Precision Determination of the Cabibbo-Kobayashi-Maskawa Element V_{cb}

Andrea Alberti, Paolo Gambino, and Kristopher J. Healey
Dipartimento di Fisica, Università di Torino and INFN, Torino, 10125 Torino, Italy

Soumitra Nandi
Physics Department, Indian Institute of Technology, Guwahati 781 039, India

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We extract the magnitude of the Cabibbo-Kobayashi-Maskawa matrix element $V_{\rm cb}$ and the most relevant parameters of the heavy quark expansion from data of inclusive semileptonic B decays. Our calculation includes the recently computed $O(\alpha_s \Lambda^2_{\rm QCD}/m_b^2)$ corrections and a careful estimate of the residual theoretical uncertainty. Using a recent determination of the charm quark mass, we obtain $|V_{\rm cb}| = (42.21 \pm 0.78) \times 10^{-3}$ and $m_b^{\rm kin}(1~{\rm GeV}) = (4.553 \pm 0.020)$ GeV.

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Introduction.—The Cabibbo-Kobayashi-Maskawa (CKM) mechanism of quark flavor violation is one of the main components of the Standard Model (SM) of fundamental interactions [1,2]. It accommodates very well all of the observed CP violation, as well as the flavor changing phenomena studied at kaon experiments, the B factories, and at high-energy colliders like the LHC (see [3,4] for recent reviews). The 3×3 unitary CKM matrix, which parametrizes flavor violation in this context, has only four independent parameters. While they are strongly constrained by present data, any improvement would be welcome as it would sharpen our tools for future tests of the SM.

In particular, more precise measurements of $|V_{\rm cb}|$, the CKM element controlling charged current $b \leftrightarrow c$ transitions, would crucially help the search for new physics in rare decays, which requires accurate SM predictions. Indeed, the present ~2% error on this single CKM element represents the dominant uncertainty on the SM prediction of important flavor-changing neutral current decays such as $B_s \to \mu^+\mu^-$ [5], $K^+ \to \pi^+\nu\bar{\nu}$, and $K_L \to \pi^0\nu\bar{\nu}$ [6], as well as of the CP violation parameter ε_K [7].

Direct information on $|V_{\rm cb}|$ can be obtained from inclusive and exclusive semileptonic B decays to charmed hadrons, which are subject to different theoretical and experimental systematics. In the first case, the operator product expansion (OPE) allows us to describe the relevant nonperturbative physics in terms of a small number of parameters that can be extracted from experiment. In the case of the exclusive decays $B \to D^{(*)}\ell\bar{\nu}$, the form factors have to be computed by nonperturbative methods, e.g., lattice QCD. The most precise recent results of each method are

$$|V_{\rm cb}| = (42.42 \pm 0.86) \times 10^{-3} \quad (1)$$

from a global fit to inclusive semileptonic moments [8], and

$$|V_{\rm cb}| = (39.04 \pm 0.49_{\rm exp} \pm 0.53_{\rm lat} \pm 0.19_{\rm QED}) \times 10^{-3} \quad (2)$$

from an unquenched lattice QCD calculation of the zero recoil form factor of $B \to D^* l\nu$ by the Fermilab-MILC Collaboration [9]. They disagree by 3σ, which remains a long-standing tension. There also exist less precise determinations of $|V_{\rm cb}|$ based on heavy quark sum rules and the decay $B \to D l\nu$ (see [4] for a review).

It is also possible to determine $|V_{\rm cb}|$ indirectly, using the CKM unitarity relations together with CP violation and flavor data, excluding the above direct information: SM analyses by the UTfit and CKMFitter collaborations give $(42.05 \pm 0.65) \times 10^{-3}$ [10] and $(41.4^{+2.4}_{-1.4}) \times 10^{-3}$ [11], which are both closer to the inclusive value of Eq. (1).

In principle, the lingering discrepancy between the values of $|V_{\rm cb}|$ extracted from inclusive decays and from $B \to D^* l\nu$ could be ascribed to physics beyond the SM, as the $B \to D^*$ transition is sensitive only to the axial-vector component of the $V - A$ charged weak current. However, the new physics effect should be sizable (8%) and would require new interactions ruled out by electroweak constraints on the effective $Zb\bar{b}$ vertex [12]. The most likely explanation of the discrepancy between Eqs. (1) and (2) is therefore a problem in the theoretical or experimental analyses of semileptonic decays.

In this Letter we focus on the inclusive extraction of $|V_{\rm cb}|$, including all contributions of $O(\alpha_s \Lambda^2_{\rm QCD}/m_b^2)$, whose calculation has recently been completed [13–15], and we discuss how this improvement affects the results.

The calculation.—Let us briefly review the calculation of the quantities that enter the inclusive analysis. The OPE allows us to write sufficiently inclusive quantities (typically the width and the first few moments of kinematic distributions) as double series in α_s and $\Lambda_{\rm QCD}/m_b$. The expansion in powers of the heavy quark mass starts at $O(1/m_b^2)$ [16–19] and involves the B-meson expectation values of local

TABLE I. Coefficients of (3) for $m_b^{\rm kin}(1\ {\rm GeV}) = 4.55$ GeV and with the charm mass in the kinetic scheme, $m_c^{kin}(1{\rm GeV}) = 1.091$ GeV (first row), and in the $\overline{\rm MS}$ scheme, $\overline{m}_c(3\ {\rm GeV}) = 0.986$ GeV (second row) and $\overline{m}_c(2\ {\rm GeV}) = 1.091$ GeV (third row).

$a^{(1)}$	$a^{(2,\beta_0)}$	$a^{(2)}$	$p^{(1)}$	$g^{(0)}$	$g^{(1)}$	$d^{(0)}$
−0.95	−0.47	0.71	0.99	−1.91	−3.51	−16.6
−1.66	−0.43	−2.04	1.35	−1.84	−2.98	−17.5
−1.24	−0.28	0.01	1.14	−1.91	−3.23	−16.6

operators of growing dimension. These nonperturbative parameters can be constrained from the measured values of the normalized moments of the lepton energy and invariant hadronic mass distributions in $B \to X_c \ell \nu$ decays:

$$\langle E_\ell^n \rangle = \frac{1}{\Gamma_{E_\ell > E_{\rm cut}}} \int_{E_\ell > E_{\rm cut}} E_\ell^n \frac{d\Gamma}{dE_\ell} dE_\ell,$$

$$\langle m_X^{2n} \rangle = \frac{1}{\Gamma_{E_\ell > E_{\rm cut}}} \int_{E_\ell > E_{\rm cut}} m_X^{2n} \frac{d\Gamma}{dm_X^2} dm_X^2,$$

where E_ℓ is the lepton energy, m_X^2 the invariant hadronic squared mass, and $E_{\rm cut}$ an experimental threshold on the lepton energy applied by some of the experiments. Since the physical information of moments of the same type is highly correlated, for $n > 1$ it is better to employ *central* moments, computed relative to $\langle E_\ell \rangle$ and $\langle m_X^2 \rangle$. The information on the nonperturbative parameters obtained from a fit to the moments enables us to extract $|V_{\rm cb}|$ from the total semileptonic width [20–22].

The expansion for the total semileptonic width is

$$\Gamma_{\rm sl} = \Gamma_0 \left[1 + a^{(1)} \frac{\alpha_s(m_b)}{\pi} + a^{(2,\beta_0)} \beta_0 \left(\frac{\alpha_s}{\pi}\right)^2 + a^{(2)} \left(\frac{\alpha_s}{\pi}\right)^2 + \left(-\frac{1}{2} + p^{(1)} \frac{\alpha_s}{\pi}\right) \frac{\mu_\pi^2}{m_b^2} + \left(g^{(0)} + g^{(1)} \frac{\alpha_s}{\pi}\right) \frac{\mu_G^2(m_b)}{m_b^2} + d^{(0)} \frac{\rho_D^3}{m_b^3} - g^{(0)} \frac{\rho_{\rm LS}^3}{m_b^3} + \text{higher orders} \right], \tag{3}$$

where $\Gamma_0 = A_{\rm ew} |V_{\rm cb}^2| G_F^2 m_b^5 (1 - 8\rho + 8\rho^3 - \rho^4 - 12\rho^2 \ln\rho)/192\pi^3$ is the tree-level free quark decay width, $\rho = m_c^2/m_b^2$, and $A_{\rm ew} = 1.014$ the leading electroweak correction. We have split the α_s^2 coefficient into a Brodsky-Lepage-Mackenzie piece proportional to $\beta_0 = 9$ (with three massless active quark flavors) and a remainder. The expansions for the moments have the same structure. The parameters $\mu_\pi^2, \mu_G^2, \rho_D^3, \rho_{\rm LS}^3$ are the B-meson expectation values of the relevant dimension 5 and 6 local operators.

In Eq. (3) and in the calculation of all the moments, we have included the complete one- and two-loop perturbative corrections [23–28], as well as $1/m_b^{2,3}$ power corrections [16–18,29]. We neglect contributions of order $1/m_b^4$ and $1/m_Q^5$ [30], which appear to lead to a very small shift in $|V_{\rm cb}|$, but we include for the first time the perturbative corrections to the leading power-suppressed contributions [13–15] to the width (see also [31] for the limit $m_c \to 0$) and to all the moments [32].

The coefficients $a^{(i)}, g^{(i)}, p^{(1)}, d^{(0)}$ in Eq. (3) are functions of ρ and of various unphysical scales, such as the one of α_s. They are given in Table 1 for specific values of the quark masses. We use the kinetic scheme [33] with a cutoff at 1 GeV for m_b and the OPE parameters and three different options for the charm mass.

The global fit.—The available measurements of the semileptonic moments [4] and the recent, precise determinations of the heavy quark masses significantly constrain the parameters entering Eq. (3), making a determination of $|V_{\rm cb}|$ whose uncertainty is dominated by our ignorance of higher order effects possible. Duality violation effects can be constrained *a posteriori*, by checking whether the OPE predictions fit the experimental data, but this again depends on precise OPE predictions.

We perform a fit to the semileptonic data listed in Table 1 of Ref. [8] with $\alpha_s(4.6\ {\rm GeV}) = 0.22$ and employ a few additional inputs. Since the moments are mostly sensitive to $\approx m_b - 0.8 m_c$, it is essential to include information on at least one of the heavy quark masses.

TABLE II. Results of the global fit in our default scenario. All parameters are expressed in GeV at the appropriate power and all, except m_c, in the kinetic scheme at $\mu = 1$ GeV. The first and second rows give central values and uncertainties; the correlation matrix follows.

$m_b^{\rm kin}$	$\overline{m}_c(3\ {\rm GeV})$	μ_π^2	ρ_D^3	μ_G^2	$\rho_{\rm LS}^3$	${\rm BR}_{c\ell\nu}$	$10^3 \vert V_{\rm cb} \vert$
4.553	0.987	0.465	0.170	0.332	−0.150	10.65	42.21
0.020	0.013	0.068	0.038	0.062	0.096	0.16	0.78
1	0.508	−0.099	0.142	0.596	−0.173	−0.075	−0.418
	1	−0.013	0.002	−0.023	0.007	0.016	−0.032
		1	0.711	−0.025	0.041	0.144	0.340
			1	−0.064	−0.154	0.065	0.201
				1	−0.032	−0.022	−0.252
					1	−0.017	0.013
						1	0.483
							1

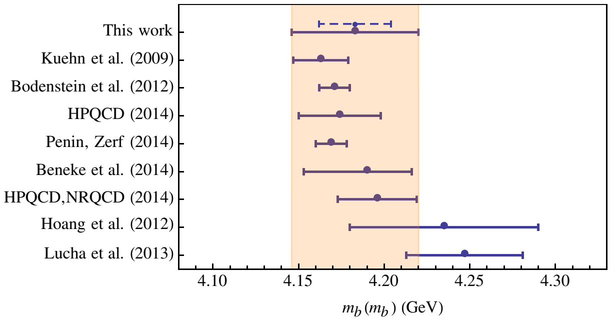

FIG. 1 (color online). Comparison of different $\bar{m}_b(\bar{m}_b)$ determinations [36,38–43]. The dashed line denotes the error before scheme conversion.

Because of its smaller absolute uncertainty, m_c is preferable. Among recent m_c determinations [34–36], we choose $\overline{m}_c(3\ \text{GeV}) = 0.986(13)$ GeV [34], although we will discuss the inclusion of m_b determinations as well. We also include a loose bound on the chromomagnetic expectation value from the B hyperfine splitting, $\mu_G^2(4.6\text{GeV}) = 0.35(7)\text{GeV}^2$. Finally, as all observables depend very weakly on ρ_{LS}^3, we use the heavy quark sum rule constraint $\rho_{LS}^3 = -0.15(10)\ \text{GeV}^3$.

As should be clear from the above discussion on higher orders in the OPE, the estimate of theoretical errors and of their correlation is crucial. We follow the strategy of [8,20] for theoretical uncertainties, updating it because of the new corrections that we include. In particular, we assign an irreducible uncertainty of 8 MeV to $m_{c,b}$, and vary $\alpha_s(m_b)$ by ± 0.018, μ_π^2 and μ_G^2 by $\pm 7\%$, and ρ_D^3 and ρ_{LS}^3 by $\pm 30\%$. This implies a total theoretical uncertainty between 2.0% and 2.6% in the semileptonic width, depending on the scheme. For the theory correlations we adopt scenario D of Ref. [8]; i.e., we assume no correlation between different *central* moments and a correlation between the same moment measured at different E_{cut}, depending on the proximity of the cuts and their magnitude. In the extraction of $|V_{cb}|$ we use the latest isospin average $\tau_B = 1.579(5)$ ps [37].

In Table II we show the results of the fit and the correlation matrix among the fitted parameters. With respect to the default fit of Ref. [8], $|V_{cb}|$ is reduced by 0.5% [see Eq. (1)], m_b^{kin} is increased by about 10 MeV, and μ_π^2 and ρ_D^3 are both shifted upward by about 10%. As the method and inputs are the same as in Ref. [8], except for the value of τ_B which is only reflected in a tiny $+0.1\%$ shift in $|V_{cb}|$, the difference can be mostly attributed to the new corrections. Because of smaller theoretical errors, the final uncertainties are slightly reduced. The χ^2/d.o.f. is very good, about 0.4.

It is interesting to compare the b mass extracted from the fit with other recent determinations, generally expressed in terms of $\overline{m}_b(\overline{m}_b)$ in the $\overline{\text{MS}}$ scheme. This is shown in Fig. 1, after converting m_b^{kin} into $\overline{m}_b(\overline{m_b})$. The scheme conversion implies an additional ~30 MeV uncertainty [28], enlarging the final error to 37 MeV, because it is known only through $O(\alpha_s^2)$. Our result, $\bar{m}_b(\bar{m}_b) = 4.183(37)$ GeV, agrees well with those reported in the figure. The combination $m_b^{\text{kin}}(1\ \text{GeV})$–$0.85\overline{m}_c(3\ \text{GeV})$ is best determined to 3.714 ± 0.018 GeV.

Table III shows the results when the fit is performed with m_c in a different scheme or at a different scale with respect to our default fit of Table II. The results are remarkably consistent and very close to the default fit, with the only partial exception of m_b, which becomes 1σ higher when $\overline{m}_c(2\ \text{GeV})$ is used as input. Table III also reports the results of a fit with an additional constraint on m_b. Even the currently most precise m_b determinations are spoiled by the uncertainty due to the scheme conversion to m_b^{kin}. Because of this, and because of the large range of m_b values given in the literature, we prefer to avoid using a m_b constraint in our default fit.

Overall, the fit results depend little on the scale of α_s. This is shown in Fig. 2 for the default fit. $|V_{cb}|$ and m_b^{kin} increase by less than 0.5% if we perform the whole analysis using $\alpha_s(m_b/2)$, while μ_π^2 and, in general, the OPE parameters are slightly more sensitive. A similar behavior is observed for the fits in Table III. Figure 3 shows instead the μ_{kin} dependence of $|V_{cb}|$ in case (a), keeping the scales of m_b and m_c distinct. In all cases, the scheme and scale

TABLE III. Results of the fit in different scenarios: (a) with m_c in the kinetic scheme, $m_c^{\text{kin}} = 1.091(20)$ GeV from [34]; (b) in the $\overline{\text{MS}}$ scheme at a lower scale, with $\bar{m}_c(2\ \text{GeV}) = 1.091(14)$ GeV from [34]; (c) same as our default fit, with an additional constraint $m_b^{\text{kin}} = 4.533(32)$ GeV, derived from [34].

	m_b^{kin}	m_c	μ_π^2	ρ_D^3	μ_G^2	ρ_{LS}^3	$\text{BR}_{c\ell\nu}$	$10^3\lvert V_{cb}\rvert$
(a)	4.561	1.092	0.464	0.175	0.333	−0.146	10.66	42.04
	0.021	0.020	0.067	0.040	0.061	0.096	0.16	0.67
(b)	4.576	1.092	0.466	0.174	0.332	−0.146	10.66	42.01
	0.020	0.014	0.068	0.039	0.061	0.096	0.16	0.68
(c)	4.548	0.985	0.467	0.168	0.321	−0.146	10.66	42.31
	0.017	0.012	0.068	0.038	0.058	0.096	0.16	0.76

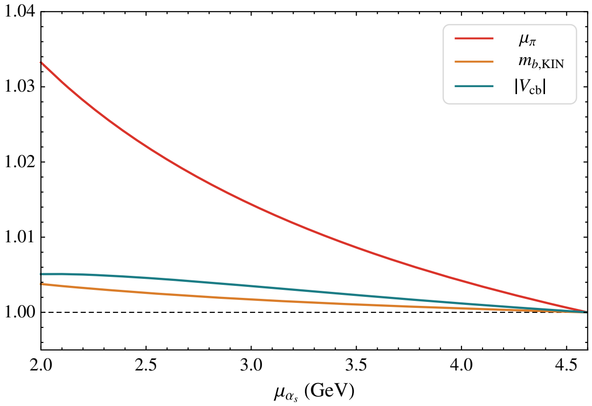

FIG. 2 (color online). Relative variation of the central values for $|V_{\text{cb}}|$, m_b^{kin}, and μ_π^2 on the scale of α_s in the default fit.

dependence confirms the size of theoretical errors employed in our analysis.

Finally, we update the value of the semileptonic phase space ratio C,

$$C = \left|\frac{V_{ub}}{V_{\text{cb}}}\right|^2 \frac{\Gamma[\bar{B} \to X_c e \bar{\nu}]}{\Gamma[\bar{B} \to X_u e \bar{\nu}]},$$

which is often used in the calculation of the branching ratio of radiative and rare semileptonic B decays, see [14] for details. Using the default fit and $\mu_{WA} = m_b/2$, we find $C = 0.574 \pm 0.008 \pm 0.014$, where the first uncertainty comes from the parameters determined in the fit and the second from unknown higher orders, estimated as explained above. Since the ratio C receives large perturbative corrections when it is expressed in terms of $\overline{m}_c(3\ \text{GeV})$ [8], we believe that using $\overline{m}_c(2\ \text{GeV})$ leads to a more reliable estimate. Including the m_b^{kin} mass constraint derived from [34] as well, we find

$$C = 0.568 \pm 0.007 \pm 0.010, \tag{4}$$

slightly higher but with a smaller error than the corresponding value in [8].

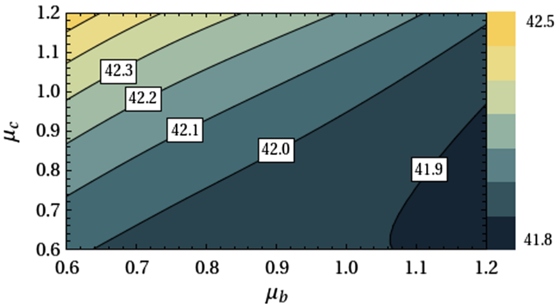

FIG. 3 (color online). Dependence of the $|V_{\text{cb}}|$ central value in fit (a) on the kinetic cutoff of the b and c masses.

Conclusion.—In summary, we have improved the inclusive determination of $|V_{\text{cb}}|$ through the inclusion of the complete $O(\alpha_s \Lambda_{\text{QCD}}^2/m_b^2)$ effects. Our final value,

$$|V_{\text{cb}}| = (42.21 \pm 0.78) \times 10^{-3}, \tag{5}$$

is compatible with previous analyses, but its uncertainty is slightly reduced thanks to the smaller theoretical errors. Equation (5) still differs at the 2.9σ level from Eq. (2). We find no sign of inconsistency in the inclusive analysis, and we adopt a conservative estimate of theory errors. The latter could be further reduced by a calculation of $O(\alpha_s \Lambda_{\text{QCD}}^3/m_b^3)$ contributions, as well as by a better understanding of higher power corrections (see [44]).

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