarized state $|\Psi_{2,0}\rangle$ to the one of the unpolarized state $|\Psi_{1,1}\rangle$ and the linearly polarized state $|\Psi_2^{(L)}\rangle$.

The composite boson many-body formalism leads to (see the Supplemental Material [28] for details)

$$\Delta_{1,1} = \frac{\sum_{\mathrm{mn}} \langle 0|B_{0,-2}B_{0,2}B_{m,2}^\dagger B_{n,-2}^\dagger|0\rangle \xi\left(\begin{smallmatrix} n & 0 \\ m & 0\end{smallmatrix}\right)}{\langle 0|B_{0,-2}B_{0,2}B_{0,2}^\dagger B_{0,-2}^\dagger|0\rangle} = \xi\left(\begin{smallmatrix} 0 & 0 \\ 0 & 0\end{smallmatrix}\right),$$

$$\Delta_{2,0} = \frac{2\xi\left(\begin{smallmatrix} 0 & 0 \\ 0 & 0\end{smallmatrix}\right) - 2\xi^{\mathrm{exch}}\left(\begin{smallmatrix} 0 & 0 \\ 0 & 0\end{smallmatrix}\right)}{2 - 2\lambda\left(\begin{smallmatrix} 0 & 0 \\ 0 & 0\end{smallmatrix}\right)}. \qquad (8)$$

$\xi\left(\begin{smallmatrix} n & j \\ m & i\end{smallmatrix}\right)$ corresponds to direct Coulomb scattering between two excitons in states $(i, j)$ ending in states $(m, n)$, the $m$ and $i$ excitons being constructed on the same electron and the same hole, while in exchange Coulomb scattering, their holes are different (see Fig. 1). The dimensionless "Pauli scatterings" $\lambda$ for fermion exchange in the absence of Coulomb process (see Fig. 1) enter the norm of the state when carrier exchange is possible as in the norm of $|\Psi_{2,0}\rangle$. $\Delta_{1,1}$ only contains direct scatterings because $(+2)$ and $(-2)$ excitons have different spins whereas $\Delta_{2,0}$ also contains carrier exchange. Since $(\xi, \xi^{\mathrm{exch}}, \lambda)$ scale as $(a_X/L)^D$, where $a_X$ is the exciton Bohr radius, the $\lambda$ part in the denominators can be neglected at first order in $(a_X/L)^D$. We then see that $\Delta_{1,1}$ and $\Delta_{2,0}$ have the same amount of direct scatterings $\xi$ but a different amount of $\xi^{\mathrm{exch}}$. The above results used for $|\Psi_2^{(L)}\rangle = |\Psi_{2,0}\rangle + |\Psi_{0,2}\rangle + 2|\Psi_{1,1}\rangle$ yield

$$\Delta_2^{(L)} = \frac{8\xi\left(\begin{smallmatrix} 0 & 0 \\ 0 & 0\end{smallmatrix}\right) - 4\xi^{\mathrm{exch}}\left(\begin{smallmatrix} 0 & 0 \\ 0 & 0\end{smallmatrix}\right)}{8 - 4\lambda\left(\begin{smallmatrix} 0 & 0 \\ 0 & 0\end{smallmatrix}\right)} \qquad (9)$$

with the same amount of $\xi$ as $\Delta_{1,1}$ and $\Delta_{2,0}$.

As shown below, in the case of spatially indirect excitons, $\xi^{\mathrm{exch}}$ changes from negative to positive when the distance between carrier planes increases. So, the lowest energy state is the "spin-fragmented" unpolarized state $|\Phi_{1,1}\rangle$ for $-\xi^{\mathrm{exch}} > 0$ but ends up by being the fully polarized state $|\Phi_{2,0}\rangle$ or $|\Phi_{0,2}\rangle$ at large distance when $-\xi^{\mathrm{exch}} < 0$. This conclusion remains true for $N \gg 2$, except that the energy difference between unpolarized and linearly polarized states is underextensive and so negligible in large sample.

(ii) Large exciton number: A rule of the thumb based on dimensional arguments gives the interaction term for $N$ excitons linear in density, by putting $N(N-1)/2$ in front of the interaction term for two excitons. As scatterings between two excitons scale as $(a_X/L)^D$, this rule gives the interaction term for $N$ excitons in $N(N-1)(a_X/L)^D \propto Nn$ where $n = N/L^D$ is the exciton density. In the same way, scatterings between $(m+1)$ excitons scale as $(a_X/L)^{mD}$; they appear with a $N^{m+1}$ prefactor which comes from the $\binom{N}{m+1}$ ways to select $(m+1)$ excitons among $N$. So, they bring a contribution in $Nn^m$. Consequently, effects at first order in density, as the ones we here consider, come from processes involving two excitons among $N$.

Exchange processes between $N$ same-spin ground-state excitons change their normalization factor from $\sqrt{N!}$ to $\sqrt{N!F_N}$ with $F_N$ exponentially small [31] since it decreases with exciton number as $e^{-N\eta}$, where $\eta = N(a_X/L)^D$ is the dimensionless many-body parameter associated with density. Yet, this exponential damping does not have dramatic consequences because $F_N$ ultimately appears through ratios like $F_{N-1}/F_N \simeq 1 + \mathcal{O}(\eta)$ which do not affect physical quantities at first order in density. By contrast, exchange processes between opposite-spin dark excitons bring them to the higher energy bright states [see Fig 1(e)]; so, these exchanges do not enter the Hamiltonian mean value in the dark exciton subspace. As $B_{0,2}^\dagger$ and $B_{0,-2}^\dagger$ are made of different carriers, $B_{0,2}B_{0,-2}^\dagger|0\rangle = 0$; so

$$\begin{aligned}\langle \Psi_{N,N'}|\Psi_{N,N'}\rangle &= \langle \Psi_{N,0}|\Psi_{N,0}\rangle\langle \Psi_{0,N'}|\Psi_{0,N'}\rangle \\ &= (N!F_N)(N'!F_{N'}),\end{aligned} \qquad (10)$$

while from Coulomb processes involving 2 excitons among $N+N'$, we get (see the Supplemental Material [28])

$$\Delta_{N,N'} \simeq \xi(X_N + X_{N'} + Y_{N,N'}) - \xi^{\mathrm{exch}}(X_N + X_{N'}) \quad (11)$$

with $\xi = \xi\binom{00}{00}$ and $\xi^{\mathrm{exch}}{=}\xi^{\mathrm{exch}}\binom{00}{00}$. The $N$ excitons $(+2)$ can have direct and exchange Coulomb scatterings. From the number of ways to choose two excitons $(+2)$ among $N$, these same-spin processes appear in $\Delta_{N,N'}$ with a prefactor $X_N \simeq N(N-1)/2 + \mathcal{O}(\eta)$. By contrast, the $N$ excitons $(+2)$ have direct Coulomb scatterings, but no exchange with the $N'$ excitons $(-2)$. So, opposite-spin processes appear in $\Delta_{N,N'}$ with a prefactor $Y_{N,N'} \simeq NN' + \mathcal{O}(\eta)$.

For a fully polarized condensate, this gives

$$\Delta_{2N,0} \simeq \frac{2N(2N-1)}{2}(\xi - \xi^{\mathrm{exch}}), \quad (12)$$

while, for unpolarized condensate, we find

$$\begin{aligned}\Delta_{N,N} &\simeq 2\frac{N(N-1)}{2}(\xi - \xi^{\mathrm{exch}}) + N^2\xi \\ &\simeq N(2N-1)\xi - N(N-1)\xi^{\mathrm{exch}}. \quad (13)\end{aligned}$$

While $\Delta_{N,N}$ and $\Delta_{2N,0}$ have the same amount of direct Coulomb scatterings, as reasonable since direct processes exist whatever the spins are, $\Delta_{N,N}$ contains less exchanges, $\Delta_{2N,0} - \Delta_{N,N} \simeq -N^2\xi^{\mathrm{exch}}$. So, the "spin-fragmented" unpolarized state $|\Psi_{N,N}\rangle$ has the lowest energy for $-\xi^{\mathrm{exch}} > 0$, while the fully polarized state $|\Psi_{2N,0}\rangle$, degenerate with $|\Psi_{0,2N}\rangle$, is the lowest energy state for $-\xi^{\mathrm{exch}} < 0$.

We can also consider the linearly polarized state

$$|\Psi_{2N}^{(L)}\rangle = (B_{0,2}^\dagger + B_{0,-2}^\dagger)^{2N}|0\rangle = \sum_{p=0}^{2N}\binom{2N}{p}|\Psi_{2N-p,p}\rangle. \quad (14)$$

To calculate the Hamiltonian mean value in this linearly polarized state is far more demanding, due to the large number of different spin states. We can however note that, for large $N$, $\binom{2N}{p}$ is very much peaked at $\binom{2N}{N}$; so (see the Supplemental Material [28]), $\Delta_{2N}^{(L)} \simeq \Delta_{N,N}$, and the unpolarized state $|\Psi_{N,N}\rangle$, which has the lowest energy for $-\xi^{\mathrm{exch}} > 0$, is quasidegenerate with the linearly polarized state $|\Psi_{2N}^{(L)}\rangle$.

We have eliminated exchange Coulomb processes between opposite spin dark excitons because they transfer them into the higher energy bright exciton subspace. Since the splitting between bright and dark states $\delta_{\mathrm{BD}}$ does not depend on sample size, the energy increase $N\delta_{bd}$ for $N$ excitons turning bright is extensive by contrast with $N$ excitons having a $2\pi/L$ momentum. So, for macroscopic samples, in contrast with the momentum fragmentation of elementary boson condensate, no additional interactions are required to drive exciton condensation toward dark states. Yet, when exciton density increases, exchange Coulomb processes cannot be neglected anymore. We have shown [12] that, above a density threshold, a bright component appears in the exciton condensate which makes it "gray"; its study becomes possible through the luminescence of its bright part.

*Spatially indirect excitons.*—Long-lived indirect excitons [16] with electrons and holes in spatially separated 2D layers [see Fig. 1(a)], are among the most promising candidates to study exciton condensation. By comparing Eqs. (5), (12), and (13), we are led to associate scatterings between arbitrary spins with direct Coulomb processes, $V = \xi$, and scattering between same-spin excitons with exchange Coulomb processes, $W = -\xi^{\mathrm{exch}}$. For carrier coordinates $(\mathbf{r}_e, d)$ and $(\mathbf{r}_h, 0)$, where $d$ is the distance between the planes confining electrons and holes and $(\mathbf{r}_e, \mathbf{r}_h)$ are 2D vectors along these planes, the direct Coulomb scattering between two ground-state excitons $(\mathbf{Q}=\mathbf{0}, \nu_0)$, read from Fig. 1(c), is given by [3]

$$\begin{aligned}\xi_d = \int \{d\mathbf{r}\}|\langle \mathbf{d} + \mathbf{r}_{e_1} - \mathbf{r}_{h_1}|\nu_0\rangle\langle \mathbf{d} + \mathbf{r}_{e_2} - \mathbf{r}_{h_2}|\nu_0\rangle|^2 \\ \times \left[\frac{e^2}{|\mathbf{r}_{e_1} - \mathbf{r}_{e_2}|} + \frac{e^2}{|\mathbf{r}_{h_1} - \mathbf{r}_{h_2}|} - \frac{e^2}{|\mathbf{d} + \mathbf{r}_{e_1} - \mathbf{r}_{h_2}|} \right. \\ \left. - \frac{e^2}{|\mathbf{d} + \mathbf{r}_{e_2} - \mathbf{r}_{h_1}|}\right]. \quad (15)\end{aligned}$$

$\xi_d$, equal to zero for $d=0$ as seen by exchanging $(\mathbf{r}_{e_1}, \mathbf{r}_{h_1})$, stays positive for finite $d$. The exchange Coulomb scattering $\xi_d^{\mathrm{exch}}$ between two same-spin excitons [see Fig. 1(d)] reads as $\xi_d$ except for the wave function part which is replaced by

$$\begin{aligned}&\langle \nu_0|\mathbf{d} + r_{e_1} - \mathbf{r}_{h_2}\rangle\langle \nu_0|\mathbf{d} + \mathbf{r}_{e_2} - \mathbf{r}_{h_1}\rangle \\ &\quad \times \langle \mathbf{d} + \mathbf{r}_{e_1} - \mathbf{r}_{h_1}|\nu_0\rangle\langle \mathbf{d} + \mathbf{r}_{e_2} - \mathbf{r}_{h_2}|\nu_0\rangle. \quad (16)\end{aligned}$$

It has been shown [3,32,33] that $\xi_{d=0}^{\mathrm{exch}} = \xi_D R_X^{3D}(a_X^{3D}/L)^D$ with $(R_X^{3D}, a_X^{3D})$ being the $3D$ exciton Rydberg and Bohr radius and $\xi_D$, a numerical factor equal to $-(8\pi - 315\pi^3/512) \simeq -6.06$ in $2D$. When $d$ increases, the electron-hole part of the bracket in Eq. (15) goes to zero. Since the wave function part stays positive, $\xi_d^{\mathrm{exch}}$ becomes positive for $d$ larger than a threshold value $d^*$ which is numerically found to be $d^* \simeq 0.5a_X^{2D}$. Accordingly, for $d < d^*$, $W = -\xi_d^{\mathrm{exch}}$ is positive and the lowest energy state is the "spin-fragmented" unpolarized state $|\Psi_{N,N}\rangle$, while for $d > d^*$, the exchange Coulomb scattering is positive and the lowest energy state is fully polarized, either $|\Psi_{2N,0}\rangle$ or $|\Psi_{0,2N}\rangle$. Most experimental works are performed in a regime where $d \sim 10$ nm [19–27]. This lies in the range of the threshold distance $d^*$ for a switch of the condensate polarization. So, observation of the present prediction is fully accessible to experiments [34]. In particular, the observation by Aichmayr *et al.* [35] of the cancellation of exchange interaction in a double quantum well setup is quite interesting. Although direct comparison with our work is not easy, in particular because our numerical calculation uses a purely 2D model, their experimental result falls in a range quite compatible with our threshold value.

The physical origin of this polarization switch, directly linked to fermion exchange, contrasts with recent theoretical studies [36–39] which concentrate on the role played by spin-orbit interaction. A similar polarization switch has already been reported [9] using the BCS-like Keldysh-Kopaev Green function approach appropriate to dense electron-hole gas—which makes direct comparison with our work difficult. The spin dependence of the resulting exciton effective interaction is associated with large wave function overlap. By contrast, overlap is not responsible for the sign change of $\xi_d^{\mathrm{exch}}$ we find. Moreover, dark excitons are not considered while they are required to eliminate exchange interaction between opposite-spin excitons, this elimination being responsible for the polarization switch we predict. Finally, the threshold found in Ref. [9] is much smaller than the one we predict.

*Conclusion.*—We have shown how fermion exchanges shape the spin structure of exciton condensates. They enrich Bose-Einstein condensation, as illustrated by spatially indirect excitons for which fermion exchanges lead to a switch of the condensate polarization from "spin-fragmented" unpolarized to fully polarized when the distance between confined carriers increases. Our findings may well relate to the low degree of polarization recently observed in experiments reporting a condensate of dark excitons [27].

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[1] L. P. Pitaevskii and S. Stringari, *Bose-Einstein Condensation* (Oxford University Press, New York, 2003).
[2] P. Nozières, in *Bose-Einstein Condensation*, edited by A. Griffin, D. W. Snoke, and S. Stringari (Cambridge University Press, Cambridge, England, 1995).
[3] M. Combescot, O. Betbeder-Matibet, and F. Dubin, Phys. Rep. **463**, 215 (2008).
[4] M. Combescot and D. W. Snoke, Phys. Rev. B **78**, 144303 (2008).
[5] S. A. Moskalenko, Fiz. Tverd. Tela (Leningrad) **4**, 276 (1962) [Sov. Phys. Solid State **4**, 199 (1962)].
[6] J. M. Blatt, K. W. Böer, and W. Brandt, Phys. Rev. **126**, 1691 (1962).
[7] L. V. Keldysh and A. N. Kozlov, Sov. Phys. JETP **27**, 521 (1968).
[8] Xuejun Zhu, P. B. Littlewood, Mark S. Hybertsen, and T. M. Rice, Phys. Rev. Lett. **74**, 1633 (1995).
[9] J. Fernandez-Rossier and C. Tejedor, Phys. Rev. Lett. **78**, 4809 (1997).
[10] B. I. Halperin and T. M. Rice, *The Excitonic State at the Semiconductor-Semimetal Transition*, Solid State Physics Vol. 21 (Academic Press, New York, 1968), p. 115.
[11] B. Laikhtman and R. Rapaport, Phys. Rev. B **80**, 195313 (2009)
[12] R. Combescot and M. Combescot, Phys. Rev. Lett. **109**, 026401 (2012).
[13] J. I. Inoue, T. Brandes, and A. Shimizu, Phys. Rev. B **61**, 2863 (2000).
[14] A. J. Leggett, *Quantum Liquids: Bose Condensation and Cooper Pairing in Condensed Matter Systems* (Oxford University Press, New York, 2006).
[15] D. M. Stamper-Kurn and M. Ueda, Rev. Mod. Phys. **85**, 1191 (2013).
[16] Yu. E. Lozovik and V. I. Yudson, JETP Lett. **22**, 274 (1975).
[17] See L. Viña, J. Phys. Condens. Matter **11**, 5929 (1999) and references therein.
[18] T. Amand, D. Robart, X. Marie, M. Brousseau, P. Le Jeune, and J. Barrau, Phys. Rev. B **55**, 9880 (1997).
[19] A. A. High, J. R. Leonard, A. T. Hammack, M. M. Fogler, L. V. Butov, A. V. Kavokin, K. L. Campman, and A. C. Gossard, Nature (London) **483**, 584 (2012).
[20] A. V. Gorbunov and V. B. Timofeev, JETP Lett. **96**, 138 (2012).
[21] A. A. High, J. R. Leonard, M. Remeika, L. V. Butov, M. Hanson, and A. C. Gossard, Nano Lett. **12**, 2605 (2012).
[22] Y. Shilo, K. Cohen, B. Laikhtman, K. West, L. Pfeiffer, and R. Rapaport, Nat. Commun. **4**, 2335 (2013).
[23] G. J. Schinner, J. Repp, E. Schubert, A. Rai, D. Reuter, A. Wieck, A. Govorov, A. Holleitner, and J. Kotthaus, Phys. Rev. Lett. **110**, 127403 (2013).
[24] M. Stern, V. Umansky, and I. Bar-Joseph, Science **343**, 55 (2014).
[25] A. A. High *et al.*, Phys. Rev. Lett. **110**, 246403 (2013).
[26] A. Violante, R. Hey, and P. V. Santos, arXiv:1408.4310.
[27] M. Alloing, M. Beian, M. Lewenstein, D. Fuster, Y. González, L. González, R. Combescot, M. Combescot, and F. Dubin, Europhys. Lett. **107**, 10012 (2014).
[28] See Supplemental Material at http://link.aps.org/supplemental/10.1103/PhysRevLett.114.090401 for details.
[29] M. Combescot and M. Leuenberger, Solid State Commun., **149**, 567 (2009).
[30] M. Combescot, O. Betbeder-Matibet, and R. Combescot, Phys. Rev. Lett. **99**, 176403 (2007).
[31] M. Combescot and C. Tanguy, Europhys. Lett. **55**, 390 (2001).
[32] J. Fernandez-Rossier, C. Tejedor, L. Muñoz, and L. Viña, Phys. Rev. B **54**, 11582 (1996).
[33] M. Combescot, M. A. Dupertuis, and O. Betbeder-Matibet, Europhys. Lett. **79**, 17001 (2007).
[34] K. Sivalertporn, L. Mouchliadis, A. L. Ivanov, R. Philp, and E. A. Muljarov, Phys. Rev. B **85**, 045207 (2012).
[35] G. Aichmayr, M. Jetter, L. Viña, J. Dickerson, F. Camino, and E. E. Mendez, Phys. Rev. Lett. **83**, 2433 (1999).
[36] M. A. Can and T. Hakioglu, Phys. Rev. Lett. **103**, 086404 (2009).
[37] M. Matuszewski, T. C. H. Liew, Y. G. Rubo, and A. V. Kavokin, Phys. Rev. B **86**, 115321 (2012).
[38] A. V. Kavokin, M. Vladimirova, B. Jouault, T. C. H. Liew, J. R. Leonard, and L. V. Butov, Phys. Rev. B **88**, 195309 (2013).
[39] D. V. Vishnevsky, H. Flayac, A. V. Nalitov, D. D. Solnyshkov, N. A. Gippius, and G. Malpuech, Phys. Rev. Lett. **110**, 246404 (2013).