Power Counting of Contact-Range Currents in Effective Field Theory

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We analyze the power counting of two-body currents in nuclear effective field theories (EFTs). We find that the existence of nonperturbative physics at low energies, which is manifest in the existence of the deuteron and the $^1S_0$ NN virtual bound state, combined with the appearance of singular potentials in versions of nuclear EFT that incorporate chiral symmetry, modifies the renormalization-group flow of the couplings associated with contact operators that involve nucleon-nucleon pairs and external fields. The order of these couplings is thereby enhanced with respect to the naive-dimensional-analysis estimate. Consequently, short-range currents enter at a lower order in the chiral EFT than has been appreciated up until now, and their impact on low-energy observables is concomitantly larger. We illustrate the changes in the power counting with a few low-energy processes involving external probes and few-nucleon systems, including electron-deuteron elastic scattering and radiative neutron capture by protons.

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Effective field theories (EFTs) describe physics at low momenta—specified by the soft scale Q—where the fields and symmetries out of which they are constructed are well defined and make sense. Q is small in comparison to the natural ultraviolet (UV) cutoff of the EFT, the hard scale M, which corresponds to the energy region where the EFT's degrees of freedom no longer describe the physics. As long as $Q < M$, the EFT provides an expansion of observables in powers of Q/M. To avoid explicit sensitivity to the physics at the hard scale M, EFTs are regularized and renormalized. In the Wilsonian formulation [1] we regularize by means of a UV cutoff Λ that serves as an explicit separation between low- and high-energy physics. The UV cutoff is not the natural cutoff M but rather a theoretical device for analyzing the EFT, and we must make sure that calculations do not depend on Λ; i.e., we renormalize the theory. By reducing the cutoff from M to Q we can analyze the evolution of the EFT couplings and determine their relative importance at low energies.

The Wilsonian renormalization group (RG) is a well-established tool in the context of EFTs where the expansion is strictly perturbative [2]—the standard model, chiral perturbation theory (χPT), QED below the weak scale—and helps to explain why these theories work [3]. However, over the last 20 years an EFT has been developed for nuclear physics, in which already at leading order (LO) the EFT two-nucleon amplitude contains poles (bound states) and the potentials are singular, behaving as $1/r^3$ as $r \to 0$. This provokes novel questions about the meaning of renormalization in this context, questions that have led to much controversy [4–8] and new approaches [9–14]. In this Letter, we examine the matrix elements of electroweak current operators in nuclear EFTs. We show how RG invariance can be used to determine the order at which these operators enter the EFT expansion for the electromagnetic or weak-nuclear currents by which nuclei couple to electrons, photons, and neutrinos. We demonstrate that naive dimensional analysis (NDA) underestimates the role of these current operators, in essence because it neglects their anomalous dimension.

This has significant implications for the theory of processes including elastic and inelastic electron-deuteron and electron-trinucleon scattering [15,16], the proton fusion reaction $pp \to de^+\nu_e$ [17] and muon capture on deuterium and ^{3}He [18–20]. Until now, the most sophisticated nuclear EFT calculations of these processes have invoked NDA, since all have employed χPT power counting to organize nuclear operators. (See Ref. [21] for a recent review.) The results of Refs. [15–20] have implications for the structure of light nuclei, solar models, and precision tests of the standard model. Here, we argue that short-distance contributions to the current operators used therein are actually significantly more important than was appreciated in these works. In particular, we show that RG invariance requires that in most Gamow-Teller or $M1$ transitions in few-nucleon systems the short-distance contribution enters at least one order earlier than is predicted in the χPT power counting originally suggested by Weinberg [22,23].

In EFT, observable quantities do not depend on the choice of the cutoff. This can be realized by imposing the cutoff independence of (here, on-shell) matrix elements

$$\frac{d}{d\Lambda}\langle\Psi'|\mathcal{O}_{\rm EFT}|\Psi\rangle = 0, \qquad (1)$$

with $\mathcal{O}_{\rm EFT}$ an EFT operator, and Ψ (Ψ') the initial- (final-) state EFT wave functions. For concreteness, we assume that $\mathcal{O}_{\rm EFT}$ is a component of a nuclear (four-) current, which depends on the momentum of the probe $\mathcal{O}_{\mathrm{EFT}} = \mathcal{O}_{\mathrm{EFT}}(\mathbf{q})$. Note that Refs. [24,25] already formulated RGs akin to Eq. (1) for NN matrix elements of $\mathcal{O}_{\mathrm{EFT}}$, although no conclusions regarding the chiral EFT (χEFT) power counting were drawn. See Refs. [6,26,27] for applications of this RG to the two-nucleon potential.

χEFT operators contain one-body, two-body, three-body, etc. contributions—separated according to how many nucleons participate directly in the interaction with the external probe. Two-body operators can be subdivided into pion-range and contact parts (but see also below)

$$\mathcal{O}_{\mathrm{EFT}} = \mathcal{O}_{1\mathrm{B}} + \mathcal{O}_{2\mathrm{B},\pi} + \mathcal{O}_{2\mathrm{B},C} + \cdots. \tag{2}$$

In what follows, we focus on the two-body operators, which in general are the dominant correction to the one-body piece. The results can be generalized to higher-body current operators. For $\mathcal{O}_{2\mathrm{B},\pi}$ the interaction among nucleons is mediated by pions. In general, the power counting of this piece is straightforward: we simply count the powers of $Q \equiv \mathbf{q}, m_\pi, \mathbf{p}$ (with m_π the pion mass and $\mathbf{p}$ any nucleon momenta on which $\mathcal{O}$ depends) in each piece of the operator and assume they are made up by powers of the breakdown scale M in the coefficient of that part of $\mathcal{O}_{\mathrm{EFT}}$. In other words, we assume NDA. This leads straightforwardly to the conclusion that $\mathcal{O}_{2\mathrm{B},\pi}$ is typically suppressed relative to the one-body contribution, as first articulated in Refs. [28,29]. It also produces a $\mathcal{O}_{2\mathrm{B},\pi}$ which—up to contact-term pieces—operates at a range $r \sim 1/m_\pi$.

If $\mathcal{O}_{2\mathrm{B},\pi}$ has divergent parts, those will appear in the final answer as contributions of contact range. They will then depend on a regularization scale. But that regularization scale can be kept distinct from the scale Λ used to regularize the Schrödinger equation. Furthermore, such contact pieces of $\mathcal{O}_{2\mathrm{B},\pi}$ will have at least the NDA order of $\mathcal{O}_{2\mathrm{B},C}$. They cannot produce the enhanced-over-NDA contact-range currents that are our concern here.

Power counting for $\mathcal{O}_{2\mathrm{B},C}$ is more subtle, but a few simplifications help us determine it. First, we only have to consider the leading-order piece of this part of the operator: subleading contact-range currents are trivially suppressed by the extra powers of Q contained in the operators. Second, $\mathcal{O}_{2\mathrm{B},C}$ cancels the cutoff dependence of $\mathcal{O}_{1\mathrm{B}}$ and $\mathcal{O}_{2\mathrm{B},\pi}$, yet the two-body piece is suppressed with respect to the one-body. Thus, we can simply ignore $\mathcal{O}_{2\mathrm{B},\pi}$ at lowest order. This is still true even if we promote the pion-exchange currents by one order (as would happen if we adopted the power counting that justifies the iteration of one-pion exchange in the leading-order NN potential in Ref. [6]). This yields the following RG equation for the leading piece of $\mathcal{O}_{2\mathrm{B},C}$:

$$\frac{d}{d\Lambda}\langle\Psi'|\mathcal{O}^{(0)}_{2\mathrm{B},C}|\Psi\rangle = -\frac{d}{d\Lambda}\langle\Psi'|\mathcal{O}^{(0)}_{1\mathrm{B}}|\Psi\rangle + \cdots, \tag{3}$$

where the dots indicate the higher-order terms. The leading $\mathcal{O}_{2\mathrm{B},C}$ most often contains no powers of the small scale (but see the example of the charge operator below), so here we write it schematically as $\mathcal{O}^{(0)}_{2\mathrm{B},C} = C_0(\Lambda)\delta^{(6)}_\Lambda(r', r)$, where the subscript indicates that the δ function is also regulated at scale Λ. The quantum numbers of the current will be carried by an operator, which we have not written here.

The (leading) renormalization-group invariance of the current matrix element is now encoded in a differential equation for $C_0(\Lambda)$, which is

$$\frac{d}{d\Lambda}[C_0(\Lambda)\langle\Psi'|\delta^{(6)}_\Lambda|\Psi\rangle] = -\frac{d}{d\Lambda}\langle\Psi'|\mathcal{O}^{(0)}_{1\mathrm{B}}|\Psi\rangle. \tag{4}$$

This is an inhomogeneous first-order differential equation, although in practice we can ignore the right-hand side. Given a boundary condition it has a unique solution, which determines the power counting of $\mathcal{O}_{2\mathrm{B},C}$ and, concomitantly, its matrix elements.

The boundary condition results from the observation that if we set the cutoff to be the size of the natural cutoff of the theory, i.e., $\Lambda = M$, then C_0 can only scale with M. We take C_0 to be a coupling of inverse mass dimension d, where that dimension is determined by the particular operator it multiplies, and so we have $C_0(M) \sim M^{-d}$. This is NDA applied at the scale M. In many EFT applications, one is interested in estimating the size of $C_0(\Lambda)$ prior to any examination of data, and this naturalness assumption provides a way forward without which the EFT power counting cannot be determined.

Once this boundary condition is chosen, Eq. (4) determines $C_0(\Lambda)$ for any Λ. Although the equation can be integrated in two directions, the standard practice is to evolve the couplings from $\Lambda \sim M$ to $\Lambda \sim Q$ to find out how integrating high-momentum modes out of the theory affects the size of the EFT operators that must compensate for their removal. In this view, the EFT results from infrared RG evolution of a more fundamental theory. In practice, χEFT calculations are carried out with a cutoff Λ that lies in between the high-energy scale of χPT, $M \approx 1$ GeV, and the soft scales $Q \approx 150$ MeV. Since the operators in question often have large inverse mass dimension, understanding their running from M to these lower scales has significant practical importance.

Now, if the wave functions $|\Psi\rangle$ and $|\Psi'\rangle$ are plane waves, then the matrix elements appearing in Eq. (4) have no dependence on Λ and it reduces to $[dC_0(\Lambda)]/d\Lambda = 0$. This, together with the assumption that $C_0(M)$ is natural with respect to the scale M means that $C_0(\Lambda) \sim M^{-d}$ for all Λ. This is the power counting on which χEFT for few-nucleon systems has been based. Such a power counting is valid if the solutions for nuclear wave functions are plane waves or reduce to plane waves in the UV region. (For example, the subleading corrections to naive dimensional analysis that are present in the UV region in the case of Coulombic wave functions do not alter the LO results that we derive here.)

However, if $|\Psi\rangle$ and $|\Psi'\rangle$ do not behave like plane waves at momenta $\gg Q$, then in general we will have

$\langle\Psi'|\delta_\Lambda^{(6)}|\Psi\rangle \sim \Lambda^a$, with the dimensions of the matrix element made up by soft scales Q. Using Eq. (4) to evolve the value of $C_0(\Lambda)$ to the soft scale, Q we find the infrared enhancement (provided that $a > 0$),

$$C_0(\Lambda \sim Q) \sim \frac{1}{M^{d-a}Q^a}. \quad (5)$$

Consequently the contact-range operator receives a promotion of a orders in the EFT expansion of $\mathcal{O}$ in powers of Q/M; $-a$ is the anomalous dimension of C_0 in this EFT and is nonzero because of the strong interactions in the nuclear wave functions. The value of a is independent of how many powers of the soft scales $\mathbf{q}$ and m_π the operator carries and so is the same for higher-order coefficients in the expansion of $\mathcal{O}_{2B,C}$ in powers of Q. For instance, nothing in the above analysis changes if the operator is $\langle\mathbf{p}'|\vec{\mathcal{O}}_C(\mathbf{q};\Lambda)|\mathbf{p}\rangle = M(\Lambda)\boldsymbol{\beta}\times\mathbf{q}$, with $\vec{\beta}$ an arbitrary vector. The coefficient $M(\Lambda)$ will then also be enhanced by the factor $1/Q^a$ with respect to the NDA result.

The power counting enhancement can be determined by examining the behavior of the distorted wave functions $|\Psi\rangle$, $|\Psi'\rangle$ at short distances. In order to demonstrate this, we choose a specific regularization of $\delta^{(6)}$

$$\langle\vec{r}'|\mathcal{O}^{(0)}_{2B,C}|\vec{r}\rangle = C_0(R)\frac{\delta(r-R)}{R^2}\frac{\delta(r'-R)}{R^2}X(\hat{r},\hat{r}'), \quad (6)$$

with X referring to the nonradial piece of the operator. Here, we have introduced a coordinate space cutoff R that is related to the Λ value of the previous paragraphs via $R \propto 1/\Lambda$. The proportionality constant does not affect the value of a.

Similarly, we are not interested here in the numerical factor (reduced matrix element) generated by the matrix element of $X(\hat{r},\hat{r}')$ between the angular pieces of the nuclear wave functions. If we write them as

$$\Psi'(\vec{r}') = \frac{u'(r)}{r}Y'(\hat{r}'), \qquad \Psi(\vec{r}) = \frac{u(r)}{r}Y(\hat{r}), \quad (7)$$

with Y' and Y nonradial pieces containing the dependence on angular momentum and other unspecified quantum numbers, then the matrix element yields

$$\langle\Psi'|\delta_\Lambda^{(6)}|\Psi\rangle \propto \frac{u(R)}{R}\frac{u'(R)}{R}. \quad (8)$$

Thus, all we need to know is the behavior of the wave at $R \ll 1/Q$ to get the anomalous dimension of $C_0(R)$. In particular, if we assume that

$$u(R)/R \sim R^b; \qquad u'(R)/R \sim R^c, \quad (9)$$

the anomalous dimension is $-a = b + c$. The problem of determining the power counting for the leading contact-operator contribution to nuclear currents is thus reduced to the simple matter of computing the UV spectral indices b and c of the EFT wave functions. Here, we analyze only the leading contact operator, so wave functions computed at LO in χEFT are adequate for this purpose.

We now illustrate these ideas by examining a few processes involving the nucleon-nucleon (NN) system. For concreteness, we will begin by considering electromagnetic reactions where the NN system interacts with a (real or virtual) photon. Consequently, the current operator has a Lorentz index and must fulfill the Ward identity (i.e., the continuity equation)

$$\langle J^\mu(\mathbf{q})\rangle = \langle\Psi'|\mathcal{O}^\mu(\mathbf{q})|\Psi\rangle, \qquad q_\mu\langle J^\mu(\mathbf{q})\rangle = 0, \quad (10)$$

a constraint that has consequences for the contact-range currents in the charge (i.e., $\mu = 0$) form factor. That quantity is defined as

$$|e|G_C(\mathbf{q}) = \overline{\langle\Psi_d|J^0(\mathbf{q})|\Psi_d\rangle}, \quad (11)$$

where $|\Psi_d\rangle$ is the deuteron wave function and the bar indicates averaging over spins. The LO operator that contributes to the charge form factor is the one-body charge, which in the plane-wave basis reads $\langle\mathbf{p}'|J^0_{1B,\mathrm{LO}}(\mathbf{q})|\mathbf{p}\rangle = |e|\delta^3(p'-p-q)$. It yields a G_C of order eQ^0. Now, Eq. (10) implies that $G_C(\mathbf{0}) = 1$, but as long as an energy-independent potential generates $|\Psi_d\rangle$ and $\langle\Psi_d|\Psi_d\rangle = 1$, then $\overline{\langle\Psi_d|J^0_{1B,\mathrm{LO}}(\mathbf{0})|\Psi_d\rangle} = 1$. Consequently all higher-order contributions to the charge operator must vanish at $\mathbf{q} = 0$. The pion-range current operator will satisfy this requirement if constructed using dimensional regularization and a mass-independent renormalization scheme (as in Refs. [30–33]), and so the lowest-order, nontrivial, contact operator that contributes to G_C is $D(\Lambda)\mathbf{q}^2$. This can be thought of as a short-distance contribution to the deuteron's charge radius. According to NDA, it affects G_C at $O(Q^5)$. However, this does not take into account the anomalous dimensions stemming from the wave functions.

In χEFT the LO nucleon-nucleon (NN) potential behaves like $1/r^3$ plus a delta-function at short distances. The singular potential is renormalized by the delta function, but results in wave functions u and $w \sim r^{3/4}$ as $r \to 0$ [34,35]. The power-law exponents in the ${}^3S_1$-${}^3D_1$ NN partial wave are thus $b = c = -1/4$. This makes the contribution of $D(\Lambda)\mathbf{q}^2$ slightly bigger: it enters J^0 at $O(eQ^{4.5})$.

If we consider the deuteron magnetic and quadrupole form factors, the lowest order contact operators take the schematic form $M(\Lambda)\vec{\beta}\times\mathbf{q}$ and $Q(\Lambda)T_2(\mathbf{q})$ (with T_2 a tensor involving two powers of $\mathbf{q}$). According to NDA, they appear at $O(Q^4)$ and $O(Q^5)$, respectively. As discussed above, their tensor structure does not affect our RG argument, so they too receive a slight enhancement, to $O(Q^{7/2})$ and $O(Q^{9/2})$, respectively. We note that the magnetic form factor starts only at $O(Q)$, and the relative importance of the chiral EFT short-distance contribution there may explain the difficulties of some models to reproduce the deuteron magnetic moment (see, e.g., the discussion in Ref. [36]). The enhancement of the short-distance part of the quadrupole operator strengthens the argument of Ref. [15] that this operator is key to accurate description of $G_Q(\mathbf{q})$ in the low-$\mathbf{q}^2$ regime.

For comparison, in the "pionless EFT," the NN potential that generates $|\Psi_d\rangle$ operates strictly at $r = 0$, and so we have $b = c = -1$. Our analysis then reproduces the well-developed power counting of electromagnetic operators in pionless EFT [37–40]. We include these results in Table I but do not discuss them further. (The pionless EFT power counting in the strong sector is derived from the Wilsonian RG in Ref. [41].)

For radiative capture of neutrons by protons (or equivalently, photodisintegration of the deuteron) at threshold the $M1$ transition dominates, and the momentum structure of the operator is as for the deuteron magnetic moment. The only difference is that the incoming NN partial wave is $^1S_0$ while the outgoing state is still a deuteron. This is, however, important, since the LO χEFT wave function in the $^1S_0$ behaves as $1/r$ at short distances; i.e., we have $b = -1$. Physically, this occurs because one-pion exchange, which is not a singular potential for spin-0 partial waves, is too weak to generate the unnaturally large scattering lengths in this channel ($a \approx -23.7$ fm), and so a contact interaction—as in the pionless theory—dominates the $r \to 0$ behavior of $u_{^1S_0}(r)$. Thus, in $np \to d\gamma$, $b = -1$ and $c = -1/4$. The contact current thus contributes to threshold np capture at $O(Q^{7/4})$ relative to leading. The enhanced importance of the short-distance contact current is borne out in explicit calculations of $np \to d\gamma$. It is even more noticeable in calculations of $nd \to t\gamma$ and $n^3\mathrm{He} \to {}^4\mathrm{He}\gamma$ [42], where suppression of the one-body piece of the matrix element renders the relative importance of different two-body currents more transparent.

This enhancement by 1.25 powers relative to the NDA estimate will affect any short-distance operator that mediates a $^3S_1 \leftrightarrow {}^1S_0$ transition. In particular, it also occurs for the solar-fusion process $pp \to de^+\nu_e$ and for related processes (e.g., muon capture) that proceed via the Gamow-Teller operator in the NN system. As in the case of threshold capture, this enhancement also affects the relative importance of short-distance pieces of two-nucleon operators when the NN system is embedded in a three- or four-nucleon system that undergoes a weak transition (cf. Refs. [17–20]). We review our results for NN system processes in Table I.

This enhancement would be even more dramatic for reactions that involved the transition $^1S_0 \to {}^1S_0$, since there the effect of the short-distance operator increases by two full orders. (The χEFT power counting for short-distance operators becomes the pionless EFT counting in this channel.) Such transitions occur inside, e.g., ^{3}He, when electrons scatter from that nucleus. The analysis of Ref. [16] could be revisited in light of this finding, to see if an improved description of the trinucleon EM form factors results when the anomalous dimensions of short-distance operators are accounted for.

We stress that RG invariance means that the modifications to the power counting we discuss here are, to a significant extent, independent of the details of the cutoff function or the numerical value of the cutoff. In particular, in all contemporary implementations of χEFT the $^1S_0$ channel at LO is dominated by the short-distance potential, and so the wave function $\psi \sim 1/r$ for distances between the breakdown scale and the cutoff. In the $S = 1$ channels the situation is more complicated, since the wave functions u and w do not seem to behave as $r^{3/4}$ for the distances at which contemporary χEFT potentials are regulated [43–45]. However, a more careful analysis [35] shows that this $r^{3/4}$ behavior is the first term in an expansion for u and w that converges up to at least 2 fm, well above the regulator range used in these potentials. Because of this, the scaling derived here should be relevant for contemporary calculations. The extent to which the corrections to u and w computed in Ref. [35] modify

TABLE I. Power counting of contact-range currents for some observables of interest involving the deuteron and/or $^1S_0$ NN state. (Note that the order given for the dominant one-body effect pertains to χEFT and not to the pionless EFT.) The reactions we are considering are electron-deuteron scattering, radiative neutron capture by protons, and proton-proton fusion. The observables we list are the squared deuteron electromagnetic radius (r^2_{em}), the deuteron magnetic dipole (μ_d) and electric quadrupole moments (Q_d), and the $M1$ matrix element for neutron capture and proton-proton fusion We also list the schematic form of the lowest order two-body contact-range current operator that contributes to each one of these observables, expressed in the plane-wave basis, where $\mathbf{q}$ is the momentum of the external probe.

Process	Matrix element	$1B$	$2B$ ($\mathcal{O}_C$)	$2B$ (NDA)	$2B$ ($\not\pi$)	$2B$ (π)
$de \to de$	r^2_{em}	LO (Q^0)	$D(\Lambda)\mathbf{q}^2$	N^{5}LO (Q^5)	N^{3}LO (Q^3)	N$^{9/2}$LO ($Q^{9/2}$)
	μ_d	LO (Q^1)	$M(\Lambda)\vec{\beta} \times \mathbf{q}$	N^{3}LO (Q^4)	NLO (Q^2)	N$^{5/2}$LO ($Q^{7/2}$)
	Q_d	LO (Q^0)	$Q(\Lambda)\underset{\sim}{T}_2(\mathbf{q})$	N^{3}LO (Q^5)	NLO (Q^3)	N$^{5/2}$LO ($Q^{4.5}$)
$np \to d\gamma$	$M1$	LO (Q^1)	$M(\Lambda)\vec{\beta} \times \mathbf{q}$	N^{3}LO (Q^4)	NLO (Q^2)	N$^{7/4}$LO ($Q^{11/4}$)
$pp \to de^-\bar{\nu}_e$	$M1$	LO (Q^0)	$A(\Lambda)\vec{\beta}$	N^{3}LO (Q^3)	NLO (Q^1)	N$^{7/4}$LO ($Q^{7/4}$)

the details of the RG flow of short-distance operators is a subject for future work.

Another avenue for future work is to use the numerical behavior of three- (or higher-) body wave functions at short distance to extend the arguments presented here so as to derive the modifications of the NDA power counting for short-range $3N$, $4N$ operators. This exemplifies the power of RG arguments in nuclear EFT [46–48]. In this work, the principle of RG invariance, applied to NN matrix elements of electroweak currents, showed the necessity of modifying the counting of short-range operators in nuclear EFT to account for anomalous dimensions. The singular nature of the potentials that bind nuclei in both χEFT and pionless EFT makes those anomalous dimensions negative, with the result that NDA underestimates how important such operators are. This implies a pressing need for revised χEFT calculations of electron-deuteron scattering, trinucleon form factors, threshold radiative capture, and weak reactions in few-nucleon systems, for such enhancements could have important consequences in observables.

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