Andreev Bound-State Dynamics in Quantum-Dot Josephson Junctions: A Washing Out of the 0-π Transition

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We consider a Josephson junction formed by a quantum dot connected to two bulk superconductors in the presence of Coulomb interaction and coupling to both an electromagnetic environment and a finite density of electronic quasiparticles. In the limit of a large superconducting gap we obtain a Born-Markov description of the relevant Andreev bound-states dynamics. We calculate the current-phase relation and we find that the experimentally unavoidable presence of quasiparticles can dramatically modify the 0-π standard transition picture. We show that photon-assisted quasiparticle absorption allows the dynamic switching from the 0 to the π state and vice versa, washing out the 0-π transition predicted by purely thermodynamic arguments. Spectroscopic signatures of Andreev bound-states broadening are investigated by considering microwave irradiation.

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Introduction.—The Josephson junction is a fundamental element of superconducting quantum nanoelectronics, with a wide spectrum of applications ranging from quantum information to medical imagery. Such a junction can be formed by contacting two superconductors by a large variety of nanostructures [1–7]. A fruitful way to describe transport through the device is to consider the formation of electronic bound states at the junction known as Andreev bound states. In thermodynamic equilibrium, at low temperatures, and for junctions shorter than the superconducting correlation length ξ, the current-phase relation is determined mainly by the phase dependence of the lowest energy Andreev bound state. A wealth of experimental and theoretical work has been devoted to investigate the current-phase dependence in Josephson junctions and leads, for instance, to the prediction [8–11] and the observation [4,12–16] of a change of sign of the current-phase relation, the so called 0-π transition. This can be induced by the presence of magnetic moments (magnetic impurities or a ferromagnetic layer) or in a nonmagnetic material by the repulsive Coulomb interaction at the quantum dot forming the junction, as is observed in carbon nanotubes [15,17–19] or semiconducting nanowire [4] Josephson junctions. At the basis of this transition is the change of the parity of the junction. In superconductors electrons are paired, but if in the quantum dot forming the Josephson junction Coulomb repulsion is sufficiently large, the ground state will accommodate only one electron. At lowest order in the tunneling, the Josephson current is suppressed, and at the next (fourth) order it changes sign [9], since Cooper pairs are recomposed by tunneling with reversed spins.

Only recently a direct detection of the excited Andreev bound states has been possible with a series of experiments that probed the Josephson junction by resonating microwave irradiation [20,21]. These experiments pointed out the importance of the coupling to the electromagnetic (EM) environment and in particular to the quasiparticles present in the superconducting leads. It is an established experimental fact that the density of quasiparticles does not vanish exponentially with the temperature as predicted by the BCS theory, but remains finite, even at the lowest temperatures [22,23]. Environment-assisted absorption of quasiparticles can modify the junction parity, since an unpaired electron can fall in the quantum dot. This process has been considered very recently for junctions where Coulomb interaction is negligible [23].

In this Letter, we investigate the effect of parity transitions induced by the quasiparticle absorption and emission in the presence of Coulomb interaction. We consider the limit for which the superconducting gap is the largest energy scale, also known as the atomic limit. We obtain an exact Born-Markov description of the system coupled to the EM environment. In this approximation, the π phase is indicated by the occupation of an odd-parity state with a vanishing of the supercurrent. This allows us to describe in a consistent way the 0-π transition by taking into account the relaxation processes that induce parity changes. We find that the presence of quasiparticles can completely wash out the 0-π transition, and invalidate the usual arguments based on the parity of the lowest energy state. The quasiparticles are necessary to let the system relax to the lowest energy ground state, but at the same time, they allow dynamic transitions between states, smoothing the transition. We also present predictions for the microwave spectroscopy of the Andreev bound states.

Model.—Let us consider a quantum dot with a single electronic level forming a Josephson junction between two

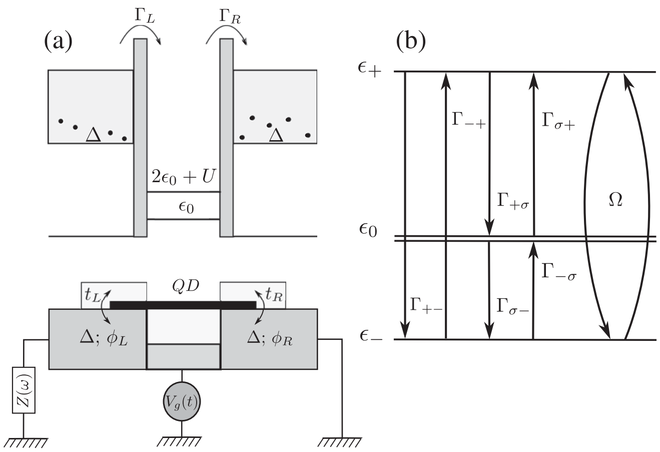

FIG. 1. (a) Representation of the Josephson junction formed, for instance, by a carbon nanotube quantum dot bridging two superconductors. (b) Schematics of the transitions between the Andreev bound states induced by incoherent Γ_{ij} and coherent Ω perturbations.

superconducting leads [see Fig. 1(a)]. We assume that the junction is phase biased, that a time-dependent gate voltage can be applied, and that the source-drain circuit is shunted on an external impedance $Z(\omega)$. This system can be modeled by the (time-dependent) Hamiltonian

$$H = H_{\rm dot}(t) + H_L + H_R + H_T + H_B + H_c. \qquad (1)$$

The first term of Eq. (1) reads $H_{\rm dot}(t) = \epsilon_d(t)(n_\uparrow + n_\downarrow) + U n_\uparrow n_\downarrow$ and it describes the single-electronic level of time-dependent energy $\epsilon_d(t) = \epsilon_0 + \epsilon_1 \cos(\omega t)$ and Coulomb repulsion U. Here $n_\sigma = d_\sigma^\dagger d_\sigma$, d_σ is the electronic destruction operator on the dot of spin projection σ, and ω is the frequency of the microwave driving. The second and third terms $H_X = \sum_{k\sigma} \xi_{Xk} c_{Xk\sigma}^\dagger c_{Xk\sigma} + \sum_k [\Delta_X e^{i\phi_X} c_{Xk\uparrow}^\dagger c_{X-k\downarrow}^\dagger + {\rm H.c.}]$ describe the left and right leads ($X = L, R$) as BCS superconductors of order parameter $\Delta_X e^{i\phi_X}$ and electronic spectrum ξ_{Xk}, with $c_{Xk\sigma}$ the related destruction operator for momentum k. The dot and the leads are coupled by the tunneling term $H_T = \sum_{Xk\sigma} t_{Xk} c_{Xk\sigma}^\dagger d_\sigma + {\rm H.c.}$ that gives rise to a rate $\Gamma_X = \pi \rho_X |t_X|^2/\hbar$, where ρ_X is the density of states at the Fermi level of the superconductor X and $\hbar$ is the reduced Planck constant. The EM modes described by $Z(\omega)$ induce fluctuations of the superconducting phase difference at the ends of the junction [24,25]. For simplicity we assume that the gate capacitance is much smaller than the symmetric left and right capacitances. Within these assumptions and for ${\rm Re}(Z) \ll R_Q = \pi\hbar/2e^2$, the quantum of resistance (e is the electron's charge), we can expand the dependence of the Hamiltonian on the phase difference fluctuations $\tilde{\phi}$, obtaining the linearized coupling term $H_c = (\hbar/e) I \tilde{\phi}$, where $I = (I_L - I_R)/2$ is the total physical current (including the displacement current). It is expressed in terms of the left and right particle current operators $I_X = (e/i\hbar)\sum_{k\sigma} t_{Xk} c_{Xk\sigma}^\dagger d_\sigma + {\rm H.c.}$ The term H_B describes the EM modes and following Ref. [25] one obtains $\langle \tilde{\phi}(t)\tilde{\phi}(0)\rangle \equiv \mathcal{C}_\phi(t) = 2\int_0^\infty d\omega {\rm Re}[Z(\omega)][\coth(\hbar\omega/2k_B T_{\rm EM}) \times \cos\omega t - i \sin\omega t]/(\omega R_Q)$, with $T_{\rm EM}$ the temperature of the EM environment and k_B the Boltzmann constant. In the following we will consider the symmetric case, for which $\Gamma_X = \Gamma/2$, and $\Delta_X = \Delta$ for $X = L$ and R. Moreover, since the final results depend only on the phase difference $\phi_L - \phi_R$ we set from the outset $\phi_L = -\phi_R = \phi/2$.

When the driving and the coupling to the environment is neglected this Hamiltonian has been widely studied in the literature [26–31] and it is known to show a rich phase diagram with a 0-π transition controlled by Kondo correlations. The problem can be treated analytically only in a few regimes, and only for the equilibrium case is an exact solution available based on the numerical renormalization group [32,33] or Monte Carlo simulations [34]. The objective of this work is to explore the fate of the 0-π transition in the presence of the quasiparticles and EM environment. The system being out of equilibrium, we choose to investigate the case $\Delta \gg |\epsilon_d|, U, \hbar\Gamma, \hbar\omega$, for which a systematic controlled approximation is possible. In this limit the four states of the isolated dot $|0\rangle$, $|\uparrow\rangle = d_\uparrow^\dagger|0\rangle$, $|\downarrow\rangle = d_\downarrow^\dagger|0\rangle$, and $|2\rangle = d_\uparrow^\dagger d_\downarrow^\dagger|0\rangle$ are only weakly coupled to the leads, and their (unperturbed) energy levels $\{0, \epsilon_0, \epsilon_0, 2\epsilon_0 + U\}$ are well separated from the quasiparticle continuum. Following a standard procedure of atomic physics [35,36] the effect of H_T can then be taken into account systematically by performing a unitary transformation that generates an effective Hamiltonian $H_d^{\rm eff}$ in the four-dimensional space of the quantum dot. At lowest order in $\hbar\Gamma/\Delta$ one obtains $H_d^{\rm eff} = \epsilon_0(|\uparrow\rangle\langle\uparrow| + |\downarrow\rangle\langle\downarrow|) + (2\epsilon_0 + U)|2\rangle\langle 2| + \hbar\Gamma\cos(\phi)(|0\rangle\langle 2| + |2\rangle\langle 0|)$, where the last off-diagonal term hybridizing the even-parity states is a manifestation of the proximity effect [37]. Performing the same unitary transformation on the current operator I one obtains at the first two nonvanishing orders $I^{\rm eff} = I^{(1)} + I^{(2)}$, where $I^{(1)} = (e/\hbar)\sum_{\sigma,\alpha=\pm} D_{\alpha\sigma} C_{\alpha\sigma}$, $I^{(2)} = -e\Gamma\sin(\phi/2)(|0\rangle\langle 2| + |2\rangle\langle 0|)$, with $D_{+\sigma} = |\sigma\rangle\langle 0| + s_\sigma |2\rangle\langle\bar{\sigma}|$, $D_{+\sigma} = D_{-\sigma}^\dagger$, $s_{\uparrow,\downarrow} = \pm 1$, $C_{\alpha\sigma} = i\alpha\sum_X (s_X t_X/2)\sum_k (u_k \gamma_{Xk\sigma}^{\bar{\alpha}} - s_\sigma v_{-k} e^{i\alpha\phi_X} \gamma_{X-k\bar{\sigma}}^{\alpha})$. The Bogoliubov operators $\gamma_{Xk\sigma}^\alpha$ diagonalize the BCS Hamiltonian of lead X: $H_X = \sum_{k\sigma} E_k \gamma_{Xk\sigma}^+ \gamma_{Xk\sigma}^-$ with $\gamma_{Xk\sigma}^+$ and $\gamma_{Xk\sigma}^-$ indicating the creation and destruction operator for energy $E_k = (\xi_k^2 + \Delta^2)^{1/2}$. Finally $u_k(v_k) = [(1/2)(1 \pm \xi_k/E_k)]^{1/2}$ and $s_{L,R} = \pm 1$.

Born-Markov description.—In order to give a quantitative description of the dynamics we proceed by treating the coupling to the environment by a Born-Markov approximation [35]. We will regard the quasiparticles in the superconductor and the EM excitations as a Markovian environment. We describe the stationary distribution of quasiparticles as an equilibrium one characterized by a temperature $T_{\rm qp} \gg T_{\rm EM}$, as it appears to be the case in several experiments [20,21,23]. Following the standard procedure and tracing out the quasiparticles and the EM fluctuations the equation for the reduced density matrix ρ for the degrees of freedom of the dot reads

$$\dot{\rho}(t) = -(i/\hbar)[H_d^{\rm eff}(t), \rho(t)] + (\hbar/e)^2 \mathcal{L}_{\mathcal{C}_\phi}[I^{(2)}, I^{(2)}] + \sum_{\alpha\sigma} \{ \mathcal{L}_{\mathcal{C}_N^{\alpha\sigma}}[D_{\alpha\sigma}, D_{\alpha\sigma}^\dagger] + \mathcal{L}_{\mathcal{C}_A^{\alpha\sigma}}[D_{\alpha\sigma}, D_{\alpha\bar{\sigma}}] \}. \tag{2}$$

Here $\hbar^2 \mathcal{L}_{\mathcal{C}}[A,B] = -\int_0^{+\infty} d\tau \{ \mathcal{C}(\tau)[A, B(t-\tau,t)\rho(t)] + \mathcal{C}(-\tau)[\rho(t)A(t-\tau,t), B] \}$, and we have defined the normal $\mathcal{C}_N^{\alpha\sigma}(t) = \langle C_{\alpha\sigma}(t) C_{\alpha\sigma}^\dagger(0) \rangle \mathcal{C}_\phi(t)$ and anomalous $\mathcal{C}_A^{\alpha\sigma}(t) = \langle C_{\alpha\sigma}(t) C_{\alpha\bar{\sigma}}(0) \rangle \mathcal{C}_\phi(t)$ quasiparticle correlation functions. The second term of the right-hand side of Eq. (2) affects the evolution of only the even-parity states, since $I^{(2)}$ has nonvanishing matrix elements only in this subspace. By contrast, the third and fourth terms, which are generated by the $(\hbar/e)I^{(1)}\tilde{\phi}$ term, allow a change in the parity of the dot. This is possible since $I^{(1)}$ describes the transfer of one electron from (to) the leads to (from) the dot by the photon assisted destruction (creation) of a quasiparticle in the leads. The correlation functions can be evaluated (see the Supplemental Material [38]) and the master equation can then be solved numerically for a given choice of $Z(\omega)$ by a projection on the Floquet basis [39]. In the following we will discuss different regimes for which the analytical and numerical results will be compared.

Nondriven case.—When the driving term is absent ($\epsilon_1 = 0$) the effective Hamiltonian can be easily diagonalized. The four states split into a degenerate doublet of odd parity at energy ϵ_0 and a nondegenerate pair of states generated by the hybridization of the even-parity states $|0\rangle$ and $|2\rangle$: $|-\rangle = \cos(\beta)|0\rangle + \sin(\beta)|2\rangle$ and $|+\rangle = -\sin(\beta)|0\rangle + \cos(\beta)|2\rangle$. Their energy reads $\epsilon_\pm = \epsilon_0 + U/2 \pm \{[\hbar\Gamma\cos(\phi/2)]^2 + (\epsilon_0 + U/2)^2\}^{1/2}$ with $\tan\beta = \epsilon_-/\hbar\Gamma\cos(\phi/2)$. In this limit, and neglecting the environment, the transition from 0 to the π phase is particularly simple. Depending on the value of U and ϵ_0 the ground state can be either the even-parity state $|-\rangle$ (for $\epsilon_- < \epsilon_0$) or the two degenerate odd-parity states $|\sigma\rangle$ (for $\epsilon_- > \epsilon_0$). The current is simply obtained by the evaluation of the current operator on the ground state and it vanishes for the odd-parity states, while it equals $I_{--} = -e\Gamma\sin(2\beta)\sin(\phi/2)(= -I_{++})$ in the $|-\rangle$ state. To the next order in Γ/Δ the current shows a small negative value in the odd-parity state (see the Supplemental Material [38] or Ref. [9]). In the following we will use the information on the parity of the occupied state to distinguish between the 0 and the π phase. We can now discuss the effect of the environment, as predicted by Eq. (2). In the absence of driving one can show that the density matrix becomes diagonal in the eigenstate basis of $H_d^{\rm eff}$ and the effect of the environment reduces to a description of incoherent tunneling between states. Neglecting the principal parts in Eq. (2) we obtain an explicit expression for the rates [see Fig. 1(b) and the Supplemental Material [38] for details]. We assume $k_B T_{\rm EM} \ll \Delta$ and ${\rm Re}[Z(\omega)] = \gamma\omega^2$ for $\omega \lesssim \Delta$ [40], and we approximate $J(\omega) = \gamma\omega$. One obtains

$$\Gamma_{+-} = 2\pi\gamma(\epsilon_+ - \epsilon_-)\Gamma^2\cos^2(2\beta)\sin^2(\phi/2), \tag{3}$$

$$\Gamma_{a\sigma/\sigma a} = \gamma\frac{\Delta^2}{\hbar^2}\Gamma\Xi\left(\frac{k_B T_{\rm qp}}{\Delta}\right)(1 \mp au)[1 \pm (\epsilon_a - \epsilon_0)/\Delta] \tag{4}$$

with $a = \pm$, $\Gamma_{-+} = 0$, $\Xi(x) = e^{-1/x}\sqrt{\pi x/8}$, and $u = \sin 2\beta\cos(\phi/2)$. (Note that β depends on ϕ and the expressions for the rates are correctly 2π periodic in ϕ.) According to our approximation the energy dependence of the rate is very weak, since $|\epsilon_a - \epsilon_0| \ll \Delta$. This implies that the energy ordering of the two states has very little effect on the parity-breaking rates. The reason is clear: the transition from one state to the other is possible thanks to a quasiparticle of energy Δ, which has to be present in the environment. An electron can then be added or removed from the dot, and the excess energy is absorbed by a photon. The relative energy of the initial and final states of the dot multiplet is small with respect to Δ, and thus in the end the energy ordering will not be important. In other terms the coupling to the environment will not allow a relaxation of the dot to its lowest energy state, but will induce instead transitions from the 0 to the π states. The average measured current becomes then simply $I_{--}\rho_{--}$: the magnetic states do not carry current, and the state $|+\rangle$ relaxes very rapidly to the state $|-\rangle$, since this transition does not need the participation of the rare quasiparticles. The final result is that the 0-π transition can be completely washed out in the average current. This is clearly visible in Fig. 2, where the numerical and analytical solution of Eq. (2) as a function of U for $\phi = \pi/2$ is compared to the prediction of the system not coupled to the environment. The former has a smooth behavior following the U dependence of I_{--}, while the latter has a sharp jump. A similar picture is obtained as a function of ϵ_0/Γ.

Effect of driving.—An experimental way of testing the state of the junction is to irradiate the gate with a microwave field. The resulting modulation in time of $\epsilon_d(t)$ is a perturbation that cannot change the parity of the junction. Since the odd-parity states are degenerate, the ac field can only induce resonant transitions between the even-parity states for small values of the detuning $\delta = \omega - \delta_{+-}$. Close to the resonance the dynamics can be described by performing a rotating-wave approximation (RWA) that gives for the density matrix in the rotating frame $\tilde{\rho}_{a\bar{a}} = \rho_{a\bar{a}}e^{ia\omega t}$, $\tilde{\rho}_{aa} = \rho_{aa}$

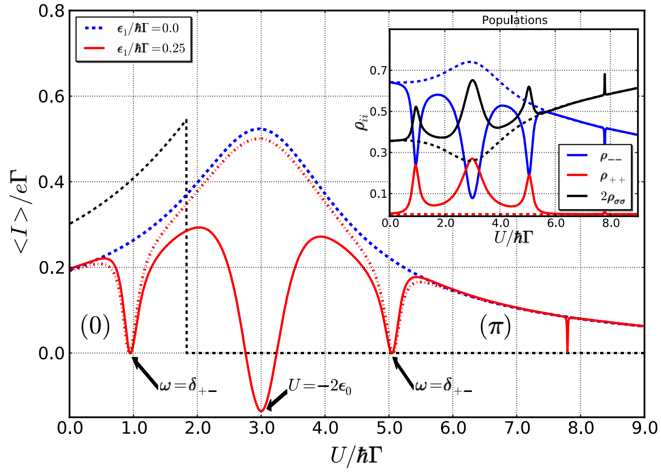

FIG. 2 (color online). Average current $\langle I\rangle$ as a function of Coulomb repulsion U. The black dashed curve results from the thermodynamic arguments in the absence of driving. The full red (dashed blue) curve is the numerical solution of the Floquet master equation in the presence (absence) of ac driving. The red dash-dotted curve is the outcome of the optical Bloch equations in the RWA approximation. Inset: the corresponding plain (dashed) numerical curves for the populations ρ_{++}, ρ_{--}, and $2\rho_{\sigma\sigma}$ of the Andreev bound states in the presence (absence) of ac driving. The parameters describing the Josephson junction are common to both plots: $\Delta/\hbar\Gamma = 10.0, \epsilon_0/\hbar\Gamma = -1.5, \epsilon_1/\hbar\Gamma = 0.25, \phi = \pi/2, \omega/\Gamma = 2.5$, $k_B T_{\rm EM}/\Delta = 0, k_B T_{\rm qp}/\Delta = 1/20$, and $\gamma\Gamma^2 = 1.4\times 10^{-4}$.

$$\dot{\tilde{\rho}}_{aa} = -i\frac{\Omega}{2}[\tilde{\rho}_{\bar{a}a} - \tilde{\rho}_{a\bar{a}}] - (\Gamma_{a\bar{a}} + 2\Gamma_{a\sigma})\tilde{\rho}_{aa} + \Gamma_{\bar{a}a}\tilde{\rho}_{\bar{a}\bar{a}} + 2\Gamma_{\sigma a}\tilde{\rho}_{\sigma\sigma}, \qquad (5)$$

$$\dot{\tilde{\rho}}_{a\bar{a}} = -i\frac{\Omega}{2}[\tilde{\rho}_{\bar{a}\bar{a}} - \tilde{\rho}_{S;aa}] + [2ia\delta - (\Gamma_{a\bar{a}} + \Gamma_{\bar{a}a} + 2\Gamma_{a\sigma} + 2\Gamma_{\bar{a}\sigma})]\tilde{\rho}_{a\bar{a}}/2 \qquad (6)$$

with $\tilde{\rho}_{\sigma\sigma} = (1 - \tilde{\rho}_{++} - \tilde{\rho}_{--})/2$, $\tilde{\rho}_{\uparrow\downarrow} = \tilde{\rho}_{\downarrow\uparrow} \approx 0$, $\tilde{\rho}_{\bar{a}a} = \tilde{\rho}^*_{a\bar{a}}$, and $\hbar\Omega = \epsilon_1 \sin(2\beta)$. As in the optical Bloch equations, when $\Omega \gg \Gamma_{+-}, \Gamma_{a\sigma}$ the coherence terms are important for the time evolution of the system. The rates implying the quasiparticles are much smaller than all the other quantities appearing in the master equation. Using this fact one can solve the equations for the block $+/-$ for a given $\rho_{\sigma\sigma}(t)$ and then solve separately the resulting equation for $\rho_{\sigma\sigma}$. This gives at vanishing $T_{\rm EM}$

$$\rho_{++} = \frac{[1 + 2\Gamma_{-\sigma}/\sum_a \Gamma_{\sigma a}]^{-1}(\Omega/2)^2}{\delta^2 + (\Gamma^2_{+-} + \Omega^2(1+\theta))/4} \qquad (7)$$

with $\theta = (2\Gamma_{+\sigma} + \sum_a \Gamma_{\sigma a})/(2\Gamma_{-\sigma} + \sum_a \Gamma_{\sigma a})$ and $\rho_{++}/\rho_{--} = (\Omega/2)^2/[\delta^2 + (\Gamma^2_{+-} + \Omega^2)/4]$. Equation (7) describes a typical resonant behavior for the populations as it can be seen in the inset of Fig. 2. At resonance ($\delta = 0$) the populations equilibrate so that $\rho_{++} = \rho_{--} = 1 - 2\rho_{\sigma\sigma} = \sum_a \Gamma_{\sigma a}/2\sum_a(\Gamma_{a\sigma} + \Gamma_{\sigma a})$. Since typically the rates are of the same order of magnitude at resonance $\rho_{++} \approx 1/4$. The average current $\langle I\rangle$ is simply $\sum_a \rho_{aa} I_{aa}$ and it is strongly modulated near the resonances. The resonance is visible in both the regions where the 0 and π phase would be stable. The narrow dip in Fig. 2 for $U/\hbar\Gamma \approx 8.0$ is a two-photon resonance described by the full numerical solution of Eq. (2).

The slow fluctuations between the σ and $\pm$ doublets induce a strong telegraph noise, since the current in the four states is very different and the fluctuations are slow. To estimate the intensity of the current noise we assume that all current fluctuations are due to the transitions among the four states, each one having a different value for the stationary current (specifically $I_{++} = -I_{--}$ and $I_{\sigma\sigma} = 0$). In the absence of driving one finds that the current noise reads $S = 4\Gamma_{\sigma-}\Gamma_{-\sigma} I^2_{--}/(\Gamma_{\sigma-} + 2\Gamma_{-\sigma})^3$, giving a very large Fano factor $F = S/2eI$ of the order of $e^{\Delta/k_B T_{\rm qp}}$. We note that a strong telegraph noise has been very recently observed in atomic point contact junctions [21]. In that experiment the Coulomb blockade plays no role, but the coupling to the quasiparticles has a very similar behavior.

The driving reveals also an unexpected maximum of the population ρ_{++} for $U = 3\Gamma$. This is the value for which $2\epsilon_0 + U = 0$ and the $|0\rangle$ and $|2\rangle$ states are degenerate. At this point the matrix element entering the rate Γ_{+-} vanishes. The population of the excited state generated by the nonresonant driving can relax to the $|-\rangle$ state only by passing through the $|\sigma\rangle$ states, with very low rates. This allows a large population of the excited state with a consequent negative contribution to the supercurrent.

Conclusion.—We have investigated the effect of a coupling to the quasiparticles and the EM environment on the 0-π transition. In a regime where the approximations can be well controlled we have shown that the quasiparticle scattering induces transitions between the 0 and π states, with a consequent washing out of the transition. We found that this induces large current fluctuations, and that the state of the junction could be investigated by driving the gate with an ac voltage. The main reason for the smoothing of the transition is the fact that the excess energy of the quasiparticles allows fluctuations from the thermodynamical ground state and the first excited state. This effect is very strong in the regime where we work, since Δ is the largest energy scale, but it will be present also for intermediate values of the gap. The theory we present indicates clearly that the effect of quasiparticles can be dramatic. The question of the crossover to the thermodynamical equilibrium when Δ is of the same order or smaller than the other energy scales remains open and calls for further investigations. The results and the methods presented are relevant also for the understanding of the tunneling spectroscopy of Andreev bound states in systems where Coulomb interaction is present [5,19,41]. The issue of the stability of Andreev bound states with respect to the quasiparticle scattering has also a strong relevance for the observation of Majorana states, which should be subject to a similar dynamics [42].

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