Testing Spontaneous Wave-Function Collapse Models on Classical Mechanical Oscillators

Lajos Diósi*
Wigner Research Center for Physics, H-1525 Budapest 114. P.O. Box 49, Hungary
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We show that the heating effect of spontaneous wave-function collapse models implies an experimentally significant increment $\Delta T_{\rm sp}$ of equilibrium temperature in a mechanical oscillator. The obtained new form $\Delta T_{\rm sp}$ is linear in the oscillator's relaxation time τ and independent of the mass. The oscillator can be in a classical thermal state, also the effect $\Delta T_{\rm sp}$ is classical for a wide range of frequencies and quality factors. We note that the test of $\Delta T_{\rm sp}$ does not necessitate quantum state monitoring just tomography. In both the gravity-related and the continuous spontaneous localization models the strong-effect edge of their parameter range can be challenged in existing experiments on classical oscillators. For the continuous spontaneous localization theory, the conjectured highest collapse rate parameter values become immediately constrained by evidences from current experiments on extreme slow-ring-down oscillators.

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Spontaneous collapse models [1] suggest that macroscopic superpositions—large spatial superpositions of quantum states of massive degrees of freedom—decay at (model dependent) universal rates. These models, the particular gravity-related (or DP) model [2–6] and the continuous spontaneous localization (CSL) model [7,8] predict the progressive violation of the quantum mechanical superposition principle for massive degrees of freedom. For atomic degrees of freedom this violation is irrelevant while for massive degrees of freedom it becomes significant though usually masked by the environmental noise. The preparation of macroscopic superpositions is extremely demanding, hence the direct experimental test of spontaneous collapse has not yet been achieved despite relentless efforts; see, e.g., [9–15] and [16,17] for the state of the art. Quite recently, Bahrami *et al.* [18] suggested a different approach, not requesting laboratory macroscopic superpositions. Nimmrichter *et al.* [19] discuss the optomechanical sensing of spontaneous momentum diffusion caused by collapse models. We further elucidate and simplify these considerations and come to new results. We emphasize that momentum diffusion is classical and this facilitates the mathematical treatment, theoretical insight, and experimental proposals. Currently available mechanical oscillators of extreme long ring-down time, e.g., in Ref. [20] by Matsumoto *et al.*, are immediately capable of sensing spontaneous heating if it exists with the strongest proposed rates.

Spontaneous collapse models are known [1] to impose spontaneous kinetic energy increase at constant rate proportional to the spontaneous collapse rate; see also [5]. This spontaneous heating is independent of the quantum state. It can be a classical state, it need not be a macroscopic superposition for being spontaneously heated.

While spontaneous collapse is a genuine quantum effect, spontaneous heating is not. This we exploit in our work: an elementary nonquantum calculation yields the spontaneous increment $\Delta T_{\rm sp}$ of the equilibrium temperature T of damped mechanical oscillators. Full quantum calculations can be safely replaced by classical calculations as long as the oscillator remains in the classical domain. Most surprisingly, it turns out that in the classical domain the current laboratory technique is already capable to test the spontaneous collapse models.

Spontaneous heating in oscillators.—Let us consider the center of mass oscillation of an extended object with mass m and frequency Ω in a harmonic potential. Its quantized Hamiltonian reads

$$\hat{H} = \frac{\hat{p}^2}{2m} + \frac{1}{2} m\Omega^2 \hat{x}^2, \tag{1}$$

where $\hat{x}, \hat{p}$ are the center-of-mass canonical variables. If the mass is subject to spontaneous collapse, model dependent stochastic Schrödinger equations are proposed for the evolution of the state vector, cf., e.g., in the review [1]. However, when it comes to calculate experimental predictions then, as observed already in [2], stochastic Schrödinger equations are redundant: deterministic master equations for the density matrix $\hat{\rho}$ suffice. The observable spontaneous decoherence is mathematically equivalent with the presence of external random forces. In our particular case, the master equation of the oscillator takes this form:

$$\frac{d\hat{\rho}}{dt} = \frac{-i}{\hbar}[\hat{H}, \hat{\rho}] - \frac{D_{\rm sp}}{\hbar^2}[\hat{x}, [\hat{x}, \hat{\rho}]]. \tag{2}$$

Its derivation can be best learned from the Supplemental Material of Ref. [19] for both CSL and DP, or from Ref. [5] for DP. Here $D_{\rm sp}$ governs the strength (rate) of spontaneous decoherence. It depends on the chosen model as well as on the features of the extended object. This $\hat{x}$ decoherence is the observable quantum effect. We add immediately that $\hat{x}$

decoherence is completely equivalent with a classical effect: $\hat{p}$ diffusion of diffusion constant $D_{\rm sp}$. To see this, observe that the decoherence term in Eq. (2) corresponds to the average influence of a random force $F(t)$ with correlation $\langle F(t)F(t')\rangle = 2D_{\rm sp}\delta(t-t')$. In classical physics, such random force corresponds to momentum diffusion at strength $D_{\rm sp}$.

From now on and through our work, we assume that the oscillator is in the classical domain. Therefore, we can describe it by the classical Liouville density $\rho(x,p)$ and the quantum master equation (2) can be replaced by the Liouville equation where the classical momentum diffusion term replaces the quantum decoherence term:

$$\frac{d\rho}{dt} = \{H,\rho\} + D_{\rm sp}\frac{\partial^2}{\partial p^2}\rho. \tag{3}$$

$H(x,p)$ is the classical Hamilton function of the oscillator, the Poisson bracket $\{H,\rho\}$ stands for $-(p/m)(\partial/\partial x)\rho + m\Omega^2 x(\partial/\partial p)\rho$. In a realistic situation, the mechanical oscillator is in a thermal environment of temperature T, which will modify the Liouville equation:

$$\frac{d\rho}{dt} = \{H,\rho\} + D_{\rm sp}\frac{\partial^2}{\partial p^2}\rho + \eta\frac{\partial}{\partial p}p\rho + D_{\rm th}\frac{\partial^2}{\partial p^2}\rho, \tag{4}$$

where η is the damping rate of oscillations and $D_{\rm th} = \eta m k_B T$ is the constant of thermal momentum diffusion. With $D_{\rm sp} = 0$ we would get the classical Fokker-Planck equation whose stationary solution is the Gibbs canonical distribution $\mathcal{N}\exp(-H/k_B T)$. It is trivial to see that with $D_{\rm sp} > 0$ the stationary solution is the Gibbs canonical distribution

$$\rho_\infty(x,p) = \mathcal{N}\exp\left(-\frac{H(x,p)}{k_B T'}\right) \tag{5}$$

at the higher temperature

$$T' = \left(1 + \frac{D_{\rm sp}}{D_{\rm th}}\right)T \equiv T + \Delta T_{\rm sp}. \tag{6}$$

This result can be interpreted as the extension of the Einstein-Smoluchowski relationship $D_{\rm th} = \eta m k_B T$ for $D_{\rm th} + D_{\rm sp} = \eta m k_B T'$, supported by the underlying Fokker-Planck equation.

The increment $\Delta T_{\rm sp} > 0$ over the environmental temperature T is the contribution of spontaneous heating, this is the very observable quantity that we wish to test. From Eq. (6), we can express it as

$$\Delta T_{\rm sp} = \frac{D_{\rm sp}}{m k_B}\tau, \tag{7}$$

where $\tau = 1/\eta$ will stand for the (energy) relaxation time of the oscillator. Our classical description is valid as long as the spontaneous heating concerns many quanta of the oscillator:

$$k_B \Delta T_{\rm sp} \gg \hbar\Omega. \tag{8}$$

Measurement.—Since we restrict ourselves for the classical domain (8) of spontaneous heating $\Delta T_{\rm sp}$, a single-shot classical (or quantum) measurement of precision $\delta T_{\rm m}$ would detect $\Delta T_{\rm sp}$ provided $\delta T_{\rm m} \lesssim \Delta T_{\rm sp}$. If this condition does not hold, we can wait until the oscillator "forgets" the backaction of the first measurement and reaches the equilibrium state again, then we do a second measurement, etc., many times. In principle, the equilibrium state of the oscillator can be fully determined on large statistics of repeated independent single shot measurements like in quantum state tomography even if the precision of the individual measurements is poor. On the contrary, in most experiments the oscillator is being continuously monitored, which is equivalent to frequently repeated measurements such that their backaction changes the original equilibrium. Therefore, tomography is the more suitable means to detect spontaneous temperature increase of the previously prepared equilibrium oscillator state. Cumulative precision of tomography is not limited quantum theoretically.

For completeness, nonetheless, let us recapitulate the features of monitoring which is usually accompanied by some classical and/or quantum noise (backaction). We characterize this backaction by a further diffusion constant $D_{\rm m}$, meaning that the backaction is modeled by a random force of correlation $2D_{\rm m}\delta(t-t')$. The complete Liouville equation (4) reads

$$\frac{d\rho}{dt} = \{H,\rho\} + \eta\frac{\partial}{\partial p}p\rho + (D_{\rm sp} + D_{\rm th} + D_{\rm m})\frac{\partial^2}{\partial p^2}\rho. \tag{9}$$

Suppose we start to measure the temperature of the oscillator at $t = 0$. The initial state of the oscillator is the Gibbs state (5) of temperature $T + \Delta T_{\rm sp}$. When the "thermometer" is switched on, the measurement noise starts to heat the oscillator towards the new stationary Gibbs state of temperature increased by

$$\Delta T_{\rm m} = \frac{D_{\rm m}}{m k_B}\tau. \tag{10}$$

Trivial dynamics of heating follows from Eq. (9) in the limit $\eta \ll \Omega$:

$$T'(t) = T + \Delta T_{\rm sp} + (1 - e^{-t/\tau})\Delta T_{\rm m}. \tag{11}$$

Observe that the temperature effect of backaction is gradually reaching its steady state value. Backaction can be ignored for times much shorter than $\Delta T_{\rm m}/\Delta T_{\rm sp}$ times τ.

There is no fundamental limitation on the measurement precision (fluctuations) $\delta T_{\rm m}$ in the classical domain. There is a quantum trade off between the spectral components of δT_m and ΔT_m at a chosen frequency ω:

$$\delta T_\mathrm{m}\Delta T_\mathrm{m} \geq \frac{\hbar^2}{4k_B^2}\frac{|\Omega^2-\omega^2+i\eta\omega/2|^2}{\eta^2}, \qquad (12)$$

as it follows from Ref. [21], cf. also Ref. [19]. The minimum of $\delta T_\mathrm{m}+\Delta T_\mathrm{m}$ is achieved when

$$\delta T_\mathrm{m} = \Delta T_\mathrm{m} = \frac{\hbar}{2k_B}\frac{|\Omega^2-\omega^2+i\eta\omega/2|}{\eta} \equiv \Delta T_\mathrm{SQL}, \qquad (13)$$

which is called the standard quantum limit. This limitation concerns the steady state spectral component of the precision and backaction, respectively. For monitoring duration much shorter than τ (yet sufficient to gather significant data on ΔT_sp) the backaction will not influence the system, we can choose finer precisions δT_m than ΔT_SQL.

Spontaneous heating: DP model.—In the gravity-related spontaneous collapse model (DP model), the spontaneous diffusion is proportional to the Newton constant G. For the oscillating object made of cubic crystal, considered in [19]:

$$D_\mathrm{DP} = \frac{\hbar}{2}m\omega_G^2 = \frac{\hbar}{2}m\frac{4\pi G\varrho}{3}\left(\frac{a}{2\sqrt{\pi}\sigma_\mathrm{DP}}\right)^3, \qquad (14)$$

where ϱ is the mass density, and a is the lattice constant, while ω_G is the effective parameter used by [3–5]. The expression (14) is valid for $\sigma_\mathrm{DP} \ll a$. Reference [19] lost a factor 2, now restored in Eq. (14) which was derived, e.g., in [5]. Accordingly, the result (14) is valid for bulk materials in general, not restricted for cubic crystals considered in [19]. The relevant parameter is the average nuclear mass (ρa^3 in cubic crystal). In the range $\sigma_\mathrm{DP} \ll a$ of validity of (14), the coefficient D_DP is independent of the shape of the mass since each nucleus contributes independently. The spatial resolution σ_DP is the free parameter of the DP model, conjectured to be in the following range:

$$10^{-12}\ \mathrm{cm} \lesssim \sigma_\mathrm{DP} \lesssim 10^{-5}\ \mathrm{cm}. \qquad (15)$$

The upper limit is borrowed from the CSL model, the lower limit is about the nuclear size which may be a finest spatial resolution nonrelativistically [3]. Using (14) for D_sp, we can write (7) as

$$\Delta T_\mathrm{DP} = \frac{\hbar\omega_G^2}{2k_B}\tau, \qquad (16)$$

where ω_G^2 is read out from (14). It is remarkable that ΔT_DP does not depend on the mass m.

Now we assume the strongest possible DP decoherence; i.e., we take the finest conjectured spatial resolution $\sigma_\mathrm{DP} = 10^{-12}$ cm, favored by certain theoretical considerations [3–5]. If the lattice constant is set to $a = 5\times 10^{-8}$ cm, for concreteness, we obtain $\omega_G \approx 1.3$ kHz for the effective parameter. The spontaneous heating effect (16) can be written as

$$\Delta T_\mathrm{DP} \approx \tau[\mathrm{s}] \times 4.0\times 10^{-5}\ \mathrm{K}. \qquad (17)$$

(Unit of measure indicated in square brackets here and henceforth.) This is a convenient expression of the effect ΔT_DP to discuss possible choices of the frequency Ω and the quality factor $Q = \Omega\tau$ of the oscillator. The mass m has, as we noticed before, canceled from ΔT_DP.

Experimental implications.—Applying Eq. (17) to a broad range of frequencies Ω and quality factors Q, we calculated the spontaneous heating ΔT_DP in Table I.

The lesson is transparent. If $\Delta T_\mathrm{DP} \gg \hbar\Omega/k_B$, and this is the case except for a few highest Ω and lowest Q examples (in brackets), the DP effect would prevent us from ground state cooling. This should be a significant detectable effect. But we do not need to try ground state cooling, the heating effect ΔT_DP equally shows up far from the ground state. Low frequency oscillators with high quality factors are the favorable test bed. If the ring-down time $\tau = Q/\Omega$ of the oscillator is chosen between 10^2 and 10^6 s, the spontaneous heating ΔT_DP scales between 1 mK and 10 K, respectively. This is a striking result. Classical (nonquantum) thermometers of precision $\delta T_\mathrm{m} \sim 1$ mK should exist in principle. Technically, nonetheless, we might need to operate the measurement device in the quantum domain especially when the oscillator itself is cooled and/or controlled via high precision quantum devices. Even in this case the oscillator is assumed to stay away from its ground state since the effect ΔT_DP is robust classical.

Following Ref. [19], and for a selection of experiments considered therein, we calculated the effect ΔT_DP; see

TABLE I. Magnitudes of the spontaneous heating effect ΔT_DP of the DP model on classical oscillators are shown at currently available or nearly available combinations of frequencies Ω (1st column) and quality factors Q (1st row). The spatial resolution $\sigma_\mathrm{DP} = 10^{-12}$ cm assumes the strongest effect. The lattice constant is set to $a = 500$ pm. Data around the upper-left corner (in brackets) are not in the classical domain $k_B\Delta T_\mathrm{DP} \gg \hbar\Omega$. Data above the millikelvin range are enhanced (typed in boldface) because their detection may not require millikelvin cooling or cooling at all.

		Q				
		10^2	10^3	10^4	10^5	10^6
Ω	10^5 Hz	[10^{-8} K]	[10^{-7} K]	[10^{-6} K]	10^{-5} K	10^{-4} K
	10^4 Hz	[10^{-7} K]	10^{-6} K	10^{-5} K	10^{-4} K	10^{-3} K
	10^3 Hz	10^{-6} K	10^{-5} K	10^{-4} K	10^{-3} K	$\mathbf{10^{-2}}$ K
	10^2 Hz	10^{-5} K	10^{-4} K	10^{-3} K	$\mathbf{10^{-2}}$ K	$\mathbf{10^{-1}}$ K
	10 Hz	10^{-4} K	10^{-3} K	$\mathbf{10^{-2}}$ K	$\mathbf{10^{-1}}$ K	**1** K
	1 Hz	10^{-3} K	$\mathbf{10^{-2}}$ K	$\mathbf{10^{-1}}$ K	**1** K	**10** K

TABLE II. Spontaneous heating ΔT_{DP} for the selection of optomechanical setups quoted in [19]. Values ΔT_{DP} are calculated from Eq. (16), assuming the largest spontaneous decoherence rates considered for the time being, corresponding to $\omega_G = 1.3$ kHz. Two of the data (in brackets) are not in the classical domain $k_B \Delta T_{\mathrm{DP}} \gg \hbar\Omega$.

System	m	$\Omega/2\pi$ (Hz)	Q	T (K)	ΔT_{DP} (K)
Gravitational wave detector [22]	40 kg	1	25 000	300	0.16
Suspended disc [20]	5 mg	0.5	5×10^5	300	6.4
SiN membrane [23]	34 ng	1.6×10^6	1100	4.9	$[4.4 \times 10^{-9}]$
Aluminium membrane [24]	48 pg	1.1×10^7	3.3×10^5	0.015	$[1.9 \times 10^{-7}]$

Table II. The experiments [22] and [20], both performed at room temperature $T = 300$ K, can promise an earliest detection of spontaneous heating. On the one hand, cooling is a reserve of higher sensitivity of detecting ΔT_{DP}. On the other hand, the experiment [20] even at room temperature might attempt to detect the 6.4 K spontaneous warming up.

As we mentioned before, monitoring may be neither convenient nor sufficient for detection. Let us consider the constraint (12) at the detection band around $\omega = 2\pi \times 500$ Hz, yielding $\delta T_{\mathrm{m}} \Delta T_{\mathrm{m}} = \Delta T_{\mathrm{SQL}}^2 = (37\ \mathrm{K})^2$. Such a standard quantum limit 37 K gives insufficient precision on the steady state: i.e., in monitoring of a duration much longer than $\tau = 1.6 \times 10^5$ s. If we choose $\delta T_{\mathrm{m}} = 1$ K the duration of monitoring must be limited to the order of hundred seconds before the backaction reaches the range of 1 K, cf. Eq. (11). This is obviously not the way to go in general. In this particular experiment measurement precisions below 1 K are not available by standard quantum monitoring. A single-pulse measurement must be considered instead, where state preparation is followed by a one-shot measurement and the preparation-detection cycle is repeated many times.

CSL model.—In the CSL model the diffusion constant is proportional to the rate parameter λ_{CSL}. For the perpendicular momentum diffusion of a disk whose thickness d and radius are much larger than $\sigma_{\mathrm{CSL}} = 10^{-5}$, we rewrite the result of Ref. [19] (times its lost factor 2) into an equivalent form displaying the ultimate $1/d$ dependence:

$$D_{\mathrm{CSL}} = \lambda_{\mathrm{CSL}} \frac{\hbar^2}{m_0^2} 4\pi\sigma_{\mathrm{CSL}}^2 \frac{\varrho m}{d}, \tag{18}$$

where m_0 is the standard atomic unit. The value of the CSL collapse rate parameter has been constrained by a lower [7] and an upper estimate [8], cf. also [1]:

$$2.2 \times 10^{-17}\ \mathrm{Hz} \lesssim \lambda_{\mathrm{CSL}} \lesssim 2.2 \times 10^{-8\pm2}\ \mathrm{Hz}. \tag{19}$$

Using D_{CSL} (18) for D_{sp} in (7) yields

$$\Delta T_{\mathrm{CSL}} = \lambda_{\mathrm{CSL}} \frac{\hbar^2}{m_0^2 k_B} 4\pi\sigma_{\mathrm{CSL}}^2 \frac{\varrho}{d} \tau. \tag{20}$$

Note that the shape (thickness) of the oscillator matters, the mass m does not.

Suppose the strongest CSL decoherence rate from the range (19), let us take the estimate $\lambda_{\mathrm{CSL}} = 2.2 \times 10^{-8\pm2}$ Hz [8]. Using this value in (20) we obtain

$$\Delta T_{\mathrm{CSL}} \approx \tau[s] \frac{\varrho[\mathrm{g/cm^3}]}{d[\mathrm{cm}]} \times 3.2 \times 10^{-6\pm2}\ \mathrm{K}. \tag{21}$$

Recall that $d \gg \sigma_{\mathrm{CSL}} = 10^{-5}$ cm; hence, the strongest heating effect is achieved when $d \approx \sigma_{\mathrm{CSL}}$, leading to

$$\Delta T_{\mathrm{CSL}} \approx \tau[\mathrm{s}] \times 6.2 \times 10^{-1\pm2}\ \mathrm{K}, \tag{22}$$

where we kept $\varrho = 2\ \mathrm{g/cm^3}$ as before. Comparing this result with (17) we conclude that, in classical oscillators, the strongest conjectured CSL effect ΔT_{CSL} would exceed the strongest conjectured DP effect ΔT_{DP} by at least 2 orders of magnitude.

Let us consider the $\Omega = 3.14$ Hz oscillator [20], also discussed in Ref. [19] in the context of the CSL model. Recall that the strongest DP effect turned out to be $\Delta T_{\mathrm{DP}} = 6.4$ K, cf. Table I. This oscillator has the high quality factor $Q = 5 \times 10^5$, the ring-down time is extremely long: $\tau = 1.6 \times 10^5$ s. The resonator is a 5 mg disk of thickness $d = 0.2$ mm, Eq. (21) yields the spontaneous heating $\Delta T_{\mathrm{CSL}} = 5.1 \times 10^{1\pm2}$ K, corresponding to the rates $\lambda_{\mathrm{CSL}} = 2.2 \times 10^{-8\pm2}$, respectively. Presumably the values $\lambda_{\mathrm{CSL}} \gtrsim 10^{-7}$ are not compatible with the experiment and the values $\lambda_{\mathrm{CSL}} \sim (10^{-8}\text{–}10^{-10})$ remain to be challenged.

Summary.—The so far hypothetic spontaneous wave-function collapse on massive degrees of freedom possesses a complementary classical effect: classical momentum diffusion. This produces a certain spontaneous increase ΔT_{sp} of the equilibrium temperature. This typical classical effect must be testable classically, without facing the standard quantum limitations of sensing. Therefore we must get spontaneous diffusion in the cross hairs instead of spontaneous collapse. We have derived the spontaneous heating ΔT_{sp} for mechanical oscillators in the classical thermal state, only using the classical Einstein-Smoluchowski relation. We found that ΔT_{sp} is proportional to the relaxation (ring-down) time and independent of the mass. Experimental implications become more transparent than before, for both leading models DP and CSL of spontaneous collapse. We conclude that currently available extreme low-loss mechanical oscillators can already confirm the presence of spontaneous diffusion if its rate is close to the conjectured maximum. Alternatively, nondetection would not yet invalidate the spontaneous collapse models but enforce the update of the current constraints, cf. in Refs. [1,25], on the collapse model's parameters. The requested measurement precisions 1 mK–1 K may not be reached in standard steady state quantum monitoring typically preferred so far. Instead, we propose that state tomography fits the demands better.

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*diosi.lajos@wigner.mta.hu
www.rmki.kfki.hu/~diosi

[1] A. Bassi, K. Lochan, S. Satin, T. P. Singh, and H. Ulbricht, Rev. Mod. Phys. **85**, 471 (2013).
[2] L. Diósi, Phys. Lett. A **120**, 377 (1987); L. Diósi, Phys. Rev. A **40**, 1165 (1989).
[3] L. Diósi, J. Phys. Conf. Ser. **442**, 012001 (2013).
[4] L. Diósi, Found. Phys. **44**, 483 (2014).
[5] L. Diósi, New J. Phys. **16**, 105006 (2014).
[6] R. Penrose, Gen. Relativ. Gravit. **28**, 581 (1996); Phil. Trans. R. Soc. A **356**, 1927 (1998); Found. Phys. **44**, 557 (2014).
[7] G. C. Ghirardi, P. Pearle, and A. Rimini, Phys. Rev. A **42**, 78 (1990); G. C. Ghirardi, R. Grassi, and F. Benatti, Found. Phys. **25**, 5 (1995).
[8] S. L. Adler, J. Phys. A **40**, 2935 (2007).
[9] W. Marshall, C. Simon, R. Penrose, and D. Bouwmeester, Phys. Rev. Lett. **91**, 130401 (2003).
[10] M. R. Vanner, I. Pikovski, G. D. Cole, M. S. Kim, Č. Brukner, K. Hammerer, G. J. Milburn, and M. Aspelmeyer, Proc. Natl. Acad. Sci. U.S.A. **108**, 16182 (2011).
[11] O. Romero-Isart, Phys. Rev. A **84**, 052121 (2011).
[12] T. Li, S. Kheifets, and M. G. Raizen, Nat. Phys. **7**, 527 (2011).
[13] B. Pepper, R. Ghobadi, E. Jeffrey, C. Simon, and D. Bouwmeester, Phys. Rev. Lett. **109**, 023601 (2012).
[14] Z. Q. Yin, A. A. Geraci, and T. Li, Int. J. Mod. Phys. B **27**, 1330018 (2013).
[15] T. Juffmann, H. Ulbricht, and M. Arndt, Rep. Prog. Phys. **76**, 086402 (2013).
[16] M. Arndt and K. Hornberger, Nat. Phys. **10**, 271 (2014).
[17] M. Aspelmayer, T. J. Kippenberg, and F. Marquardt, Rev. Mod. Phys. **86**, 1391 (2014).
[18] M. Bahrami, M. Paternostro, A. Bassi, and H. Ulbricht, Phys. Rev. Lett. **112**, 210404 (2014).
[19] S. Nimmrichter, K. Hornberger, and K. Hammerer, Phys. Rev. Lett. **113**, 020405 (2014).
[20] N. Matsumoto, Y. Michimura, G. Hayase, Y. Aso, and Tsubono, arXiv:1312.5031.
[21] C. M. Caves, K. S. Thorne, R. W. P. Drever, R. W. P. Drever, V. D. Sandberg, and M Zimmermann, Rev. Mod. Phys. **52**, 341 (1980); V. B. Braginsky and F. Y. Khalili, *Quantum Measurement* (Cambridge University Press, Cambridge, England, 1999); V. B. Braginsky, JETP **26**, 831 (1968); S. L. Danilishin and F. Y. Khalili, Living Rev. Relativity **15** (2012).
[22] H. Müller-Ebhardt, H. Rehbein, R. Schnabel, K. Danzmann, and Y. Chen, Phys. Rev. Lett. **100**, 013601 (2008).
[23] T. P. Purdi, R. W. Peterson, and C. A. Regal, Science **339**, 801 (2013).
[24] J. D. Teufel, T. Donner, D. Li, J. W. Harlow, M S. Allman, K. Cicak, A. J. Sirois, J. D. Whittaker, K. W. Lehnert, and R. W. Simmonds, Nature (London) **475**, 359 (2011).
[25] W. Feldmann and R. Tumulka, J. Phys. A **45**, 065304 (2012).