Excitation of Flow-Stabilized Resistive Wall Mode by Coupling with Stable Eigenmodes in Tokamaks

Nobuyuki Aiba
Japan Atomic Energy Agency, 2-166, Omotedate, Rokkasho, Aomori 039-3212, Japan

Makoto Hirota
Tohoku University, 2-1-1, Katahira, Aoba-ku, Sendai, Miyagi 980-8577, Japan
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In a rotating toroidal plasma surrounded by a resistive wall, it is shown that linear MHD instabilities can be excited by couplings between the resistive wall mode (RWM) and stable ideal MHD modes. In particular, it is shown that the RWM can couple not only with stable external kink modes but also with Alfvén eigenmodes that are ordinarily in the stable continuum of a toroidal plasma. The RWM growth rate is shown to peak whenever the Doppler shift caused by the plasma rotation cancels the frequency of an ideal MHD mode, so that the mode appears to have zero frequency in the laboratory frame. At these values of the rotation frequency, the RWM can overcome the stabilizing effects of plasma rotation, continuum damping, and ion Landau damping.

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In order to realize tokamak-type fusion reactors, it is necessary to stabilize magnetohydrodynamic (MHD) instabilities that cause disruptive events and terminate the plasma discharge. In particular, it is crucial to stabilize the ideal kink-ballooning mode by surrounding the plasma with a conducting wall. Even when the ideal mode is stabilized, however, the finite electric conductivity of any realistic wall gives rise to another instability known as the resistive wall mode (RWM). Although RWMs grow at a much slower rate than ideal kink-ballooning modes, they do trigger disruptive events, eventually. It is important to understand the physics of RWM stabilization in order to produce a steady-state high-β plasma, where β is the ratio of plasma thermal pressure to magnetic pressure.

Numerical studies have shown that RWMs in tokamaks can be stabilized by toroidal plasma rotation in combination with energy dissipation mechanisms [1]. Such rotational stabilization has been confirmed in many experiments [2–4]. These observations have motivated much research activity with the goal of identifying the key energy dissipation mechanisms that need to be taken into account in order to make precise predictions for the minimum rotation frequency required to stabilize RWMs. For example, recent theoretical and numerical studies have shown that kinetic effects play an important role [5,6].

On the other hand, experiments have also shown that MHD instabilities are occasionally capable of terminating a discharge disruptively, even when the plasma rotation frequency should have been sufficient to stabilize the RWMs [7,8]. These experimental results imply that another MHD mode becomes unstable in a rotating tokamak plasma. Such a destabilization has also been reported in numerical studies [9,10], but the underlying physics remained unclear.

The connection between ordinarily stable external kink modes and the RWM has already been made in a rotating cylindrical plasma [11,12]. When a resistive wall is located near the plasma surface, not only the RWM but also stable external kink modes (with frequencies $\pm\omega_{\mathrm{kink}}$ in a static plasma) exist. Since these stable kink modes are ideal modes, their frequencies are Doppler shifted by plasma rotation. When the rotation frequency is increased, mode frequencies successively cross through zero in the laboratory frame. Whenever this occurs, the RWM is strongly destabilized by coupling with the Doppler-shifted kink mode. After the Doppler-shifted mode frequency passes through zero, the RWM is stabilized since the sign of its energy switches to positive [12]. When this occurs, however, the kink mode attains negative energy and thus becomes weakly unstable due to finite wall resistivity. The destabilization of negative-energy modes by wall resistivity is a generic effect that applies to any ordinarily stable ideal MHD modes [13]. It should be noted, however, that the negative-energy kink mode can be stabilized when its mode structure and frequency overlaps with sound and shear Alfvén continua [14,15]. Other damping effects, such as ion Landau damping, also have a stabilizing influence.

In the present work, using numerical methods, the excitation of MHD instabilities through couplings between RWMs and stable MHD modes is demonstrated for the first time for a rotating toroidal plasma. These instabilities exist even after the plasma rotation frequency enters a regime that is stable with respect to RWMs. The plasma stability is analyzed with the linear MHD stability code MINERVA/RWMaC [16,17]. MINERVA [18] solves the Frieman-Rotenberg equation [19], including a parallel sound wave damping force $\boldsymbol{F}_{\mathrm{SD}}$ that represents ion Landau damping [1,20]:

$$\rho\frac{\partial^2\boldsymbol{\xi}}{\partial t^2}+2\rho(\boldsymbol{u}\cdot\nabla)\frac{\partial\boldsymbol{\xi}}{\partial t}=\boldsymbol{F}(\boldsymbol{\xi})+\boldsymbol{F}_{\rm SD}(v_\parallel)+\frac{\partial D_w}{\partial t},\quad (1)$$

$$\boldsymbol{F}_{\rm SD}=-\kappa_\parallel\sqrt{\pi}|k_\parallel v_{{\rm th},i}|\rho v_\parallel\boldsymbol{b},\quad (2)$$

$$v_\parallel=\frac{\partial\xi_\parallel}{\partial t}+(\boldsymbol{u}\cdot\nabla)\boldsymbol{\xi}|_\parallel.\quad (3)$$

Here, ρ is the plasma mass density, $\boldsymbol{\xi}$ is the plasma displacement, $\boldsymbol{u}$ is the plasma rotation velocity, and $\boldsymbol{F}$ is the force operator including rotation effects. The last term in Eq. (1) with D_w is the contribution from the resistive wall, which is calculated with RWMaC. The parameters in Eq. (2) are the parallel wave number $k_\parallel=(n-m/q)/R$, the ion thermal velocity $v_{{\rm th},i}$, the strength parameter $\kappa_\parallel$, and the unit vector in the direction parallel to magnetic field $\boldsymbol{b}$, where n and m are the toroidal and poloidal mode numbers, q is the safety factor, and R is the major radius. Note that $\boldsymbol{F}_{\rm SD}$ was originally derived for a cylindrical plasma [21], and is implemented here for a torus plasma with $\kappa_\parallel$.

The parameters of the numerically constructed equilibrium are as follows. The cross section of the last closed magnetic surface is circular with minor radius a. The aspect ratio is $A=R/a=3.0$, with R evaluated at the axis $R_0=1.0$ [m]. The poloidal beta value is $\beta_p=0.3$, the toroidal magnetic field on axis is $B_{t0}=1.0$ [T], and the plasma current is $I_p=0.6$ [MA]. An ideal external kink mode becomes marginally stable when an ideal conducting wall is located at radius $d/a=1.25$, where d is the wall minor radius. Hence, a resistive wall with wall diffusion time $\tau_{\rm wall}=1.6\times10^{-2}$ is located at $d/a=1.2$. Figure 1 shows the profile of q, and the spectra of shear Alfvén continua; the ion (deuterium) number density is taken to be constant with $n_i=1\times10^{20}[1/{\rm m}^3]$. For a static equilibrium, there are three eigenmodes in the gap between the $m=2$ and $m=3$ shear Alfvén continua: one is the external kink mode with $\omega/\omega_{A0}=0.155$, and the other two are the reversed shear Alfvén eigenmode (RSAE) [22] and the global Alfvén eigenmode (GAE) [23] with $\omega/\omega_{A0}=0.085$ and 0.310, where ω_{A0} is the toroidal Alfvén frequency on axis.

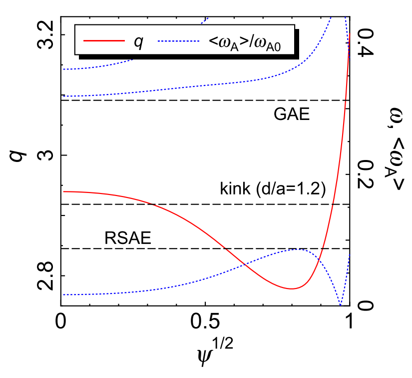

FIG. 1 (color online). Profile of q and spectra of shear Alfvén continua; ψ is the poloidal flux normalized to be 0 (1) at the axis (surface), $\langle A\rangle$ is the value of A averaged on flux surface, and the frequency is normalized with ω_{A0}. There are three eigenmodes in the gap between the $m=2$ and $m=3$ shear Alfvén continua: the RSAE, the external kink mode, and the GAE with $\omega=0.085$, 0.155, and 0.310, respectively.

The plasma is rotated rigidly in the toroidal direction and the RWM stability is analyzed for a range of rotation frequencies $\Omega_{\phi0}$. In the following, the frequencies ω and $\Omega_{\phi0}$ are normalized by ω_{A0}. Note that the effect of the centrifugal force on the equilibrium is neglected for simplicity. As a first step, the RWM stability is analyzed in the absence of sound waves by letting $\Gamma=0.0$, where Γ is the ratio of specific heats. The result shown in Fig. 2 indicates that the growth rate γ of the RWM decreases when $\Omega_{\phi0}$ is increased up to 0.07. This is due to the well-known stabilizing effect of toroidal rotation. However, for $\Omega_{\phi0}>0.07$, one can see that γ exhibits three local maxima near $\Omega_{\phi0}\simeq0.085$, 0.155, and 0.310, which correspond to the frequencies of the RSAE, the external kink mode and the GAE, respectively. As shown in Fig. 3, the radial structure of the unstable mode at each of these local maxima is nearly identical to that of the corresponding stable eigenmode whose frequency is Doppler shifted to zero. In addition, the dependence of the mode frequency on $\Omega_{\phi0}$ in Fig. 2 shows that the unstable modes are Doppler shifted when $\Omega_{\phi0}$ becomes larger than the respective frequency at which γ has a local maximum. These results prove that the MHD instabilities are caused by couplings between RWMs and stable MHD modes.

The new and remarkable finding from our analysis of a toroidal plasma is that RWMs can not only couple to external kink modes but also Alfvén eigenmodes. In contrast, in a cylindrical plasma, RWMs couple only with external kink modes [11,12]. These results imply that a RWM may couple to many other modes as well: for example, RSAE/GAE as in Fig. 2, toroidicity-induced Alfvén eigenmodes [24], beta-induced Alfvén eigenmodes

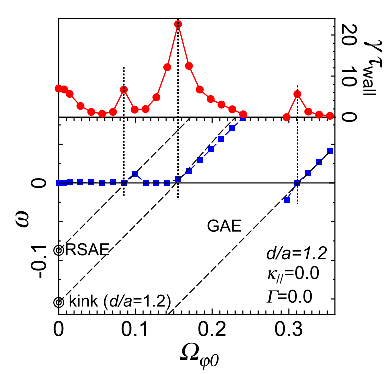

FIG. 2 (color online). Dependencies of γ and ω on $\Omega_{\phi0}$. There are three local maxima of γ near $\Omega_{\phi0}\simeq0.085$, 0.155, and 0.310, which correspond to the frequencies of the RSAE, the external kink mode, and the GAE. In addition, the unstable mode is Doppler shifted after $\Omega_{\phi0}$ passes the above frequencies.

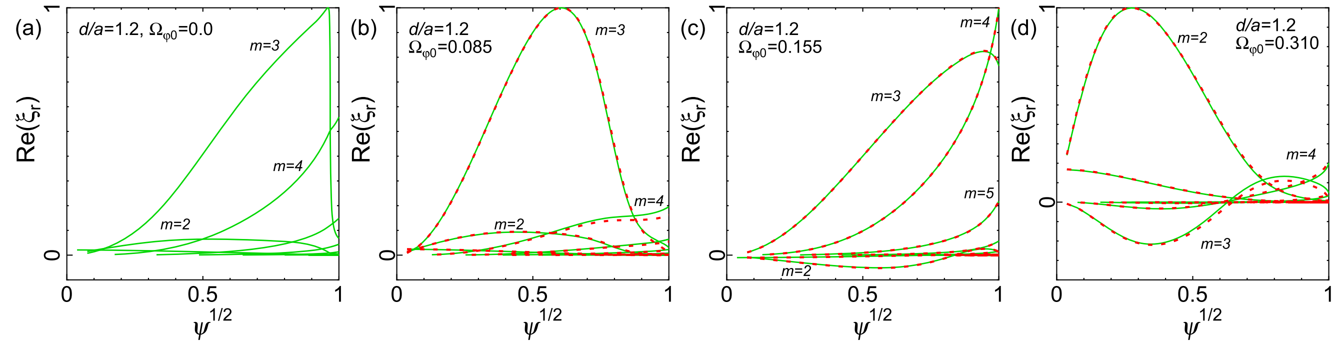

FIG. 3 (color online). Eigenfunctions of the modes which are destabilized when $\Omega_{\phi 0} =$ (a) 0.0, (b) 0.085, (c) 0.155, and (d) 0.310 (green lines). The red dotted lines in (b), (c), and (d), respectively, show the eigenfunctions of the RSAE, the external kink mode and the GAE, which are stable when the plasma is static ($\omega|_{\Omega_{\phi 0}=0} = \Omega_{\phi 0}$). Clearly, each of these ideal MHD modes is identical to the respective unstable mode in (b), (c), and (d).

[25], and so on. Moreover, note that these couplings lead to the successive excitation of different MHD instabilities as the rotation frequency increases, even after the RWM is fully stabilized.

In a conventional aspect ratio $A \simeq 3$ tokamak with $R_0 \geq 1.0$, the maximum toroidal rotation frequency is usually less than $0.05\omega_{A0}$. In this range, however, sound wave damping has a strongly stabilizing effect on RWMs [20,26]. In order to determine whether or not the RWM excitation is effective at such a low value of the rotation frequency, RWM stability is analyzed numerically in a case where the minimum q value is decreased to 2.07. The q profile and the spectra of the shear Alfvén and sound continua are shown in Fig. 4. The lower value of the minimum of q reduces the frequency of the RSAE, which is expected to exist near the local minimum of the $m = 2$ shear Alfvén continuum. In addition, plasma compressibility is included ($\Gamma = 5/3$) in order to take into account eigenmodes related to sound wave continua.

The shape and the equilibrium parameters of the new equilibrium are the same as those of the equilibrium analyzed above, except for the q profile and the value of

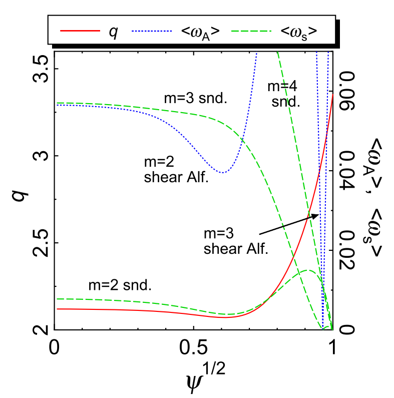

FIG. 4 (color online). Profile of q and spectra of shear Alfvén and sound continua in a plasma with weakly reversed shear. The minimum of q is 2.07.

$\beta_p = 0.7$. The q profile was changed by adjusting only the plasma current profile. The impact of the sound wave damping force on plasma stability is investigated by varying $\kappa_{\parallel}$ between 0.0 (ideal MHD) and 0.5. Relatively low values of $\kappa_{\parallel}$ are used because the strength of the damping force in the toroidal case is weak compared with the cylindrical case when the plasma rotation frequency is lower than the sound wave frequency [26,27]. We note that the ideal mode is marginally stable when an ideally conducting wall is located at $d/a = 1.38$, so that $d/a = 1.3$ is chosen as a suitable resistive wall position for the RWM stability analysis.

Figure 5 shows the dependencies of γ and ω on the rigid rotation frequency $\Omega_{\phi 0}$. First, let us focus on the result of the ideal MHD case ($\kappa_{\parallel} = 0.0$). In this case, the dependence of γ on $\Omega_{\phi 0}$ shows four local maxima in the range $\Omega_{\phi 0} \leq 0.05$. The rotation frequencies at these maxima seem to correspond to the two local maxima of the $m = 2$ sound continuum, the crossing point between the $m = 2$ sound

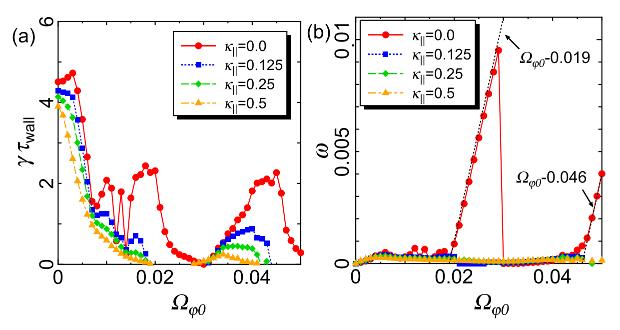

FIG. 5 (color online). Dependencies of (a) γ and (b) ω on $\Omega_{\phi 0}$ for several values of $\kappa_{\parallel}$. In the ideal MHD case ($\kappa_{\parallel} = 0.0$), there are four local maxima of γ near $\Omega_{\phi 0} \simeq 0.01$, 0.013, 0.019, and 0.046, and the unstable modes are Doppler shifted. By increasing $\kappa_{\parallel}$, the RWM is stabilized when $\Omega_{\phi 0} > 0.019$, but an MHD mode is again destabilized when $\Omega_{\phi 0} > 0.03$ as in the ideal MHD case. All the MHD modes that are destabilized for $\kappa_{\parallel} \neq 0.0$ are regarded as RWMs because these modes are hardly Doppler shifted.

continuum and the $m = 3$ shear Alfvén continuum, and the local minimum of the $m = 2$ shear Alfvén continuum, respectively. It is well known that discrete modes can exist near local minima or maxima and intersections of the continua, and such ordinarily stable MHD modes may be responsible for the observed excitations. In particular, the unstable modes in the ranges $0.019 < \Omega_{\phi 0} < 0.03$ and $\Omega_{\phi 0} \geq 0.046$ are negative-energy ideal modes because the mode frequencies are Doppler shifted. This indicates that the RWM indeed couples with these ideal modes, and can make them unstable.

Next, we consider the effect of the sound wave damping force ($\kappa_{\parallel} \neq 0$) on the instability excited by the coupling. As shown in Fig. 5, the growth rate γ of the RWM decreases monotonically as $\kappa_{\parallel}$ increases, and the RWM is fully stabilized in the range $0.019 \leq \Omega_{\phi 0} \leq 0.03$. However, the unstable mode still remains for $\Omega_{\phi 0} > 0.03$, even when $\kappa_{\parallel}$ is increased to 0.5. This result demonstrates that the MHD instabilities due to couplings between RWMs and stable MHD modes can be excited in the low-rotation-frequency range, where they are subject to not only continuum damping but also ion Landau damping.

It is also interesting to note that, in Fig. 5(a), the plasma is stable in the ranges $0.019 \leq \Omega_{\phi 0} \leq 0.03$ and $\Omega_{\phi 0} \geq 0.046$ when $\kappa_{\parallel} \neq 0.0$, whereas the negative-energy ideal modes are unstable in the same ranges when $\kappa_{\parallel} = 0.0$. In addition, the dependence of ω on $\Omega_{\phi 0}$ in Fig. 5(b) shows that the unstable modes are not Doppler shifted when $\kappa_{\parallel} \neq 0.0$. These results indicate that the sound wave damping force effectively stabilizes the negative-energy ideal modes that have nonzero frequencies.

In summary, we have demonstrated the existence of a new type of MHD instabilities in toroidally rotating tokamak plasmas. The instabilities are a flow-stabilized RWM and a negative energy ideal MHD mode which are excited through the coupling between a RWM and a stable ideal MHD mode. This coupling becomes effective when the frequency of the stable ideal mode in the laboratory frame becomes zero due to the Doppler shift caused by the rotation. Unlike in a cylindrical plasma, the RWM in a torus can couple with not only the external kink mode but also various shear Alfvénic and slow magnetosonic eigenmodes. As a consequence, the RWM is excited at various values of the plasma rotation frequency, overcoming stabilizing effects associated with continuum damping and ion Landau damping. However, our numerical results also show that ion Landau damping is capable of suppressing negative-energy ideal modes with nonzero frequencies, thus, opening windows of stability with respect to RWMs. We remark that excitations of RWMs through couplings with stable Alfvén eigenmodes also occur below the no-wall β limit, where RWMs are stable even for a static plasma. In this case, the couplings to Alfvén eigenmodes occur in the same way as coupling with an external kinks mode that were discussed in Refs. [11,12].

In order to check whether the RWM excitation mechanism described above plays a role in experiments, it is necessary to take into account other damping mechanisms, which are not included in our model. For instance, it has been reported that resonant interactions with energetic particles can stabilize RWMs [9,28], and the effect of such interactions on the excitation mechanism described in this Letter should be clarified. As discussed in [9], the resonant interaction of RWMs with energetic particles depends on the relationship between the rotation frequency and the characteristic frequencies of energetic particles, and this interaction becomes ineffective in some cases. Since the RWM is basically an MHD mode, the verification of its stability in the MHD model should be of primary importance, and the RWM excitation mechanism described here may constitute a crucial component of RWM physics that was not considered until now.

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