Wang *et al.* Reply The main objections of the previous Comment by Baranov *et al.* [1] to our Letter [2] are (i) that the linear theory is not applicable in the gain systems having the singular points (SPs) in the upper-half complex frequency ($\tilde{\omega}$) plane, and (ii) that the causality is violated when there are SPs in the upper-half $\tilde{\omega}$ plane in the reflection and transmission coefficients. We disagree, and in the following we present our arguments.

The main argument of Baranov *et al.* that "the movement of SPs … indicates the onset of lasing" implies that the onset of lasing does not depend on the field strength but relies on the slab's thickness. This incorrect argument is different from the conventional view on lasing (e.g., see a recent reference, Ref. [3]). This argument is also contradictory to their own simulation in Ref. [4] (this paper is by some of the authors of the Comment) where they showed that the linear method is very good in a gain medium as long as the input field is sufficiently weak. On the basis of these simulations, it was pointed out in Ref. [4] that the linear approximation is valid for the slab width d, which satisfies the condition $d_{\mathrm{cr}} < d < d_{\mathrm{las}}$. Here d_{cr} is the critical slab width, and d_{las} is the slab width where we have lasing. As pointed out in Ref. [3], d_{las} is dependent on the intensity of the incident light. The lower the intensity of the incident light is, the larger the width d_{las} will be. For a certain slab width d, we can use low intensity for the incident light, so that no lasing will happen; that is to say, the condition $d_{\mathrm{cr}} < d < d_{\mathrm{las}}$ is still valid. Therefore, our linear approximation can still be a good one.

Furthermore, the argument that "a nonzero solution in the absence of incident field arises" is also incorrect. It is evident from Maxwell's equations that $\mathbf{P}$ must be zero without the presence of $\mathbf{E}$. The nonzero solution is unphysical and does not exist if one correctly deals with the SPs in the transmission and reflection coefficients within the linear approach. This refutes the first point by Baranov *et al.*

Next, we show that the second point raised by Baranov *et al.* is also incorrect. Although Baranov *et al.* agree with our statement that "an appearance of SPs in the upper half plane leads to the violation of the conventional Kramers-Kronig (CKK) relation", they misunderstand the Titchmarsh theorem [5] and mistakingly think that "causality and the validity of CKK relations are equivalent." It is known that Kramers-Kronig (KK) relations may have different forms [6] and certain KK relations may not automatically imply causality. In our work, the presence of SPs in the upper-half $\tilde{\omega}$ plane in the reflection and transmission coefficients is not related to causality [7]. It has been proven in Ref. [8] that in gain systems, when the electric field $E(t)$ increases without exceeding the exponential growth for large time t, $|E(t)| \le E_0 e^{\gamma_0 t}$ (here, E_0 and γ_0 are positive constants), the function $F(t) = E(t)e^{-\gamma t}$ with $\gamma > \gamma_0$ satisfies the Titchmarsh theorem and also satisfies the causality. This is due to the fact that $F(t)$ satisfies the precondition of the Titchmarsh theorem (the integral of $|F(t)|^2$ is finite), while $E(t)$ does not (the integral of $|E(t)|^2$ may go to infinity). $F(t)$ satisfies the causality; consequently, $E(t)$ satisfies it too. But the CKK relations related to $E(t)$ are not satisfied in this case. Actually, we have pointed out [2] that the modified KK relations may be recovered by using the "Blaschke product." For example, the CKK relations are not satisfied without breaking the causality in the system of the Gires-Tournois interferometer [9]. However, modified KK relations may be satisfied in such a system [10]. Most importantly, any impulse output (i.e., optical precursors) cannot occur earlier than the input impulse [11]. The claim in the Comment is therefore incorrect.

Hence, in summary, we conclude that the linear theory in our Letter [2] is valid as long as the electric field is sufficiently weak within a finite thickness. The causality is also preserved by correctly dealing with the SPs in the reflection and transmission coefficients. We emphasize the main result in our Letter [2], which is the existence of the counterintuitive dispersion. This is proposed to be tested by measuring the Hartman effect in the reflection from the gain slab. The purpose of stating that "… the causality of the gain slab system is always preserved" in our Letter is to eliminate the wrong impression that the violation of the CKK relations always break the causality. Thus, we believe that the statements in the Comment are misleading and incorrect.

This research is supported by NPRP Grant No. 5-102-1-026 by the Qatar National Research Fund (QNRF) and a grant from King Abdulaziz City for Science and Technology (KACST). This work is also supported by grants of the National Basic Research Program of China (Grants No. 2012CB921602,3 and No. 2011CB922203), and by NSFC Grants No. 11274275, No. 11174026.

Li-Gang Wang,[1,2,3] Lin Wang,[1,4] M. Al-Amri,[2,3,4]
Shi-Yao Zhu[4,5] and M. Suhail Zubairy[2,3,4]

[1]Department of Physics
Zhejiang University
Hangzhou 310027, China
[2]Institute for Quantum Science and Engineering (IQSE)
and Department of Physics and Astronomy
Texas A&M University
College Station, Texas 77843-4242, USA
[3]The National Center for Applied Physics
KACST, P.O. Box 6086, Riyadh 11442, Saudi Arabia
[4]Beijing Computational Science Research Center
Beijing 100084, China
[5]Synergetic Innovation Center of Quantum Information
and Quantum Physics
University of Science and Technology of China
Hefei, Anhui 230026, China

Received 6 January 2015; published 26 February 2015
DOI: 10.1103/PhysRevLett.114.089302
PACS numbers: 42.50.Nn, 11.55.Fv, 42.25.Bs, 42.25.Gy

[1] D. G. Baranov, A. A. Zyablovsky, A. V. Dorofeenko, A. P. Vinogradov, and A. A. Lisyansky, Preceding Comment, Phys. Rev. Lett. **114**, 089301 (2015).
[2] L. G. Wang, L. Wang, M. Al-Amri, S. Y. Zhu, and M. S. Zubairy, Phys. Rev. Lett. **112**, 233601 (2014).
[3] A. Fang, T. Koschny, and C. M. Soukoulis, J. Opt. **12**, 024013 (2010).
[4] A. V. Dorofeenko, A. A. Zyablovsky, and A. A. Pukhov A. A. Lisyansky, and A. P. Vinogradov, Phys. Usp. **55**, 1080 (2012).
[5] H. M. Nussenzveig, *Causality and Dispersion Relations*, (Academic, New York, 1972).
[6] J. S. Toll, Phys. Rev. **104**, 1760 (1956).
[7] B. Nistad and J. Skaar, Phys. Rev. E **78**, 036603 (2008).
[8] H. O. Hågenvik, M. E. Malema, and J. Skaar, arXiv:1407.0187v1.
[9] M. Beck, I. A. Walmsley, and J. D. Kafka, IEEE J. Quantum Electron. **27**, 2074 (1991).
[10] R. H. J. Kop, P. de Vries, R. Sprik, and A. Lagendijk, Opt. Commun. **138**, 118 (1997).
[11] L. Brillouin, *Wave Propagation and Group Velocity*, (Academic, New York, 1960).